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NONLINEAR RANDOM WAVES IN FLUID, AND THE MAIN MECHANISM OF THEIR EXCITATION

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To describe the problem of the random nonlinear waves in fluid, we must know, exactly or approximately, how occurs the process of the vortex separation. For this it is conveniently to use models based on physical considerations and (or) some experimental data. The main attention in this review will be attended to random waves, emerging, for example, at stall flutter. Such waves often appear in fluid, and they are the main cause of many disasters in seas and oceans.

As a rule, stall flutter is connected with the pulling phenomenon, and observed in systems with two and (or) more degrees of freedom. In principle, in such systems both approximately one-frequency (synchronous) mode, and many-frequency (asynchronous) modes (when each mode oscillates with its natural frequency) are possible. But in the case of the pulling phenomenon only one-frequency mode, corresponding to its natural frequency (see [1]) is stable. Unlike to usual turbulence stall flutter is a self-oscillatory process.

The feedback in this process appears due to interaction between the fluid and the streamline body.

It should be noted that wave motions in fluid can be of very complex character. In last years a great interest appears to waves of an anomalously high amplitude – so called **freak-waves**, and **rogue-waves**.

We assume that the main cause of such waves is also vortex separation.

Keywords: Nonlinear waves in fluid, vortex separation, stall flutter, disasters in seas and oceans, pulling phenomenon, degrees of freedom, freak-waves, rogue waves, using the mathematical models for approximate solution of the problem.

1. Introduction

It is well known that systems with random forces having a strong effect on the investigated system are difficult both for analytical and numerical solution. In some cases such solutions are impossible. The very important example is the phenomenon of vortex separation and connected with it phenomenon of stall flutter. In this case to find even average motion, as a rule, is impossible. Just such problems are considered in this paper in detail. The consideration with using models allows us to understand the causes of many disasters possible in such systems.

There is a very important type of models, called by us «models of a phenomenon» (see [2]). As a rule, equations describing these models are not isomorphic to the equations for the simulated system, but they are possessed of a very important property: they are based on experimental data which are known to us. Just such models are considered in this paper. The existence of such models is conditioned by the universality of oscillatory and wave processes. An assumption, that this universality exists really, and some reasonings showing that it can exist, were made by L. Mandelshtam, S. Strelkov, and me. Although these assumptions are not proved, numerous experiments show that they are valid. We do not know any experiments showing that it is absent. The universality allow us to understand the causes of the considered phenomena and find the means for attenuation of their undesirable influence.

Due to the universality of oscillatory and wave laws (the most of experiments and a great number of argumentative considerations show that this universality exist really) [2–4], these disasters are of a similar character in different systems of such a kind. That is why the main purpose of this paper lies in the consideration of causes of such disasters and methods of controlling by them.

Special attention to these problems is attended last time in connection with a great number of disasters caused by the stall flow of the lengthy bodies: wires, suspension bridges, steel factory pipes, helicopter propellers, periscopes of submarines, compressor blades, turbines in turbo-jet engines, ropes drawn down from ships into ocean and so on. The vortex separation occurs under streamline of the blunt notched bodies. At such streamline vortices are reflected from the inequalities of the body surface. It is evident that such reflection is of a random character, and hereupon the reflected wave is random. Owing to multiple reflection we obtain the bunch of random waves. Since we do not know this randomness, and its behavior depends on time, we cannot describe this wave analytically, i.e. we cannot write equations describing our problem. Many examples of similar phenomena we can indicate in hydrodynamics and biology. That is why we have called the corresponding problems **unsolvable** [5]. It is very important that we have often a possibility to observe different manifestations of these phenomena in real life. Below we give several real examples of the stall flutter resulted in catastrophes.

1. Not far from Lancaster there is a sea, where very fast tides exist. In these tides the flow velocity changes randomly. It is known that these tides cause sometimes the loss of people and cars. We assume that due to large velocity of there stall flutter is excited. It is evident that the same might be in Ocean when there is strong wind.

2. As another example of real stall flutter was described in [6], where it is shown that stall flutter of electrical conductors is excited due to wind.

3. Stall flutter of the helicopter' screw. Such stall flutter is very unsafe. There is an assumption that a famous Russian oculist S.N. Fedorov was met with a helicopter accident in 2000 and died.

It should be noted that in last several years a great interest appears to waves of a high amplitude — freak waves and Rogue waves [7–9]. We assume that the main cause of freak and Rogue waves is also vortex separation. Our assumption may be conditioned by universality of wave processes and some pictures of these waves. Further we will consider some of such waves.

2. Aeolian tones, vortex separation and stall flutter

The phenomena of vortex separation when lengthy bodies are flowed around by fluid and connected with this sound emission are known many years as Aeolian tones [10, 11]. They are described in many books and papers (see, for example, [1, 5, 12–19]). It was found that these sounds are resulted from the reaction to the vortex-shedding that creates a Kármán wake [20] downstream of the body, as sketched in Fig. 1.

This wake is precisely that reason which causes oscillations of the streamlined body and the surrounding fluid. The phenomenon of the excitation of such oscillations we will call stall flutter, in spite of the fact that the authors of some books (see, for example, [18, 21]) distinguish the stall flutter, when oscillations of aerodynamical forces are induced by the body oscillations, and forced oscillations (wind resonance), when the Kármán wake exists also in the case of the immovable streamlined body.

It should be noted that vortices excite the sounds, and vice versa, sounds generate the vortices, because, as shown in [22], acoustic and hydrodynamic waves represent two wave solutions of the same equations of fluid dynamics in a moving medium.

More often the stall flutter excites when one of the natural frequencies of the body oscillations is close or divisible to the frequency of vortex separation from this body when it is immovable. Stall flutter was observed repeatedly for a diversity of technical constructions, for example, many suspended bridges, steel factory pipes, helicopter's propellers, periscopes of submarines, compressor blades, turbines in turbo-jet engines, ropes drawn down from ships into ocean and so on [1, 14–19]. We believe that stall flutter is one of the main reasons resulted in the sway of wires in the presence of wind. It may be shown that the stall flutter may cause rotatory oscillations of wires which are similar in its form to thermo-mechanical self-oscillations considered in [2, 23]. It is also known that stall flutter is the main cause of many technical disasters. The most known from them is the so called Tacoma catastrophe that happened in 1940 [1, 16]. That is why the study of stall flutter is an important theoretical and practical problem.

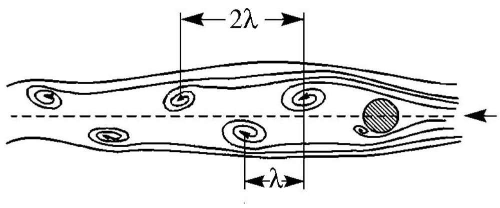

Рис. 1. Schematic diagram of the Kármán wake for streamlined flow around a cylinder

Experimental studies of stall flutter and photos of some disasters caused by him, for example, the shot from the film about the catastrophe of Tacoma bridge demonstrating its state for half hour before its failure [1, 16] show that for this flutter torsional oscillations are predominated, whereas bending constituent is small (see fig. 2). By this stall flutter differs markedly from more known bending-torsion flutter that was the main cause of the airplane catastrophes at the earliest stage of their appearance. However, as shown in [24], taking into account of even very small

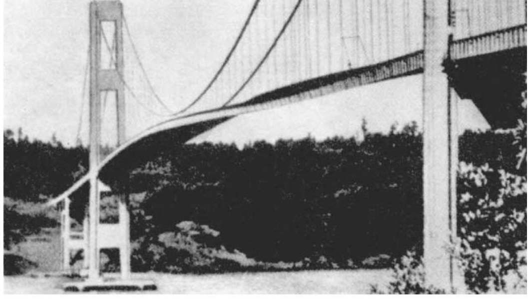

Рис. 2. The shot from the film about the catastrophe of Tacoma bridge demonstrating its state for half hour before its failure

bending vibrations can change essentially critical velocity and character of the stall flutter. This confirms once more that the stall flutter phenomenon is very complicated, far more complicate than more known flexure-torsion flutter. It is also seen that near the right bridge footing many vortices are stalled. We assume that just these vortex separations (each vortex separation is an impact) excite the bridge self-oscillations.

It should be noted that there are many works devoted to the stall flutter phenomenon, but all of them are of a partial character, whereas a sequential description is absent. Apparently, this may be explained by a great complexity of the problem and by the fact that in airplane flight this form of flutter was primarily rare in occurrence [18], and therefore it was studied not very intensively.

In detail monograph [18] it is written: «Classical type of the flutter is associated with a potential flow and usually, but not necessarily, involves the interaction between two or more degrees of freedom. Nonclassical type of the flutter, which is theoretically analyzed with difficulty, may take place at stalling flow: periodical separation of the flow with its adjacency again.» It should be noted that here the term «potential flow» is lame because in a potential boundless flow of perfect liquid any flutter is impossible owing to the fact that the sum of all aerodynamical forces is equal to zero (d'Alamber paradox) [25]. It is our opinion that the main difficulty of the stall flutter theory lies in the description of the random backwash behind the streamlined body that appears at vortex separation.

3. Modeling of stall flutter

Any exact mathematical model of stall flutter is impossible. But we can use any approximate model of some phenomenon attendant to stall flutter. It follows from experimental data that such a phenomenon is the synchronization of the stall flutter frequency by some periodic forcing. For an oscillator with additional circuit ours numerical calculations have shown that such forcing can be caused by oscillations of the additional circuit. Because the synchronization is possible only in self-oscillatory systems (see [26]), this means that we will use a self-oscillatory model.

General principles of the construction of mathematical models for different systems and their classification. General principles of the construction of mathematical models for different systems are described in some Russian textbooks, mainly in [27,28]. There the following types of models are considered: geometrical, physical, analogous mathematical and simulated. It is asserted that all these models must have a general foundation — isomorphism. It is evident that the definition given above is based on that our object can be described by dynamical equations, i.e. that it is a dynamical system. But up-to-date oscillation theory and nonlinear dynamics research often not dynamical systems, but stochastic ones. Many systems cannot be described sufficiently by dynamical equations and therefore they cannot be solved exactly neither analytically nor numerically.

In book by Blekhman, Myshkis and Panovko [29] some principles of model construction are also considered. There the following definition of a model is given: «an object a' is a model of an object a with respect to some system S of characteristics (properties), if a' is constructed (or is selected) for the simulation of a in according with these characteristics.» In this book it is emphasized that «the modeling lies in the base of all sciences.» This assertion is undoubtedly quite right, since any science cannot study nature, as is written in school textbooks. It can study only models of natural phenomena.

Another classification of models and their role in the nature study is given in [2]. There a very important type of models, called «models of a phenomenon» was introduced. As a rule, equations describing these models are not isomorphic to the equations for the simulated system, but they possess of a very important property: they are based on experimental data.

4. «Unsolvable problems» as a special class

The impossibility to obtain equations for the process studied by us is precisely that feature which separates the «unsolvable problems» from other complicated problems into a special class. Such separation and the approach to the solution of these problems are new. We do not know any similar works. Although models used by us and considered systems are known in oscillation theory and engineering, the results obtained by us and their explanation are new.

In this review we consider in detail an example of systems from this class: vortex separation and connected with this the stall flutter phenomenon.

In the most of existing works the stall flutter is explained as the excitation of resonant oscillations under the action of a periodic force caused by the vortex separation and having a given frequency depending on the flow velocity, the size and shape of the streamlined body. The force frequency is defined by the Strouhal number [30]. First of all such an explanation comes into conflict with known experimental facts concerning the stall flutter of sufficiently long ropes which are streamlined by flow with different velocities in different sections of the rope. This conflict is connected with the fact that stall flutter is not forced oscillations but self-oscillations.

Because of the similarity between stall flutter and self-oscillations, we can consider a model of stall flutter as self-oscillations in a system containing both active and passive oscillatory elements. A classic example of such a system is a van der Pol oscillator with an additional oscillatory circuit [26, 31–35].

We have a good reason to believe that all peculiar properties of this process: frequency pulling and characteristic dependencies of the oscillation amplitudes in the oscillator and additional circuit as the frequency mismatch varies [11, 32, 35] have to be observed in the case of the stall flutter too. Unfortunately, experimental comparisons of these phenomena are unknown to us.

To signs the analogy between self-oscillations and the vortex separation process from a streamlined body we indicate the following.

1. Pulsations of the velocity and pressure in the Kármán wake have rather narrow spectrum with well-defined maximum.

2. The vortex separation process from a streamlined body may be synchronized on the average if we excite oscillations of this body at a certain frequency that is approximately divisible by the frequency of vortex separation from the immovable body [12, 36–38]. The dependence of the vortex separation from the surface of oscillating body on the mismatch between the mean body oscillation frequency f and the frequency of vortex separation from the rest body is of the same character as well known dependence of the frequency of synchronized oscillations of van der Pol oscillator on the mismatch between the free self-oscillation frequency and the external force frequency [26] (see fig. 3 constructed from the experimental data of

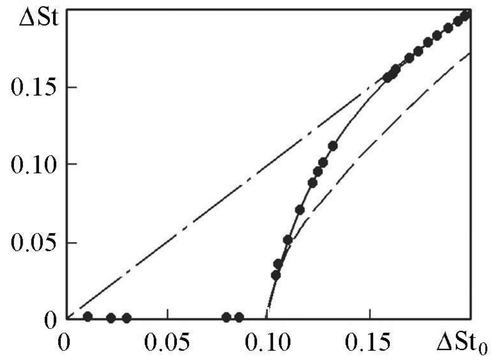

Рис. 3. The dependence of the Strouhal numbers ΔSt (oscillating cylinder) on ΔSt_0 (immovable cylinder). (Solid line) For the mismatch between the frequency of vortex separation from oscillated cylinder f and frequency of cylinder oscillations f_0 ($\Delta St = (f - f_0)D/U_0$) on the mismatch between frequencies of vortex separation from a rest cylinder f_{st} and f_0 ($\Delta St_0 = (f_{st} - f_0)D/U_0$). (Dashed line) For $\Delta St = \sqrt{(\Delta St_0)^2 - (\Delta St_0)_s^2}$, where $(\Delta St_0)_s$ is the half-width of the synchronization region. (Dash-dot line) For $\Delta St = \Delta St_0$.

[12]). In this figure $\Delta St = (f - f_0)D/U_0$ and $\Delta St_0 = (f_{st} - f_0)D/U_0$ are the mismatches between the frequencies of vortex separation from the oscillating f and the immovable f_0 cylinders expressed in terms of the Strouhal numbers, f is the frequency of the cylinder oscillations, D is the cylinder diameter and U_0 is the flow velocity. For comparison the dependence $\Delta St = \sqrt{(\Delta St_0)^2 - (\Delta St_0)_s^2}$, where $(\Delta St_0)_s$ is the half-width of the synchronization region, is shown in the same figure by dashed line. Such a dependence should be valid in the case of synchronization of a van der Pol oscillator by small harmonic external force. However, judging by the fact that the synchronization region is rather wide, the amplitude of the cylinder oscillations was significant, that resulted in the more steep dependence of ΔSt on ΔSt_0.

It should be noted that, as in van der Pol oscillators, synchronization in average can occur not only on the main frequency but on its harmonics and subharmonics [37]. We note that the synchronization of a van der Pol oscillator on the harmonics and subharmonics of an external force is considered in [26, 39].

5. Synchronization of the vortex separation frequency by the cylinder oscillations

The stall flutter phenomena may be conveniently considered by using the simplest model of oscillating body in the form of an elastic circular cylinder of length l with fixed ends and placed transversely to the flow directed along x-axis. In fluid flow on such a cylinder transversal forces act in the plane orthogonal to the flow [19, 37, 40]. We can set that one of third forces is directed along axis z. In this case the cylinder will be displaced along x-axis.

As was noted, behind a streamlined cylinder the Kármán vortex wake is formed [20], which for the Reynolds numbers $40 < Re < 150$ is regular (the Reynolds number is defined by the following formula: $Re = VD/\nu$, where V is the flow velocity, D is the cylinder diameter, ν is the kinematic viscosity), and for $150 < Re < 300$ – turbulent. For $300 < Re < 2 \cdot 10^5$ the Kármán wake becomes again close to regular but with turbulent bursts. After this for $Re > 5 \cdot 10^6$ in the wake spectrum a dominating frequency of vortex separation is also observed [41].

It should be emphasized that such alternation of the regions of different behavior as a parameter changes is typical for chaotic self-oscillatory systems (the definition of chaotic self-oscillatory systems and their dissimilarity from stochastic ones is given in books [39, 42]). In particular, the region $300 < \mathsf{Re} < 2 \cdot 10^5$ may be considered as the region of intermittency. The aforesaid testifies once more that forming the Kármán wake may be modeled by a weakly noisy self-oscillation process.

When stalling streamline occurs in the direction of axis x with mean velocity V, identical for all cylinder sections (the assumption that all sections of the cylinder are streamlined with the same mean velocity is not principal). The problem may be solved with using, for example, Galerkin method [43] in the case when this assumption does not valid. On this section a periodic (or close to periodic) lift force $F_z(t)$ directed along axis z acts. This force represents a sum of two components: regular, independent of time, and random, slowly changing with time.

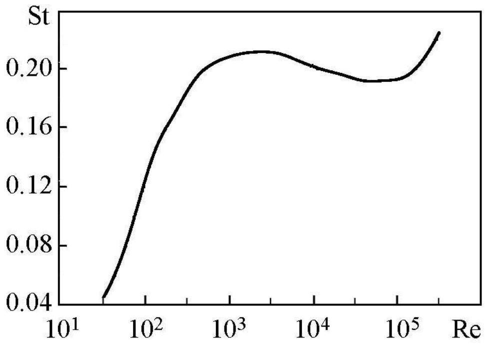

Рис. 4. The dependence of the Strouhal number on the Reynolds number for transversal streamline of an immovable circular cylinder [19]

In the case of immovable cylinder, for a wide range of the Reynolds numbers, the frequency of the lift force is close to the vortex separation frequency $\omega_{\rm stall}$, and the frequency of drag force is close to $2\omega_{\rm stall}$ [37]. The difference between mean periods of lift and drag forces is conditioned by the fact that the mean lift force frequency is defined by the distance between vortices only along one of the sides of the Kármán wake (2λ), whereas the mean drag force frequency is defined by the distance between vortices irrespective of the side of the Kármán wake (λ) (see fig. 1).

The mean frequency $\omega_{\rm stall}$ is connected with the Strouhal number St and the flow velocity V by formula

$$\omega_{\rm stall} = 2\pi \mathsf{St}/(DV), \tag{5.1}$$

where D is the cylinder diameter.

Experiments [19] show that for $40 < \mathsf{Re} < 150$ the Strouhal number initially increases, then becomes approximately constant and equal to 0.2, and further again begins to increase. In the range $2 \cdot 10^5 < \mathsf{Re} < 5 \cdot 10^6$ vortex wake is strongly turbulent, so that the Strouhal number cannot be determined. For $\mathsf{Re} > 5 \cdot 10^6$, the Strouhal number $\mathsf{St} \approx 0.3$. All of the said is illustrated by fig. 4, taken from [19].

As we know, any equations allowing us to describe rigorously the lift and drag forces in the case of stalling streamline are absent in literature. Only the expressions for peak-to-peak values of these forces are given [1, 19, 37, 40]. In all known books on aeroelasticity (see, for example, [14, 19]) the amplitudes of these forces are found from dimensionality considerations. For example, the amplitudes of the drag and lift forces (A_x and A_z) may be written as

$$A_x = c_x(\mathsf{Re}) S_{yx} \frac{\rho V^2}{2}, \quad A_z = c_z(\mathsf{Re}) S_{yz} \frac{\rho V^2}{2}, \tag{5.2}$$

where ρ is the density of medium where the body moves, $c_z(\mathsf{Re})$ are factors depending on the extent of the body streamlining (the worse is streamlining, the greater values of c_z), and on the Reynolds number. In 5.2 S_{yx} is the area of the body projection on the plane yx, S_{yz} is the area of the body projection on the plane yz. We suppose that S_{yx} and S_{yz} are independent of y.

6. Model equations for stall flutter without regard for random forces

In our work [11] we have considered the oscillating cylinder as a string and retained only a single oscillation frequency. In so doing we can change the cylinder by a material point of mass m and write the equations of its oscillations along axes x and z in the form:

$$\ddot{U}_x + 2\alpha_x \dot{U}_x + \omega_x^2 U_x = \frac{F_x}{m}\,, \quad \ddot{U}_z + 2\alpha_z \dot{U}_z + \omega_z^2 U_z = \frac{F_z}{m}\,, \qquad (6.1)$$

where U_x and U_z are the cylinder displacements along axes x and z, $F_z(t)$ and $F_x(t)$ are the forces acting on the cylinder at the expense of the vortex separation in the directions of x and z axes; ω_x and ω_z are the natural frequencies of the cylinder in the directions of x and ω_z, α_x and α_z are the damping factors in x and z directions. In the case of difference of the oscillation frequencies in the directions x and z, resulting oscillations of the cylinder may have rather complex form. It should be noted that force F_x is a drag force, whereas F_z is a lift force.

It should be noted that because of the backwash behind the streamlined body that always appears at vortex separation, on the cylinder always act random forces caused by this backwash. We will denote these forces $\eta_x(t)$ and $\eta_z(t)$. These forces must be added in the right side of Eqs. (6.1). Subject to these forces, Eqs. (6.1) becomes

$$\ddot{U}_x + 2\alpha_x \dot{U}_x + \omega_x^2 U_x = \frac{F_x}{m} + \eta_x(t), \quad \ddot{U}_z + 2\alpha_z \dot{U}_z + \omega_z^2 U_z = \frac{F_z}{m} + \eta_z(t). \qquad (6.2)$$

Because any equations describing the vortex separation phenomenon and allowing to find an expression for forces $F_z(t)$ and $F_x(t)$ are absent in the literature, we will use for our calculations model equations for self-oscillations. Considering van der Pol oscillator [39] as a self-oscillation system, taking account of the experimental fact that the cylinder oscillations can synchronize the vortex separation, and neglecting the influence of the backwash, we write the following model equations for forces $F_x(t)$ and $F_z(t)$ acting on the cylinder:

$$\begin{aligned} &\ddot{F}_x - \mu\left(1 - a_1 F_x^2\right)\dot{F}_x + \omega_{\rm stallx}^2 F_x = m_x U_x,\\ &\ddot{F}_z - \mu\left(1 - a_2 F_z^2\right)\dot{F}_z + \omega_{\rm stallz}^2 F_z = m_z U_z, \end{aligned} \qquad (6.3)$$

where μ is a parameter responsible for the excitation of self-oscillations, $a_{1,2}$ are coefficients determining the amplitudes of forces F_x and F_z, $m_{x,z}$ are the coupling coefficient which determines the influence of the cylinder oscillations on the vortex separation in x and z directions, $\omega_{\rm stallx}$ and $\omega_{\rm stallz}$ are frequencies of vortex separation in the directions of axes x and z.

We note that members $m_x U_x$ and $m_z U_z$ are responsible for the synchronization of vortex separation by cylinder oscillations in x and z directions. It follows from Eqs. (6.3) that, in the case of the immovable cylinder and small μ the stationary values of $F_x(t)$ and $F_z(t)$ are

$$F_x(t) \approx A_x \cos \omega_{\rm stallx} t, \quad F_z(t) \approx A_z \cos \omega_{\rm stallz} t, \tag{6.4}$$

where A_x and A_z are the oscillatory amplitudes.

It should be noted that formulas (6.4), in view of (5.2), are in full accordance with the expressions for drag and lift forces given in [37]. In the simplest case the forces $F_x(t)$ and $F_z(t)$ can be described by expressions (6.4).

Eqs. (6.2) and (6.3) define two self-oscillatory systems, each with two degree of freedom. In particular, such systems are considered in [35, 39].

Further we will make examples of self-oscillatory systems with two degree of freedom, which may be taken as models of stall flutter.

Versus the Reynolds number in x and z directions oscillations of systems described by Eqs. (6.2) and (6.3) can be periodical, quasi-periodical or random.

In the next section we consider self-oscillatory systems with two degree of freedom without regard for random forces.

6.1. Self-oscillatory systems with the main and additional circuits and pulling phenomenon without regard for random forces. It is known that oscillators with two degree of freedom may be described either two differential equations of the second order either a single equation of the fourth order. Mandelshtam in his lectures [3] considered different systems of the second order, but without friction. In well known works [31–34] authors considered systems described by two second order equations corresponding to their experimental installations. Schematic image of the installation with inductive coupling between circuits, studied in [32] and [26] is given in [32]. Another installation with capacitive coupling between circuits, studied by Teodorchik [34], is shown in [34]. Both these installations consist from two oscillatory circuits and an amplifier. It should be noted that equations describing oscillations in both installations differ from one another. This difference results in the fact that the dependencies of frequency and amplitude on the frequency mismatch are essentially different (compare Figs. 7 and 10, and also 8 and 11).

In this section oscillators with capacitive and inductive couplings between the circuits will be studied as applied to stall flutter.

We consider a system consisting from two oscillators, i.e. a system with two degrees of freedom. A consideration of such systems without friction was first conducted in famous lectures on oscillations delivered by Russian academician L. Mandelshtam in 20–30th years of last century [3]. In these lectures he showed that the behavior of such systems is conditioned by so called **connectedness** but not by coupling. The connectedness cd is defined as

$$cd = \frac{\omega_1 \omega_2}{\left|\omega_1^2 - \omega_2^2\right|}, \tag{6.5}$$

where ω_1 and ω_2 are partial frequencies of the system considered. It is seen from here that the closer partial frequencies the stronger is the connectedness. It should be noted that the notion of the connectedness is also was given by Strelkov in textbook [4].

Simultaneously the problems of pulling phenomenon were investigated by Andronov and Witt for oscillators with two inductively coupled circuits. For calculations authors have used the Poincaré small parameter method [32]. Almost at the same time similar investigations were conducted by Strelkov and Skibarko with using the qualitative methods

[33]. Both in [32] and in [33] authors have considered only one-frequency mode. It was found an area of pulling and discovered the phenomenon of quenching self-oscillations in some region of the parameters. It should be noted that the term «pulling» was first introduced in work [3] and until now it is used in all textbooks on oscillations (see, for example, [34, 35]). Similar results, but by the averaging method, proposed by [44] and developed by Mitropol'skii [45], were obtained later in [5, 6, 26].

6.2. The triode generator with additional circuit inductively coupled with the main one. We consider the triode generator with an additional circuit studied by Andronov and Witt [32] and described later in [35]. Its schematic image is shown in Fig. 5. We see that in this generator the coupling between circuits is inductive.

Setting $I_a = S_0U_1 - S_1U_1^3/3$, we can write the equations for such a generator in dimensionless coordinates. They may be easily transformed to the following:

$$\frac{d^2x}{dt^2} - \mu(1-\alpha x^2)\frac{dx}{dt} + \omega_1^2 x = m_1\frac{d^2y}{dt^2}, \quad \frac{d^2y}{dt^2} + \varepsilon\mu\frac{dy}{dt} + \omega_2^2 y = m_2\frac{d^2x}{dt^2}, \qquad (6.6)$$

where

$$x = \sqrt{\frac{M_1S_1}{M_1S_0 - R_1C_1}}\,U_1, \quad y = \sqrt{\frac{M_1S_1}{M_1S_0 - R_1C_1}}\,U_2$$

are dimensionless voltage drops across the triode grid and the capacitor C_2, respectively,

$$\mu = \frac{M_1S_0 - R_1C_1}{\sqrt{L_1C_1}}, \quad \varepsilon = \frac{R_2L_1C_1}{L_2(M_1S_0 - R_1C_1)},$$

$m_{1,2} = M_2C_{2,1}/(L_{1,2}C_{1,2})$ are the coupling coefficients, ω_1 and ω_2 are the natural frequencies of the main and additional circuits, $M_{1,2}$ are the coefficients of mutual induction between the coils L_1 and L and L_1 and L_2, respectively.

We will assume that the coupling coefficients $m_{1,2}$ are small, of order of a conditional small parameter. In this case we can use for approximate solution of Eqs. (6.6) the averaging method proposed by Bogolyubov [44] and developed by Mitropol'skii [45, 46]. We apply this method in the form set forth in [26]. For using this method we rewrite Eqs. (6.6) introducing dimensionless time $\tau = \omega t$, where frequency ω is unknown frequency of self-oscillations and another conditional small parameter $\mu_1 = \mu/\omega$.

In this case Eqs. (6.6) can be conveniently rewritten as

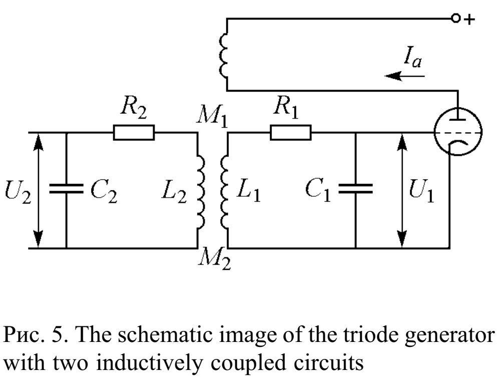

Рис. 5. The schematic image of the triode generator with two inductively coupled circuits

$$\frac{d^2x}{d\tau^2} + x - \mu_1(1-\alpha x^2)\frac{dx}{d\tau} = m_1\frac{d^2y}{d\tau^2},$$
$$\frac{d^2y}{d\tau^2} + \varepsilon\mu_1\frac{dy}{d\tau} + \xi y = m_2\frac{d^2x}{d\tau^2}, \qquad (6.7)$$

where $\mu_1 = \mu/\omega$,

$$\xi = \frac{\omega_2^2}{\omega_1^2} \qquad (6.8)$$

is the mismatch frequency squared.

6.2.1. Zero approximation. In zero approximation with respect to small parameters μ_1 Eqs. (6.7) become

$$\frac{d^2x_0}{d\tau^2} + x_0 - m_1\frac{d^2y_0}{d\tau^2} = 0,$$
$$\frac{d^2y_0}{d\tau^2} + \xi y_0 - m_2\frac{d^2x_0}{d\tau^2} = 0. \tag{6.9}$$

It should be noted that system (6.9) is a conservative system. Its solution can be set as

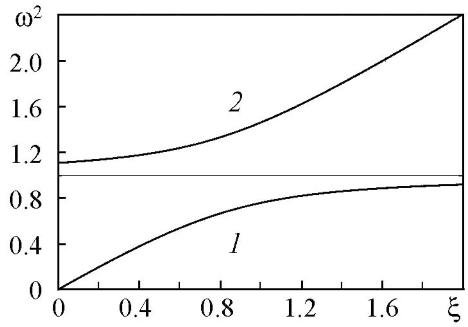

Рис. 6. The dependencies of two oscillation frequencies squared on the frequency mismatch ξ in zero approximation for $m_1m_2 = 0.1$ (the Vien diagram)

$$x_0(\tau) = A_x\cos\omega\tau, \quad y_0(\tau) = A_yk\cos\omega\tau, \tag{6.10}$$

where ω is the unknown frequency of self-oscillations, k is the distribution coefficient.

System (6.9) allows us to calculate its determinant D and the distribution coefficient $k = A$:

$$D = (1 - m_1m_2)\omega^4 - (1+\xi)\omega^2 + \xi, \tag{6.11}$$
$$k = \frac{A_y}{A_x} = \frac{\omega^2 - 1}{m_1\omega^2} = \frac{m_2\omega^2}{\omega^2 - \xi}\,. \tag{6.12}$$

Systems of equations (6.7) and (6.9) describe the interaction between the self-oscillatory system defined by function $x(\tau)$ and passive oscillatory system defined by function $y(\tau)$. If oscillatory frequencies of these systems differ strongly from each other then the systems will be weakly interacting, i.e. they will be sensibly independent. But if the frequencies are sufficiently close, synchronization, i.e. one-frequency mode, can appear.

Andronov and Witt [32] solved Eqs. (6.7) in one-frequency mode by using the Poincaré method of small parameter [47] at the assumption that parameter μ_1 is sufficiently small. However, here we will solve equations (6.7) by the Krylov–Bogolyubov method, as more preferable.

In zero approximation a solution of Eqs. (6.9) is

$$x \equiv x_0 = A\cos(\omega\tau + \varphi_x), \quad y \equiv y_0 = kA\cos(\omega\tau + \varphi_y), \tag{6.13}$$

where A is the amplitude of variable x, and φ_x and φ_y are phases of variables x and y, and ω is the frequency defined from the condition of vanishing determinant of system (6.9) defined by Eq. (6.11). Equation for the frequency ω has two real roots:

$$\left(\omega^{(1,2)}\right)^2 = \frac{1+\xi}{2\xi(1-m_1m_2)}\left(1 \mp \sqrt{1 - 4\xi^2(1-m_1m_2)/(1+\xi)^2}\right). \tag{6.14}$$

The dependencies of these roots on ξ are shown in Fig. 6. Such a graph is often called the Vien diagram.

To consider the pulling phenomenon we must take into account the nonlinear terms at least in the first approximation.

6.2.2. First approximation. We will assume that the coefficient of nonlinearity α is small, so that the term $(1-\alpha x^2)\, dx/d\tau$ can be linearized under the assumption that x changes according to harmonic law. In addition we will suppose that frequencies ω_1, ω_2 and ω are sufficiently close, so that their differences are of order of a conditional small parameter ε_1. This can be possible only for ξ closed to 1.

In the first approximation we will seek a solution of Eqs. (6.7) in the complex form:

$$x = A_x \exp\left(i\omega\tau + \varphi_x\right), \quad y = A_y \exp\left(i\omega\tau + \varphi_y\right). \tag{6.15}$$

Substituting (6.15) into Eqs. (6.9) we obtain the following equations:

$$\begin{gathered}(1-\omega^2)A_x - i\mu_1\omega\left(1-\frac{\alpha A_x^2}{4}\right)A_x + m_1\xi\omega^2 A_y e^{i\varphi} = 0,\\ i\varepsilon\mu_1\omega A_y + (\xi-\omega^2)A_y + m_2\omega^2 A_x e^{-i\varphi} = 0,\end{gathered} \tag{6.16}$$

where $\varphi = \varphi_y - \varphi_x$.

The condition of the equality to zero for the determinant of linear system (6.16) gives us an approximate complex characteristic equation in the first approximation. Real and imaginary parts of this equation are

$$(1-\omega^2)(\xi-\omega^2) + \varepsilon\mu_1^2\omega^2\left(1-\frac{\alpha A_x^2}{4}\right) - m_1 m_2\xi\omega^4 = 0, \tag{6.17}$$

$$\varepsilon(1-\omega^2) - (\xi-\omega^2)\left(1-\frac{\alpha A_x^2}{4}\right) = 0. \tag{6.18}$$

From Eq. (6.18) we can find αA_x^2:

$$\alpha A_x^2 = 4\left(1-\varepsilon\,\frac{1-\omega^2}{\xi-\omega^2}\right). \tag{6.19}$$

Substituting (6.19) into (6.17) we obtain the following bicubic equation for ω:

$$\omega^6 - \left(1+2\xi-\varepsilon\mu_1^2+m_1m_2\right)\omega^4 + \left((2+m_1m_2)\xi+\xi^2-\varepsilon\mu_1^2\right)\omega^2 - \xi^2 = 0. \tag{6.20}$$

Further, by using Eqs. (6.19) and (6.20) we calculate examples of the dependencies of ω^2 and αA_x^2 on ξ for $m_1m_2 = 0.1$, $\mu_1 = 0.1$ and four values of ε (see Figs. 7, 8).

We see from Eq. (6.20) that it can have either one or three real positive roots. Our calculations showed that for $\varepsilon < \varepsilon_4$ and $\xi \le \xi_1(\varepsilon)$ Eq. (6.20) has a single such root, and for $\xi > \xi_1(\varepsilon)$ it has three such roots (see Fig. 7 *a*, *b*, *c*). For ε more some value and all ξ Eq. (6.20) has a single root (see Fig. 7 *d*).

We can see from Fig. 8 that for $\varepsilon > 1$ there exist the domains of non-existence of the solutions found. In these domains the equilibrium states are stable for $\varepsilon < m_1m_2/\mu_1^2$, i.e. for sufficiently large coupling factors.

It can be seen that the amplitude dependencies are of the same character as calculated by Andronov and Witt [26, 32]. It should be noted that these dependencies are shown in Fig. 8 were calculated nonmetering the stability.

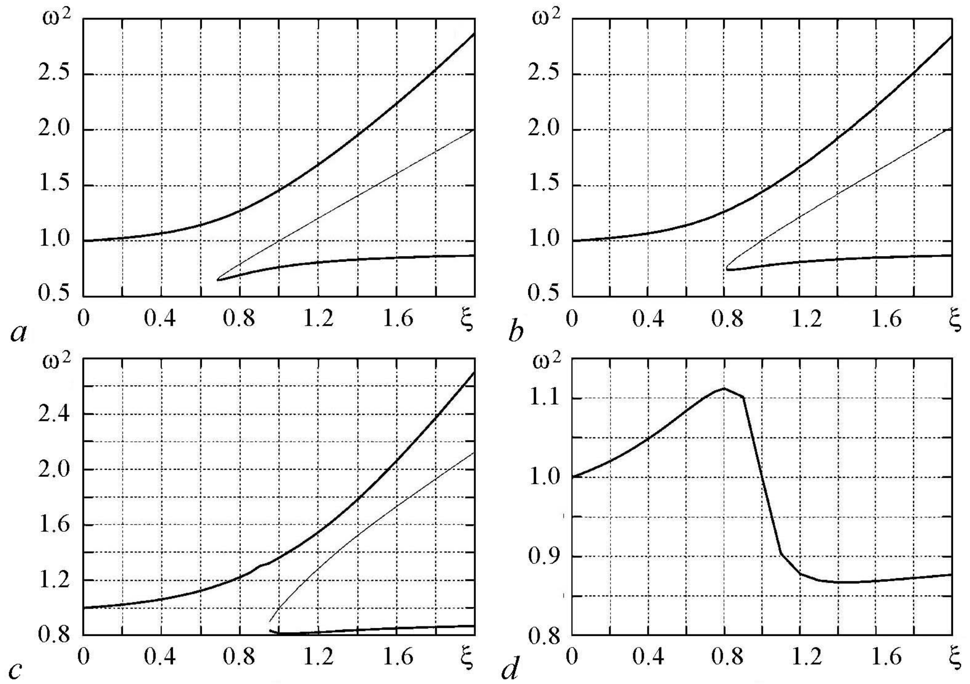

Рис. 7. Examples of the dependencies of ω^2 on the frequency mismatch ξ for $\mu_1 = 0.1$, $m_1 m_2 = 0.1$, $\varepsilon = \varepsilon_1 = 0.5$ (*a*), $\varepsilon = \varepsilon_2 = 1$ (*b*), $\varepsilon = \varepsilon_3 = 2$ (*c*) and $\varepsilon = \varepsilon_4 = 4$ (*d*) in the case of an oscillator with inductive coupling between circuits

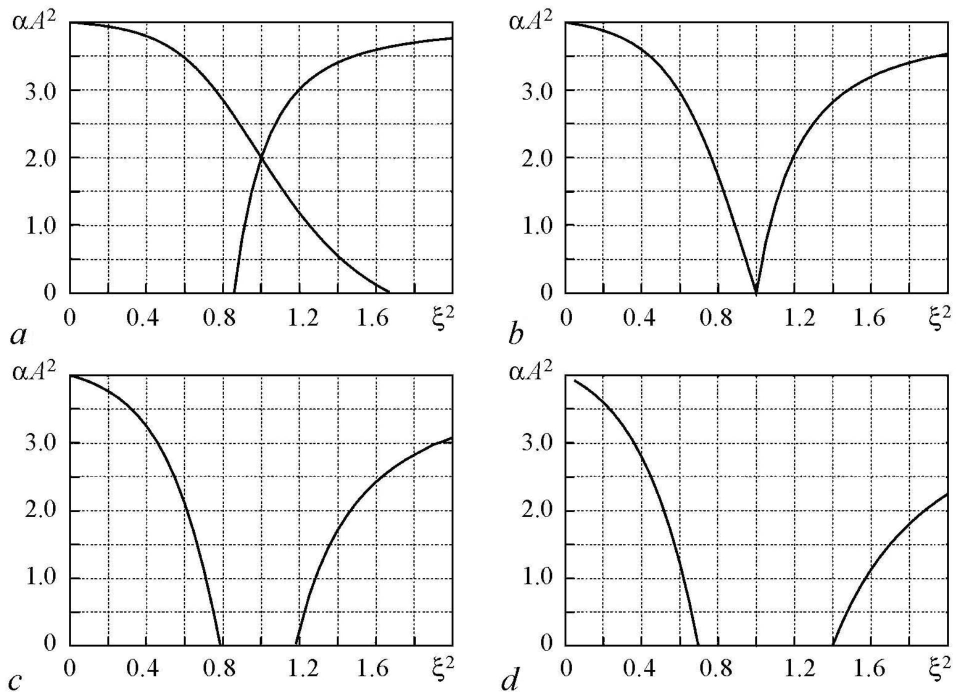

Рис. 8. Examples of the dependencies of oscillation amplitudes squared αA_x^2 on the frequency mismatch ξ for $\mu_1 = 0.1$, $m_1 m_2 = 0.1$ and the values of ε the same as in Fig. 7 ($\varepsilon = \varepsilon_1$ (*a*), $\varepsilon = \varepsilon_2$ (*b*), $\varepsilon = \varepsilon_3$ (*c*) and $\varepsilon = \varepsilon_4$ (*d*)) in the case of an oscillator with inductive coupling between circuits

6.3. The triode generator with the capacitive coupling between circuits without regard for random forces. A detailed consideration of the pulling phenomenon in oscillators with capacitive coupling between circuits was first pursued by Teodorchik [34]. In Fig. 9 the same schema of an oscillator that was analyzed by Teodorchik in [34] is shown. Setting $dz/dt = (S_0 - S_1x^2)\,dx/dt$, where z is the anode current, we obtain the following equations for the oscillations in this schema [34]:

$$\left(1+\frac{C_1}{C_0}\right)x + (R_1C_1 - MS_0 + MS_1x^2)\,\frac{dx}{dt} + L_1C_1\,\frac{d^2x}{dt^2} = \frac{C_2}{C_0}\,y,$$
$$\left(1+\frac{C_2}{C_0}\right)y + R_2C_2\,\frac{dy}{dt} + L_2C_2\,\frac{d^2y}{dt^2} = \frac{C_1}{C_0}\,x. \qquad (6.21)$$

Equations (6.21) may be conveniently transformed to the following:

$$\frac{d^2x}{dt^2} - \mu(1-\alpha x^2)\,\frac{dx}{dt} + \omega_1^2 x = m_1 y, \quad \frac{d^2y}{dt^2} + 2\varepsilon\mu\,\frac{dy}{dt} + \omega_1^2\xi y = m_2 x, \qquad (6.22)$$

where

$$x = \sqrt{\frac{M_1S_1}{M_1S_0 - R_1C_1}}\,U_1, \quad y = \sqrt{\frac{M_1S_1}{M_1S_0 - R_1C_1}}\,U_2$$

are dimensionless voltage drops across the triode grid and the capacitor C_2, respectively,

$$\mu = \frac{M_1S_0 - R_1C_1}{L_1C_1}, \quad \alpha = \frac{MS_1}{MS_0 - R_1C_1}, \quad \omega_1 = \sqrt{\frac{1}{L_1}\left(\frac{1}{C_1}+\frac{1}{C_0}\right)},$$

$$\omega_2 = \sqrt{\frac{1}{L_2}\left(\frac{1}{C_2}+\frac{1}{C_0}\right)}, \quad m_1 = \frac{C_2}{L_1C_1C_0}, \quad m_2 = \frac{C_1}{L_2C_2C_0}, \quad \delta = \frac{R_2}{2L_2},$$

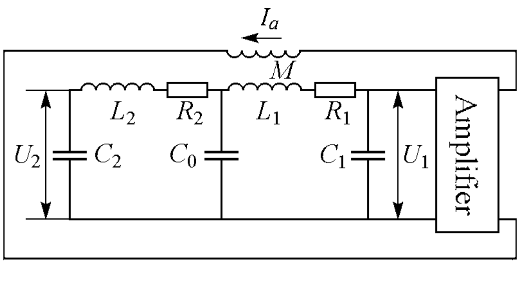

Рис. 9. Schematic image of an oscillator with capacitive coupling between circuits

$m_{1,2} = M_{1,2}C_{2,1}/(L_{1,2}C_{1,2})$ are the coupling coefficients, $M_{1,2}$ are the coefficients of mutual induction between the coils L_1 and L and L_1 and L_2, respectively; the dots mean the differentiation with respect to dimensionless time $\tau = \omega t$, where ω is a supposed frequency of self-oscillations, $\xi = \omega_2^2/\omega_1^2$ is the frequency mismatch squared.

Comparing Eqs. (6.22) with Eqs. (6.7) for oscillator with inductive coupling between the circuits, we see that they differ from one another. The main difference consists in that in Eqs. (6.7) the coupling is realized via the second derivatives $\ddot{x}$ and $\ddot{y}$, whereas in Eqs. (6.22) it is realized via the variables x and y. This distinction results in different equations for self-oscillatory frequencies and amplitudes.

As before, for approximate solution of Eqs. (6.22) we use the averaging method proposed by Bogolyubov [44] and developed by Mitropol'skii [45, 46]. We apply this method in the form set forth in [26]. For using this method let us rewrite Eqs. (6.22) introducing dimensionless time $\tau = \omega t$, where ω is a complex self-oscillation frequency, and another conditional small parameter ε_1 characterizing the difference between frequencies ω and ω_1:

$$\begin{aligned} &\ddot{x} - \mu_1(1 - \alpha x^2)\dot{x} + x - \tilde{m}_1 y = \varepsilon_1 \left(1 - \frac{\omega_1^2}{\omega^2}\right) x, \\ &\ddot{y} + \varepsilon\mu_1\dot{y} + \xi y - \tilde{m}_2 x = \varepsilon_1\xi \left(1 - \frac{\omega_1^2}{\omega^2}\right) y, \end{aligned} \tag{6.23}$$

where dots mean differentiation with respect to τ, $\mu_1 = \mu/\omega$, $\tilde{m}_{1,2} = m_{1,2}/\omega^2$. It should be noted that parameters μ_1 and $\tilde{m}_{1,2}$ are dimensionless.

Further, as in the last section, we reject in these equations terms of order of ε_1. As a result, we obtain the following equations:

$$\begin{aligned} &\ddot{x} - \mu_1(1 - \alpha x^2)\dot{x} + x - \tilde{m}_1 y = 0, \\ &\ddot{y} + \varepsilon\mu_1\dot{y} + \xi y - \tilde{m}_2 x = 0. \end{aligned} \tag{6.24}$$

We will assume in Eqs. (6.24) that the coefficient of nonlinearity α is small, so that the term $(1 - \alpha x^2)\dot{x}$ can be linearized subject to that x and y change according to harmonic low with complex frequency ω, i.e.

$$x = A_x \exp\left(i(\omega\tau + \varphi_x) + \text{c.c.}\right), \quad y = A_y \exp\left(i(\omega\tau + \varphi_y) + \text{c.c.}\right), \tag{6.25}$$

where A_x, A_y are amplitudes of variables x and y, and φ_x, φ_y are their phases.

Substituting (6.25) into (6.24) we obtain the following equations:

$$\begin{aligned} &(1 - \omega^2 - i\omega\mu_1)\left(1 - \frac{\alpha A_x^2}{4}\right) A_x - \tilde{m}_1 A_y e^{-i\varphi} + \text{c.c.} = 0, \\ &(\xi - \omega^2 + i\omega\varepsilon\mu_1)A_y - \tilde{m}_2 A_x e^{i\varphi} + \text{c.c.} = 0, \end{aligned} \tag{6.26}$$

where $\varphi = \varphi_x - \varphi_y$.

The condition of the vanishing of the determinant of linear system (6.26) gives a complex equation coupling ω^2, ξ and $\alpha A_x^2/4$:

$$\left[1 - \omega^2 - i\omega\mu_1\left(1 - \frac{\alpha A_x^2}{4}\right)\right]\left(\xi - \omega^2 + i\varepsilon\omega\mu_1\right) - \tilde{m}_1\tilde{m}_2 = 0. \tag{6.27}$$

Real and imaginary parts of this equation are:

$$\begin{aligned} &(1 - \omega^2)(\xi - \omega^2) + \varepsilon\omega^2\mu_1^2\left(1 - \frac{\alpha A_x^2}{4}\right) - \tilde{m}_1\tilde{m}_2 = 0, \\ &(1 - \omega^2)\varepsilon\omega\mu_1 - (\xi - \omega^2)\omega\mu_1\left(1 - \frac{\alpha A_x^2}{4}\right) = 0. \end{aligned} \tag{6.28}$$

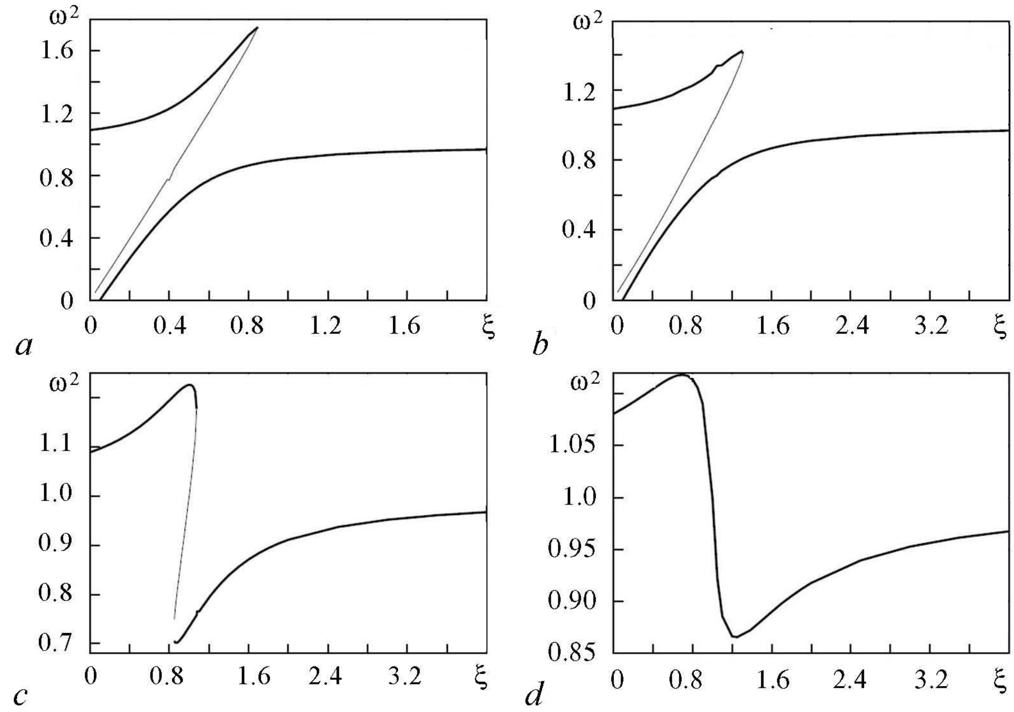

Рис. 10. Examples of the dependencies of ω^2 on ξ for $\mu_1 = 0.1$, $m_1 m_2 = 0.1$, $\varepsilon = 0.5$ (*a*), $\varepsilon = 1$ (*b*), $\varepsilon = 2$ (*c*) and $\varepsilon = 4$ (*d*) in the case of an oscillator with capacitive coupling between circuits

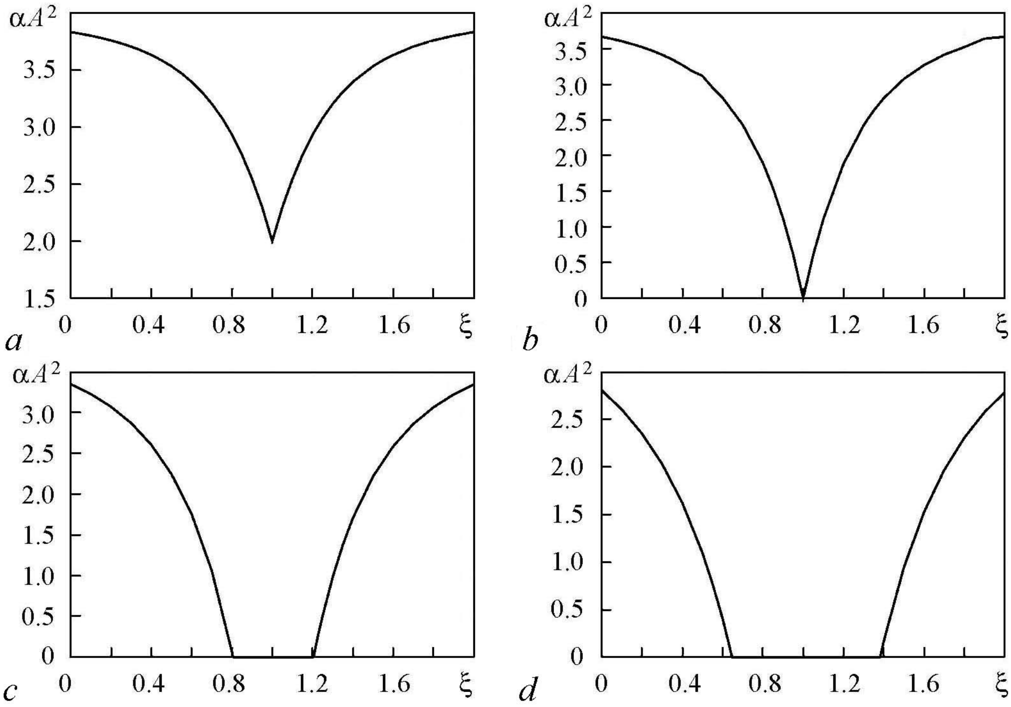

Рис. 11. Examples of the dependencies of oscillation amplitudes on the frequency mismatch ξ for $\mu_1 = 0.1$, $m_1 m_2 = 0.1$, $\varepsilon = 0.5$ (*a*), $\varepsilon = 1$ (*b*), $\varepsilon = 2$ (*c*) and $\varepsilon = 4$ (*d*) in the case of an oscillator with capacitive coupling between circuits

From Eqs. (6.28) we obtain equation for frequency ω:

$$\omega^6 - (1 + 2\xi - \varepsilon^2\mu_1^2)\omega^4 + (2\xi + \xi^2 - \varepsilon^2\mu_1^2 - \tilde{m}_1\tilde{m}_2)\omega^2 - \xi^2 + \tilde{m}_1\tilde{m}_2\xi = 0. \quad (6.29)$$

We see that Fig. 10 differs significantly from Figs. 7 for oscillator with inductively coupled circuits, whereas Figs. 11 and 8 are similar.

As for a generator with inductive coupling between the circuits, the dependencies shown in Fig. 10 and 11 are constructed without taking into account the stability of the solutions found. That is why they contain unstable pieces. Eqs. (6.23), (6.24) allow us to investigate the stability.

7. Model equations for stall flutter

As a model for stall flutter we have taken a generator with additional circuit coupled with the main by capacitor. In so doing for the vortex separation we have used model equations with a noise sources $\eta_x(t)$, $\eta_z(t)$ in equations for F_x, F_z and model equations with a noise sources $\zeta_x(t)$, $\zeta_z(t)$ in equations for U_x, U_z. These equations are

$$\ddot{U}_x + 2\alpha_x\dot{U}_x + \omega_x^2 U_x = \frac{F_x}{m} + \eta_x(t), \quad \ddot{F}_x - \mu\Big(1 - a_1F_x^2\Big)\dot{F}_x + \omega_{\text{stallx}}^2 F_x = = m_1U_x + \zeta_x(t), \quad (7.30)$$

$$\ddot{U}_z + 2\alpha_z\dot{U}_z + \omega_z^2 U_z = \frac{F_z}{m} + \eta_z(t), \quad \ddot{F}_z - \mu\Big(1 - a_2F_z^2\Big)\dot{F}_z + \omega_{\text{stalls}}^2 F_z = = m_2U_z + \zeta_z(t), \quad (7.31)$$

where U_x and U_z are displacements of the additional circuit in x and z directions, F_x and F_z are aerodynamical forces in x and z directions, ω_x and ω_z are the natural frequencies of the additional circuit in x and z directions, ω_{stallx} and ω_{stallz} are the vortex stall frequencies in x and z directions, $\eta_x(t)$ and $\eta_z(t)$ are white noises of intensity K_u, $\zeta_x(t)$ and $\zeta_z(t)$ are white noises of intensity K_f.

Equations (7.30) and (7.31) describe two independent self-oscillatory systems each of which possess two degree of freedom. In terms of oscillation theory these systems can be considered as generators with additional oscillatory circuits connected with the main ones by capacitors. It should be noted that terms $m_1U(x)$ and m_2U_z are responsible for the synchronization of vortex separation frequency by the additional circuits oscillations $U_x(t)$ and $U_z(t)$.

For brevity we will consider a system described by Eqs. (7.30). This system describes the interaction between the self-oscillatory system defined by $F_x(t)$ and a passive oscillatory system defined by $U_x(t)$. If natural frequencies of each of this systems (ω_x and ω_{stallx}) differ essentially, then the system will be weakly interacting one, i.e. variables U_x and F_x will be practically independent. But if the frequencies are sufficiently close then synchronization may be appear. Similar equations we can obtain for the system described by Eqs. (7.31).

We will assume that $\omega_x^2 U_x \ll \ddot{U}_x$ and $\omega_z^2 U_z \ll \ddot{U}_z$. In this case the first of Eqs. (7.30) can be rewritten approximately as

$$\dot{V}_x + 2\alpha_x V_x = \frac{F_x}{m} + \eta_x(t), \quad (7.32)$$

where $V_x = \dot{U}_x$ is the velocity of the additional circuit in x-direction.

Equation for self-oscillations of force F_x can be solved approximately for small μ in the absence of noise. In the case of immovable additional circuit, when $U_x = 0$, its approximate solution is:

$$F_x \approx A_x \cos\Big(\omega_{\mathrm{stallx}} t + \varphi_x\Big), \tag{7.33}$$

where A_x is an oscillatory amplitude of force F_x, φ_x is its phase. It should be noted that formula (7.33) is in the full accordance with the expression for lift force given in [37].

The first case was considered in two previous sections. Here we will consider the case, when the frequencies differ essentially, i.e. the variables U_x and F_x are practically independent. We will suppose that the oscillatory process in the additional circuit (U_x) is incomparably more rapid than the vortex separation process (F_x), i.e. that $\omega_x \gg \omega_{\mathrm{stallx}}$. A solution of Eq. (7.32) may be conveniently found as sums of two constituents: fast constituent V_x and slow constituent defined by function F_x.

The following Fokker–Planck equation corresponds to Eq. (7.32) (see [26]):

$$\frac{\partial w(V_x, F_x, t)}{\partial t} = \frac{\partial}{\partial V_x}\left[\left(2\alpha_x V_x - \frac{F_x}{m}\right) w(V_x, F_x, t)\right] + \frac{K_u}{2}\,\frac{\partial^2 w(V_x, F_x, t)}{\partial V_x^2}. \tag{7.34}$$

We will assume that solution Eq. (7.34) differs slightly from $w_0(V_x)$, i.e.

$$w(V_x, F_x, t) = w_0(V_x) + \varepsilon w_1(V_x) F_x(t), \tag{7.35}$$

where $w_0(V_x)$ and $w_1(V_x)$ are described by equations

$$2\alpha_x V_x w_0(V_x) + \frac{K_u}{2}\,\frac{\partial w_0(V_x)}{\partial V_x} = 0, \tag{7.36}$$

$$w_1(V_x)\,\frac{\partial F_x(t)}{\partial t} - F_x(t)\,\frac{\partial}{\partial V_x}\left[2\alpha_x V_x\, w_1(V_x) + \frac{K_u}{2}\,\frac{\partial w_1(V_x)}{\partial V_x}\right] =$$

$$= -\frac{\partial}{\partial V_x}\left(\frac{F_x(t)}{m}\, w_0(V_x)\right), \tag{7.37}$$

ε is a small parameter.

A solution of (Eq. 7.36) is

$$w_0(V_x) = C \exp\left(-\frac{2\alpha_x V_x^2}{K_u}\right), \tag{7.38}$$

where C is an arbitrary constant.

As followed from (7.33) and (7.37), functions $F_x(t)$ and $w_1(V_x)$ are periodic functions of time with period $T = 2\pi/\omega_{\mathrm{stallx}}$. Amplitude of function $w_1(V_x)$ is defined by the right-hand member of Eq. (7.37). The greater is the derivative of $w_0(V_x)$ over velocity V_x, the greater is $w_1(V_x)$. It follows from here that the location of maximum of a wave and its value are defined by not the maximal value of the wave amplitude, but mainly by the derivative of $w_0(V_x)$ over velocity V_x. It follows from here that the explanation of the freak and rogue waves given in work [9] is not right for all waves.

8. Conclusion

As followed from our results that for systems with two and more numbers od degree of freedom the small parameter methods can give wrong results even for rather small coupling between the degrees of freedom. However, Russian researcher Poznyak [48] has shown that using the method of harmonic linearization gives results more close to numerical and experimental.

Using of model equations for vortex separation from the surface of the oscillating body allow us to calculate amplitudes and frequencies of self-oscillations excited due to this vortex separation, i.e. to solve the problem on stalling flatter. In so doing the effect of synchronization of the vortex separation frequency by natural oscillations of the streamlined body was discovered. This effect requires experimental examination.

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НЕЛИНЕЙНЫЕ СЛУЧАЙНЫЕ ВОЛНЫ В ЖИДКОСТИ И ОСНОВНОЙ МЕХАНИЗМ ИХ ВОЗБУЖДЕНИЯ

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Чтобы описать явление случайных нелинейных волн в жидкости, мы должны знать точно или приблизительно, как происходит процесс срыва вихрей. Для этого удобно использовать модели, основанные на физических соображениях и некоторых экспериментальных данных. Основное внимание в этом обзоре будет уделено случайным волнам, возникающим, например, при срывном флаттере. Такие волны часто возбуждаются в жидкости, и они являются одной из основных причин катастроф в морях и океанах. Как правило, срывной флаттер связан с явлением затягивания, и наблюдается в системах с двумя и (или) более степенями свободы. В принципе, в таких системах возможны, как примерно одночастотный (синхронный) режим, так и мультичастотные (асинхронные) режимы (когда каждая мода колеблется с собственной частотой). Но в случае явления затягивания только режим с одной частотой,

соответствующей собственной частоте (см [1]) является устойчивым. В отличие от обычной турбулентности срывной флаттер это автоколебательный процесс. Обратная связь в этом процессе возникает из-за взаимодействия между жидкостью и обтекаемым телом. Следует отметить, что волновые движения в жидкости могут иметь очень сложный характер. В последние годы большой интерес представляют волны аномально высокой амплитуды – так называемые **аномальные волны** и **волны-убийцы**. Мы полагаем, что основной причиной таких волн является срыв вихрей.

Ключевые слова: Нелинейные волны в жидкости, срыв вихрей, срывной флаттер, катастрофы в морях и океанах, явление затягивания, степени свободы, блуждающие волны, катастрофические волны, использование математической модели для приближенного решения задачи.

Ланда Полина Соломоновна – окончила физический факультет МГУ. Защитила диссертацию на соискание ученой степени кандидата физико-математических наук в МГУ и доктора физико-математических наук в Горьковском университете в области теории колебаний и волн. Профессор, ведущий научный сотрудник МГУ. Область научных интересов – теория колебаний и волн, радиофизика, применение методов нелинейной динамики в различных областях науки. Автор и соавтор десяти монографий по колебаниям и волнам, в том числе монографии «Стохастические и хаотические колебания», переведенной на английский язык, а также монографии «Нелинейные колебания и волны в динамических системах», вышедшей в издательстве «Kluwer», «Регулярные и хаотические колебания», вышедшей в издательстве «Springer» в 2001 году, и нескольких обзоров, в том числе в УФН и «Physics Reports», Член Национального комитета по механике (Россия). Опубликовала много научных статей по направлениям, указанным выше. Член редакционной коллегии журналов «Chaos, Solitons and Fractals» и «Изв. вузов. Прикладная нелинейная динамика».

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