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On trajectory tracking control of prismatic and revolute joined robotic manipulators

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Abstract. The *purpose* of this paper is to construct a trajectory tracking feedback controller for prismatic and revolute joined multi-link robotic manipulators using a new form of sliding modes. *Methods*. In this paper, the Lyapunov functions method has been applied to establish the stability property of the closed-loop system. *Results*. Due to the presence of rotational joints, the motion equations of the manipulator are periodic in the angular coordinates of the corresponding links. A control law is constructed which is also periodic in the angular coordinates of the links. Thus, a closed-loop system has not one, but a whole set of equilibrium positions that differ from each other by a multiple of the system period. The dynamics mathematical model of a complex five-link manipulator with cylindrical and prismatic joints has been constructed on the basis of the Lagrange equations. Simulation results on a 5-degree-of-freedom robotic arm demonstrate the applicability of the proposed control scheme. *Conclusion*. We obtain a relay controller such that the set of all equilibrium positions of the closed-loop system is uniformly asymptotically stable. The novelty of the obtained control law is based on a new approach that takes into account the periodicity of the model in angular variables with the solution of the tracking problem in the cylindrical phase space. The simulation results for a 5-degree-of-freedom robotic manipulator clearly show the good performance of our controller. The applied significance of the results obtained in the paper is as follows. At present, in connection with the widespread introduction and mass production of manipulators, it seems important to develop the mathematical foundations for designing a control structure that has a universal character, namely, allowing to perform the required process without additional adjustment of control parameters with simple and convenient algorithms and programs of their implementation.

Keywords: trajectory tracking control, robot manipulator, revolute and prismatic joints, Lyapunov functions method, sliding mode, nonlinear system.

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Об управлении движением роботов-манипуляторов с призматическими и вращательными шарнирами

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Аннотация. *Цель* настоящего исследования – с использованием новой формы скользящих режимов построить закон управления для отслеживания траектории многозвенных роботов-манипуляторов с призматическими и вращательными шарнирами. *Методы*. В данной работе для установления свойства устойчивости положений равновесия замкнутой системы применяется метод функций Ляпунова и его развитие для неавтономных систем. *Результаты*. Из-за наличия вращательных шарниров уравнения движения манипулятора являются периодическими по угловым координатам соответствующих звеньев. Построен закон управления, также являющийся периодическим по угловым координатам звеньев. Таким образом, замкнутая система имеет не одно, а целое множество положений равновесия, которые отличаются друг от друга на величину, кратную периоду системы. На основе уравнений Лагранжа построена математическая модель динамики сложного пятизвенного манипулятора с цилиндрическим и призматическим шарнирами. Результаты моделирования на примере роботизированной руки с 5 степенями свободы демонстрируют применимость предложенной схемы управления. *Заключение*. Для многозвенных роботов-манипуляторов с призматическими и вращательными шарнирами получен релейный закон управления, такой что множество всех положений равновесия замкнутой системы равномерно асимптотически устойчиво. Новизна полученного закона управления основана на новом подходе, учитывающем периодичность модели в угловых переменных с решением задачи слежения в цилиндрическом фазовом пространстве. Результаты моделирования робота-манипулятора с пятью степенями свободы ясно показывают хорошие характеристики нашего закона управления. Прикладное значение полученных в статье результатов состоит в следующем. В настоящее время в связи с повсеместным внедрением и массовым производством манипуляторов представляется актуальной разработка математических основ проектирования управляющей структуры, имеющей универсальный характер, а именно позволяющей без дополнительной настройки управляющих параметров выполнять требуемый процесс с помощью простых и удобных алгоритмов и программ их реализации.

Ключевые слова: отслеживание траектории, робот-манипулятор, вращательный и призматический шарниры, метод функций Ляпунова, скользящий режим, нелинейная система.

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Introduction

The trajectory tracking control problem for robotic arms has been considered in great detail in the literature (see the survey [1] and the monograph [2]). Many control schemes have been proposed by the researches to solve the control problem for serial robot manipulators. A feedback linearization method has been used in [1, 3], a sliding mode control methodology has been considered in [4–8], a passivity-based technique has been applied in [9], and a robust adaptive control design has been implemented in [10]. In [11], a robust control scheme has been proposed based on the combination of both proportional derivative (PD) and sliding mode terms for the global trajectory tracking of robotic manipulators with uncertain dynamics. In [12], a PD with a feedforward scheme has been proposed to provide the global tracking of robot-manipulators under the bounded constantly acting disturbances.

The main contribution of this paper consists in the following. We propose a solution to the trajectory tracking control problem for prismatic and revolute joined robotic manipulators using a discontinuous bounded controller without a feedback linearization.

We denote by $\|\cdot\|$, both the Euclidean vector norm and the operator matrix norm. We use the symbol $\mathrm{lgn}\|\cdot\|$ as a logarithmic matrix norm which corresponds to the Euclidean vector norm and can be calculated as $\mathrm{lgn}\|\mathrm{A}\| = \lambda_{\max}(\mathrm{A}^{\mathrm{T}} + \mathrm{A})/2 \ \forall A \in \mathbb{R}^{n\times n}$, where $\lambda_{\max}(\cdot)$ is the maximum eigenvalue. The symbol $E \in \mathbb{R}^{n\times n}$ denotes the identity matrix.

1. Robot Model and Preliminaries

The dynamics of an n-degree-of-freedom (DOF) serial rigid robotic manipulator with revolute and prismatic joints is defined by the following equations

$$A(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + d(q,\dot{q}) = u, \tag{1}$$

where the vector of joint positions is represented by $q \in \mathbb{R}^n$, the inertia matrix $A(q) \in \mathbb{R}^{n\times n}$ is positive definite, the Coriolis and centrifugal forces are expressed by the term $C(q,\dot{q})\dot{q}$ with $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$, the gravitational forces are expressed by the vector $g(q) \in \mathbb{R}^n$, the vector $d(q,\dot{q}) \in \mathbb{R}^{n\times n}$ is the viscous damping forces of the manipulator links, $u \in \mathbb{R}^n$ is the vector of control forces.

Proposition 1. *Let J_R and J_P be two subsets of the set $J_n = \{1, 2, \ldots, n\}$ such that $J_R \cup J_P = J_n$ and $J_R \cap J_P = \emptyset$. Let the generalized coordinates $q_i \ \forall i \in J_R$ be the angular displacements of the revolute joints, and let the others $q_i \ \forall i \in J_P$ be the linear displacements of the prismatic joints.*

Remark 1. *The matrices $A(q)$ and $C(q,\dot{q})$ have some properties such as follows [2].*

1. The following inequalities hold

$$\lambda_1^2||q||^2 \leqslant q^{\mathrm{T}}A(q)q \leqslant \lambda_2^2||q||^2 \quad \forall q \in \mathbb{R}^n, \tag{2}$$

where λ_1 and λ_2 are some positive constants.

2. The matrix $\dot{A}(q) - 2C(q,\dot{q})$ is skew symmetric.

Define the set of the reference trajectories for the robot-manipulator (1) as follows

$$X = \{q^{(0)}(t) : [0, +\infty) \to \mathbb{R}^n : \ ||\dot{q}^{(0)}(t)|| \leqslant g_1, \ ||\ddot{q}^{(0)}(t)|| \leqslant g_2\}, \tag{3}$$

where the time functions $q^{(0)}(t)$ are twice continuously differentiable, g_1 and g_2 are some constants.

The problem of trajectory tracking control consists in constructing a feedback controller $u = u(t, q, \dot{q})$ such that the reference trajectory $q^{(0)}(t) \in X$ of the manipulator (1) is uniformly asymptotically stable.

2. Problem Solution

For some reference trajectory $q^{(0)}(t) \in X$, define the feedforward term of the controller $u = u(t, q, \dot{q})$ as follows

$$u^{(0)}(t) = A(q^{(0)}(t))\ddot{q}^{(0)}(t) + C(q^{(0)}(t), \dot{q}^{(0)}(t))\dot{q}^{(0)}(t) + g(q^{(0)}(t)) + d(q^{(0)}(t), \dot{q}^{(0)}(t)). \tag{4}$$

The tracking errors are given by $x = q - q^{(0)}(t)$ and $\dot{x} = \dot{q} - \dot{q}^{(0)}(t)$. The error dynamics equations can be expressed as

$$A^{(1)}(t,x)\ddot{x} + C^{(1)}(t, x, 2\dot{q}^{(0)}(t) + \dot{x})\dot{x} + R_1(t,x) + R_2(t,x,\dot{x}) = u^{(1)}, \tag{5}$$

where $A^{(1)}(t,x) = A(q^{(0)}(t)+x)$, $C^{(1)}(t,x,\dot{x}) = C(q^{(0)}(t)+x,\dot{x})$, $R_1(t,x) = (A(q^{(0)}(t)+x) - -A(q^{(0)}(t)))\ddot{q}^{(0)}(t) + (C^{(1)}(t,x,q^{(0)}(t)) - C^{(1)}(t,0,q^{(0)}(t)))\dot{q}^{(0)}(t) + g(q^{(0)}(t)+x) - g(q^{(0)}(t)) + +d(q^{(0)}(t)+x,\dot{q}^{(0)}(t)) - d(q^{(0)}(t),\dot{q}^{(0)}(t))$, $R_2(t,x,\dot{x}) = d(q^{(0)}(t)+x,\dot{q}^{(0)}(t)+\dot{x}) - d(q^{(0)}(t)+x,\dot{q}^{(0)}(t))$, $u^{(1)} = u - u^{(0)}(t)$.

Proposition 2. *Assume that there exist positive reals k_A, k_C, k_g, and k_d such that $\forall(t,x) \in \mathbb{R}^+ \times \mathbb{R}^n$ the following conditions hold*

$$\begin{aligned}
&||A(q^{(0)}(t)+x) - A(q^{(0)}(t))|| \leqslant k_A||p(x)||,\\
&||C^{(1)}(t,x,\dot{q}^{(0)}(t)) - C^{(1)}(t,0,\dot{q}^{(0)}(t))|| \leqslant k_C||p(x)||,\\
&||g(q^{(0)}(t)+x) - g(q^{(0)}(t))|| \leqslant k_g||p(x)||,\\
&||d(q^{(0)}(t)+x,\dot{q}^{(0)}(t)) - d(q^{(0)}(t),\dot{q}^{(0)}(t))|| \leqslant k_d||p(x)||,
\end{aligned} \tag{6}$$

where the function $p : \mathbb{R}^n \to \mathbb{R}^n$ is such that $p(0) = 0$ and $p = (p_1, p_2, \ldots, p_n)^{\mathrm{T}}$, $p_i = p_i(x_i)$ ($i = 1,2,\ldots,n$) are continuously differentiable functions, and the following holds

$$\begin{aligned}
&||p(x)|| = ||p(x+2\pi l)|| \ \forall l \in \mathbb{Z}^n : \ \ l_i = 0 \ \forall i \in J_P, \quad \forall x \in \mathbb{R}^n,\\
&||p(x)|| \leqslant p_{\max} = \text{constant} > 0.
\end{aligned} \tag{7}$$

Also assume that the functions $R_1(t,x)$ and $R_2(t,x,y)$ satisfy the equalities

$$\begin{aligned}
&R_1(t,x) = F(t,x)p(x) \ \ \forall (t,x) \in \mathbb{R}^+ \times \mathbb{R}^n,\\
&R_2(t,x,y) = B(t,x,y)y \ \ \forall (t,x,y) \in \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n,
\end{aligned} \tag{8}$$

where $F : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$ $B : \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$ are some continuous matrix functions.

Remark 2. *Note that the inequalities in (6) with $p(x) = x$ represent the well-known important properties of the robot-manipulators [2]. We generalize these inequalities by introducing a periodic function $p(x)$ satisfying the conditions (7). Thus, we took into account the following property of the robotic manipulators with revolute joints: they are described by the equations with periodic functions of x with period 2π. For example, the motion equation of the pendulum is given by $\ddot{\varphi} + b\dot{\varphi} + a\sin\varphi = cU$, where φ is the angle between the pendulum bar and the vertical axis, $a = g/L$, $b = k/m$, $c = 1/(mL^2)$, L is the pendulum length, m is the pendulum mass, k is the coefficient of the damping torque, U is the torque applied to the pendulum. The function considered as a controller is given by $u = cU$. The aim of the controller is to stabilize the desired motion $\varphi^{(0)}(t)$ of the pendulum. The equality that the feedforward controller satisfies has the form $u^{(0)}(t) = \ddot{\varphi}^{(0)}(t) + b\dot{\varphi}^{(0)}(t) + a\sin\varphi^{(0)}(t)$. The error dynamics equation is given by $\ddot{x} + b\dot{x} + a\sin(x+\varphi^{(0)}(t)) - a\sin(\varphi^{(0)}(t)) = u^{(1)}$, where $x = \varphi - \varphi^{(0)}(t)$. This equation can be represented as follows $\ddot{x} + b\dot{x} + 2a\cos(x/2 + \varphi^{(0)}(t))\sin(x/2) = u^{(1)}$. If one chooses $p(x) = \sin(x/2)$, then the constant k_g of (6) is such as $k_g = 2a$. Note that the function $p(x)$ has a period of 4π, and the function $|p(x)|$ has a period of 2π.*

Construct the feedback term of the controller u as follows

$$u^{(1)} = K\,\mathrm{Sign}\,(\dot{x} + r(x)), \tag{9}$$

where $K \in \mathbb{R}^{n\times n}$ is the constant gain matrix, $\mathrm{lgn}||K|| \leqslant -k_0 = \text{constant} < 0$, the vector function $r : \mathbb{R}^n \to \mathbb{R}^n$ is continuous, $r(0) = 0$, and $r(x) = (r_1(x_1), r_2(x_2), \ldots, r_n(x_n))^{\mathrm{T}}$ $\forall x \in \mathbb{R}^n$, $r_i(y) = r_0 d(p_i^2(y))/dy$ $\forall y \in \mathbb{R}$, $\forall i \in \{1,2,\ldots,n\}$, $r_0 = \text{const} > 0$, the vector function $\mathrm{Sign}\,(x)$ is given by $\mathrm{Sign}\,(x) = (\mathrm{sign}\,(x_1), \mathrm{sign}\,(x_2), \ldots, \mathrm{sign}\,(x_n))^{\mathrm{T}}$ $\forall x \in \mathbb{R}^n$.

Remark 3. *One can easily see that* $r_i(y) = r_i(y + 2\pi l)$ $\forall i \in J_R$, $\forall y \in \mathbb{R}$, $\forall l \in \mathbb{Z}$.

Proposition 3. *There does not exist a vector* $x \in \mathbb{R}^n$ *such that* $R_1(t, x) = 0$ $\forall t \in \mathbb{R}^+$ *and* $p(x) \neq 0$.

By substituting (9) into (6) and using (8), we get the closed-loop system such as

$$A^{(1)}(t,x)\ddot{x} + C^{(1)}(t,x,2\dot{q}^{(0)}(t) + \dot{x})\dot{x} + F(t,x)p(x) + B(t,x,\dot{x})\dot{x} - K\,\mathrm{Sign}(\dot{x} + r(x)) = 0. \quad (10)$$

Note that under Assumption 3 the closed-loop system (10) has the set of all equilibrium positions such as

$$H = \{(y, \dot{y}) \in \mathbb{R}^n \times \mathbb{R}^n : \ p(y) = 0, \quad \dot{y} = 0\}. \quad (11)$$

One can easily see that the set H can be written as

$$H = \{(y, \dot{y}) \in \mathbb{R}^n \times \mathbb{R}^n : \ y_i = 2\pi l_i, \quad y_j = 0, \quad \dot{y} = 0, \quad l_i \in \mathbb{Z}, \quad i \in J_R, \quad j \in J_P\}. \quad (12)$$

Theorem 1. *Consider the closed-loop system (10) under Propositions 1, 2 and 3. Let the positive constants* m_1, m_2, *and* δ_0 *exist such that the following holds*

$$\begin{aligned}
&k_A + k_C + k_g + k_d + r_0 \left\| \left(C^{(1)}(t,x,r(x) - 2\dot{q}^{(0)}(t)) - A^{(1)}(t,x)\frac{\partial r(x)}{\partial x} \right) \frac{\partial p(x)}{\partial x} \right\| \leqslant m_1, \\
&\mathrm{lgn} \left\| C^{(1)}(t,x,\dot{q}^{(0)}(t) - r(x)) + A^{(1)}(t,x)\frac{\partial r(x)}{\partial x} \right\| \leqslant m_2, \\
&m_1 p_{\max} + m_2\delta_0/\lambda_1 < k_0.
\end{aligned} \quad (13)$$

Then, the set (11) of (10) is uniformly asymptotically stable.

Proof 1. *Consider the Lyapunov function candidate* $V = V(t, x, \dot{x})$ *as follows*

$$V(t, x, \dot{x}) = \sqrt{(\dot{x} + r(x))^{\mathrm{T}} A^{(1)}(t,x)(\dot{x} + r(x))}\,. \quad (14)$$

Note that the function $V(t, x, \dot{x})$ *is periodic on* x_i, $i \in J_R$ *with period* 2π. *Note also that the Lyapunov vector function candidate* V *is continuous but not continuously differentiable. We will use the right hand time derivative of* V *along the trajectories of the closed-loop system (10).*

The time derivative of the function $V^2(t, x, \dot{x})/2$ *is given by*

$$\begin{aligned}
V\dot{V} &= (\ddot{x} + \dot{r}(x))^{\mathrm{T}} A^{(1)}(t,x)(\dot{x} + r(x)) + (\dot{x} + r(x))^{\mathrm{T}} \dot{A}^{(1)}(t,x)(\dot{x} + r(x)) = \\
&= (C^{(1)}(t,x,\dot{x} + 2\dot{q}^{(0)}(t)) + B(t,x,\dot{x})\dot{x} + F(t,x)p(x) + \\
&\quad + K\,\mathrm{Sign}\,(\dot{x} + r(x)))^{\mathrm{T}} (A^{(1)}(t,x))^{-1} A^{(1)}(t,x)(\dot{x} + r(x)) + \\
&\quad + \left(\frac{\partial r(x)}{\partial x}\dot{x} \right)^{\mathrm{T}} A^{(1)}(t,x)(\dot{x} + r(x)) + (\dot{x} + r(x))^{\mathrm{T}} \dot{A}^{(1)}(t,x)(\dot{x} + r(x)) = \\
&= (\dot{x} + r(x))^{\mathrm{T}} \left(C^{(1)}(t,x,\dot{x} + \dot{q}^{(0)}(t)) + \frac{1}{2}\dot{A}^{(1)}(t,x) \right) (\dot{x} + r(x)) + \\
&\quad + (\dot{x} + r(x))^{\mathrm{T}} \left(F(t,x) - r_0 \left(B(t,x,\dot{x}) + C^{(1)}(t,x,2\dot{q}^{(0)}(t) - r(x)) + \right.\right. \\
&\qquad\qquad \left.\left. + A^{(1)}(t,x)\frac{\partial r(x)}{\partial x} \right) \frac{\partial p(x)}{\partial x} \right) p(x) + \\
&\quad + (\dot{x} + r(x))^{\mathrm{T}} \left(B(t,x,\dot{x}) + C^{(1)}(t,x,\dot{q}^{(0)}(t) - r(x)) + \right. \\
&\qquad\qquad \left. + A^{(1)}(t,x)\frac{\partial r(x)}{\partial x} \right) (\dot{x} + r(x)) + (\dot{x} + r(x))^{\mathrm{T}} K\,\mathrm{Sign}\,(\dot{x} + r(x)).
\end{aligned}$$

Using Remark 1, one can easily obtain the following estimate

$$\begin{aligned}\dot{V} \leqslant \Bigg\| F(t,x) - r_0 \big(B(t,x,\dot{x}) + C^{(1)}(t,x,2\dot{q}^{(0)}(t) - r(x)) + \\ + A^{(1)}(t,x)\frac{\partial r(x)}{\partial x}\Big) \frac{\partial p(x)}{\partial x}\Bigg\| \|p(x)\|/\lambda(t,x,\dot{x}) + \\ + \operatorname{lgn}\Bigg\| B(t,x,\dot{x}) + C^{(1)}(t,x,\dot{q}^{(0)}(t) - r(x)) + \\ + A^{(1)}(t,x)\frac{\partial r(x)}{\partial x}\Bigg\| V/(\lambda(t,x,\dot{x})^2) + \operatorname{lgn}\|K\|/\lambda(t,x,\dot{x}),\end{aligned} \tag{15}$$

where the function $\lambda(t,x,\dot{x})$ satisfies the following relationship

$$\lambda(t,x,\dot{x})\|\dot{x} + r(x)\| = \lambda_1 V, \quad \lambda_1 \leqslant \lambda(t,x,\dot{x}) \leqslant \lambda_2. \tag{16}$$

From (15) using (13), we get the following inequality

$$\dot{V}(t,x,\dot{x}) \leqslant \frac{m_1}{\lambda(t,x,\dot{x})}\|p(x)\| + \frac{m_2}{\lambda(t,x,\dot{x})^2}V(t,x,\dot{x}) - \frac{k_0}{\lambda(t,x,\dot{x})}. \tag{17}$$

From (17) using (13), we get the following inequality

$$\dot{V}(t,x,\dot{x}) \leqslant -\varepsilon < 0 \quad \forall (t,x,\dot{x}) \in \mathbb{R}_{\geqslant 0} \times \mathbb{R}^n \times \mathbb{R}^n, \tag{18}$$

where $\varepsilon > 0$ is some constant.

Using (18), one can obtain

$$V(t,x(t),\dot{x}(t)) \leqslant V(t_0,x(t_0),\dot{x}(t_0)) - \varepsilon t \quad \forall t \geqslant t_0. \tag{19}$$

From (19) one can easily see that there exists the time moment $t_1 > t_0$ such that

$$V(t,x,\dot{x}) = 0 \quad \forall t \geqslant t_1. \tag{20}$$

Consider now the Lyapunov function candidate $W = W(x)$ such as

$$W = \|p(x)\|. \tag{21}$$

The time derivative of the function $W^2/2$ is given by

$$\begin{aligned}W\dot{W} = p(x)\frac{\partial p(x)}{\partial x}\dot{x} = p(x)\frac{\partial p(x)}{\partial x}(\dot{x} + r(x) - r(x)) = \\ = -r_0 p(x)\left(\frac{\partial p(x)}{\partial x}\right)^2 p(x) + p(x)\frac{\partial p(x)}{\partial x}(\dot{x} + r(x)).\end{aligned} \tag{22}$$

From (22) one can easily obtain the following estimate

$$\dot{W}(t,x,\dot{x}) \leqslant \operatorname{lgn}\left\| -r_0\left(\frac{\partial p(x)}{\partial x}\right)^2\right\| W(x) + \frac{1}{\lambda(t,x,\dot{x})}\left\|\frac{\partial p(x)}{\partial x}\right\| V(t,x,\dot{x}). \tag{23}$$

Using (20) and (23), one can obtain that the estimation of the right hand time derivative of the function $W(x)$ is given by

$$\dot{W}(t,x,\dot{x}) \leqslant \operatorname{lgn}\left\| -r_0\left(\frac{\partial p(x)}{\partial x}\right)^2\right\| W(x) \leqslant 0 \quad \forall t \geqslant t_1. \tag{24}$$

Using Theorem from [13], we get that the positive limit set $\Omega^+(t_0, y_0)$ *of the solution* $(x(t, t_0, x_0, \dot{x}_0), \dot{x}(t, t_0, x_0, \dot{x}_0))^{\mathrm{T}}$ *of (10) is such that the following holds*

$$\Omega^+(t_0, y_0) \subset \{\dot{W}^*(t, x, \dot{x}) = 0\} = \Lambda, \tag{25}$$

where $y_0 = (x_0, \dot{x}_0)^{\mathrm{T}}$, $\dot{W}^*(t, x, \dot{x})$ *is a limiting function to* $\dot{W}(t, x, \dot{x})$.

From (25) one can conclude that the set of all equilibrium points (11) of the closed-loop system (10) is uniformly attractive.

Using Assumptions 1 and 2 note now that the function $V(t, x, \dot{x})$ *satisfies the inequality*

$$\omega_1(\|(r(x), \dot{x})^{\mathrm{T}}\|) \leqslant V(t, x, \dot{x}) \leqslant \omega_2(\|(r(x), \dot{x})^{\mathrm{T}}\|), \tag{26}$$

where ω_1 *and* ω_2 *are Hahn functions.*

Then, using (18) and (26) we obtain that the set (11) is uniformly stable.

Thus, the set of all equilibrium points (11) of the closed-loop system (10) is uniformly asymptotically stable.

3. Solution to the trajectory tracking for a 5-DOF robotic manipulator

In this section, the simulation test for a 5-DOF serial robotic manipulator (see Fig. 1) is illustrated. The robot has one prismatic joint and four revolute ones.

Each link of the manipulator is represented as a solid body. The kinematic pairs of the manipulator are assumed to be single-link, their geometric centers are denoted by O_k $(k = 1, 2, \ldots, 5)$. The centers of mass C_k $(k = 1, 2, \ldots, 5)$ of the links lie on the axes O_kO_{k+1}, the axes O_kO_{k+1} $(k = 1, 2, \ldots, 5)$ are their axes of symmetry. The first base link is vertical, it rotates around O_1O_2, its rotation angle is θ_1. The second kinematic pair allows rotation of the second link around the horizontal axis passing through O_2. The third kinematic pair allows rectilinear movement of the third link along O_2O_4 $(O_3 \in O_1O_4)$, let us introduce the notation for the displacement of the third link $x = d_3 = O_1O_4$. The fourth link can rotate around a horizontal axis passing through O_4 with the angle of rotation θ_4. The fifth link simulating the gripper allows the rotation around O_4O_5, its rotation angle is denoted by θ_5. We will assume that the centers of mass C_k of the links lie on the axes O_kO_{k+1} and these axes are the axes of symmetry of the corresponding links $(k = 1, 2, \ldots, 5)$. Let's introduce the main central axes C_kx_k, C_ky_k, C_kz_k of the links. We will assume that for links 1 and 5 the axes C_1z_1 and C_5z_5 are the axes of symmetry. For links 2, 3 and 4 such axes are C_2x_2, C_3x_3 and C_4x_4 respectively. Let us assume that the axes C_2z_2, C_3z_3 and C_4z_4 are horizontal. The masses of the links are denoted by m_k $(k = 1, 2, \ldots, 5)$, and their main central moments of inertia are denoted by $I_x^{(k)}$, $I_y^{(k)}$ and $I_z^{(k)}$. Accordingly, we have $I_x^{(1)} = I_y^{(1)}$, $I_x^{(5)} = I_y^{(5)}$, $I_y^{(2)} = i_z^{(2)}$, $I_y^{(3)} = I_z^{(3)}$, $I_y^{(4)} = I_z^{(4)}$. Let's introduce the lengths $O_2C_2 = l_2$, $C_3O_4 = l_3$, $O_4O_5 = 2O_4C_4 = 2l_4$, and $O_5C_5 = l_5$.

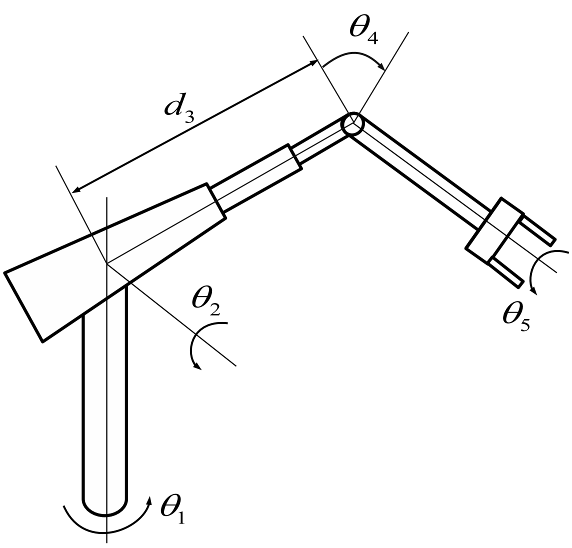

Fig. 1. Model of a 5-DOF robotic manipulator

By using Koenig theorem we can find the kinetic energy T_i of each link $i = 1, 2, \dots, 5$ as the kinetic energy of an absolutely rigid body.

$$
\begin{aligned}
T_1 &= \frac{1}{2} I_z^{(1)} \dot{\theta}_1^2,\\
T_2 &= \frac{1}{2} m_2 l_2^2 (\sin^2\theta_2 \dot{\theta}_1^2 + \dot{\theta}_2^2) + \frac{1}{2}(I_x^{(2)} \dot{\theta}_1^2 \cos^2\theta_2 + I_z^{(2)}(\dot{\theta}_1^2 \sin^2\theta_2 + \dot{\theta}_2^2)),\\
T_3 &= \frac{1}{2} m_3 (\dot{x}^2 + (x - l_3)^2 \dot{\theta}_z^2 + (x - l_3)\sin^2\theta_2 \cdot \dot{\theta}_1^2) + \frac{1}{2}(I_x^{(3)} \dot{\theta}_1^2 \cos^2\theta_2 + I_z^{(3)}(\dot{\theta}_1^2 \sin^2\theta_2 + \dot{\theta}_2^2)),\\
T_4 &= \frac{1}{2} m_4 ((\dot{x} + l_4\dot{\theta}_4 \sin\theta_4)^2 + (x\dot{\theta}_2 - l_4\dot{\theta}_4\cos\theta_4)^2 + (x\cos\theta_4 - l_4\sin(\theta_2+\theta_4))^2\dot{\theta}_1^2 +\\
&\qquad + \frac{1}{2}(I_x^{(4)}\dot{\theta}_1^2\cos^2(\theta_2+\theta_4) + I_z^{(4)}(\dot{\theta}_1^2\sin^2(\theta_2+\theta_4) + (\dot{\theta}_2+\dot{\theta}_4)^2)),\\
T_5 &= \frac{1}{2} m_5 ((\dot{x} + (2l_4 + l_5)\dot{\theta}_4\sin\theta_4)^2 + (x\dot{\theta}_2 - (2l_4+l_5)\dot{\theta}_4\cos\theta_4)^2 +\\
&\qquad +(x\cos\theta - (2l_4+l_5)\sin(\theta_2+\theta_4))^2\dot{\theta}_1^2 +\\
&\qquad + \frac{1}{2} I_x^{(5)}(\dot{\theta}_1\cos(\theta_2+\theta_4) + \dot{\theta}_5)^2 + \frac{1}{2} I_z^{(5)}(\dot{\theta}_1^2\cos^2(\theta_2+\theta_4) + (\dot{\theta}_2+\dot{\theta}_4)^2).
\end{aligned}
$$

The kinetic energy of the manipulator is given by

$$T = T_1 + T_2 + T_3 + T_4 + T_5.$$

The potential energy of the manipulator has the form

$$
\begin{aligned}
\Pi = -m_2 g l_2 \cos\theta_2 - m_3 g (x - l_3)\cos\theta_2 - m_4 g (x\cos\theta_2 + l_4\cos(\theta_2+\theta_4)) -\\
- m_5 g (x\cos\theta_2 + (2l_4 + l_5)\cos(\theta_2+\theta_4)).
\end{aligned}
$$

The motion equations of the manipulator are as follows

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} = -\frac{\partial \Pi}{\partial q} + u,$$

where $q = (\theta_1, \theta_2, x, \theta_4, \theta_5)^{\mathrm{T}}$.

The dynamics for the manipulator can be expressed by (1).

The vector functions $p = p(x)$ and $r = r(x)$ are chosen such as

$$
\begin{aligned}
p &= \left(\sin\frac{x_1}{2}, \sin\frac{x_2}{2}, x_3, \sin\frac{x_4}{2}, \sin\frac{x_5}{2}\right)^{\mathrm{T}},\\
r &= (\sin x_1, \sin x_2, x_3, \sin x_4, \sin x_5)^{\mathrm{T}}.
\end{aligned}
\tag{27}
$$

Choose the reference trajectory of the manipulator as

$$
\begin{aligned}
&q_1^{(0)}(t) = \frac{t}{2}\ \text{rad}, \quad q_2^{(0)}(t) = 2\cos\frac{t}{2}\ \text{rad},\\
&q_3^{(0)}(t) = 0.5 + 0.1\sin\frac{t}{2}\ \text{m},\\
&q_4^{(0)}(t) = 4\cos\frac{t}{2}\ \text{rad}, \quad q_5^{(0)}(t) = 3\sin\frac{t}{2}\ \text{rad}.
\end{aligned}
\tag{28}
$$

We fixed the control gains to $K = -k_0 E$, $k_0 = 600$.

Let the initial position of the manipulator be such as

$$
\begin{aligned}
q_1(0) = -3.1\ \text{rad}, \quad q_2(0) = -2.8\ \text{rad}, \quad q_3(0) = 0.6\ \text{m}\\
q_4(0) = -3.1\ \text{rad}, \quad q_5(0) = 2.9\ \text{rad}
\end{aligned}
\tag{29}
$$

In Figures 2–6, one can see the time evolution of the link positions as well as the references. From the simulation tests, it can be seen that the controller

$$u = u^{(0)}(t) + K\,\mathrm{sign}\,(\dot{q} - \dot{q}^{(0)}(t) + r(q - q^{(0)}(t))) \tag{30}$$

provides the asymptotic stability property of the reference trajectory.

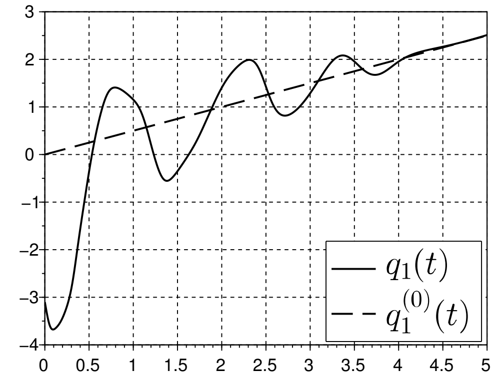

Fig. 2. Time evolution of the first link angular position and reference

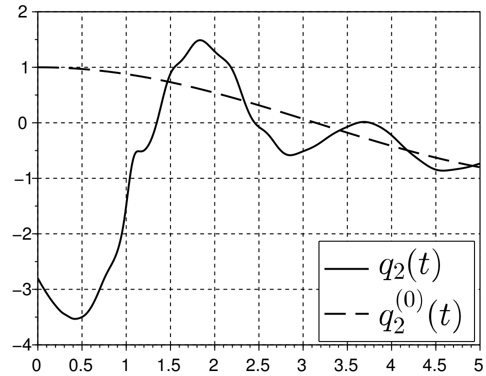

Fig. 3. Time evolution of the second link angular position and reference

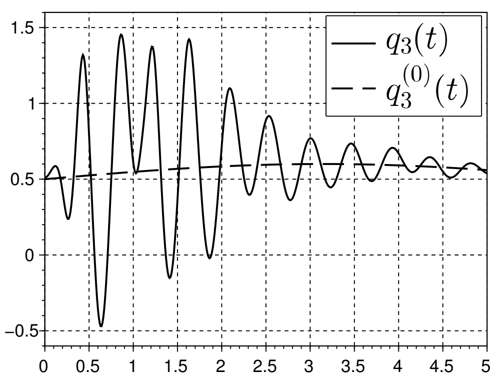

Fig. 4. Time evolution of the third link linear position and reference

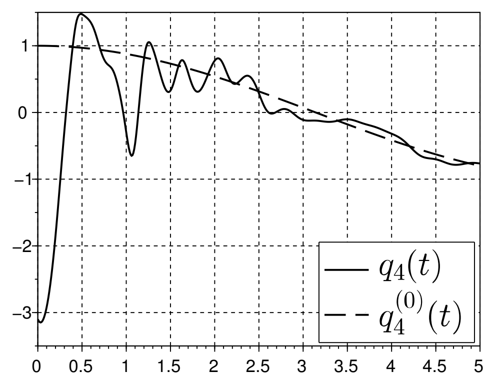

Fig. 5. Time evolution of the forth link angular position and reference

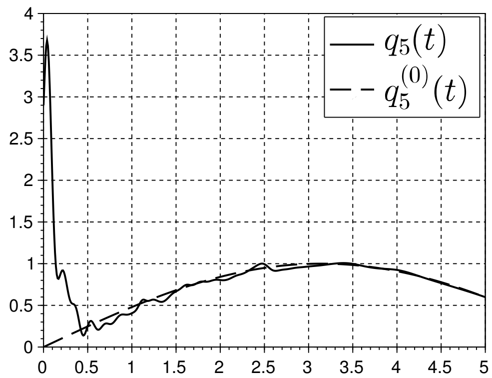

Fig. 6. Time evolution of the fifth link angular position and reference

Conclusions

We consider the trajectory tracking control problem for serial robotic manipulators equipped by revolute and prismatic joints. The generalized coordinates of such mechanical systems include the rotation angles of the manipulator links which makes it possible to consider the trajectory tracking control problem in a cylindrical phase space. The paper proposes and substantiates a technique for constructing a relay control law consisting of the sum of two components: the feedforward term and the sign function of a feedback component. The asymptotic stability property of the set of all equilibrium positions of the closed-loop system is obtained by constructing a Lyapunov function which is periodic in angular coordinates with a semi-definite time derivative and using the principle of quasi-invariance for non-autonomous systems of ordinary differential equations. The numerical simulation of a five-link robotic manipulator demonstrates the good applicability of the controller.

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