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Development of the Russian state in the 20th and 21st centuries: Mathematical modeling based on the socio-energy approach

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Abstract. *Purpose.* The article is devoted to modeling the socio-political development of Russia in 1910–2009 based on the author's socio-energy approach. In this paper, we briefly talk about the basics of the proposed approach, its principles and basic equations. *Methods.* The mathematical model is based on the Langevin diffusion equation. We also introduce the concepts of social energy, coefficients of the state of society and give them definitions. *Results.* Based on its, computer modeling is carried out and the characteristic regularity of the development of society in Russia in 1910–2009 is derived. *Conclusion.* We can notice the spiral pattern of the Russian society in the 20th century. Depending on the density, scale and diversity of events «the flow» of history is accelerating, and the movement in the coil becomes «faster». On its basis, the further development of the Russian society and the state as a whole can be predicted.

Keywords: socio-energy approach, mathematical modeling, nonlinear dynamical systems, socio-political development of Russia.

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Развитие Российского государства в XX и XXI веках: математическое моделирование на основе социально-энергетического подхода

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Аннотация. *Цель.* Статья посвящена моделированию социально-политического развития России в 1910–2009 гг. на основе авторского социально-энергетического подхода. В этой статье мы кратко рассказываем об основах предлагаемого подхода, его принципах и основных уравнениях. *Методы.* Математическая модель основана на уравнении диффузии Ланжевена. Мы также вводим понятия социальной энергии, коэффициентов состояния общества и даем им определения. *Результаты.* На основе данной модели проведено компьютерное моделирование и выведена характерная закономерность развития общества в России в 1910–2009 гг. *Заключение.* Выявлена спиралевидная закономерность развития российского (советского) общества в XX веке. В зависимости от плотности, масштаба и разнообразия событий «поток» истории ускоряется, а движение в отдельном витке становится «быстрее». На основе данного подхода можно прогнозировать дальнейшее развитие российского общества и государства в целом.

Ключевые слова: социально-энергетический подход, математическое моделирование, нелинейные динамические системы, социально-политическое развитие России.

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Introduction

Even though mathematical models are so widely used in the natural sciences, they are rather rare in sociological, political, and historical research. However, in recent years it has successfully developed the models of social and political history [1]. The models available to date can be divided into three groups.

1. Conception models, which are based on the identification and analysis of the general historical patterns and present them in the form of cognitive schemas describing the logical connection between different factors affecting the historical processes (G. Goldstein, I. Wallerstein, L. N. Gumilev, N. S. Rozov, etc.). These models have a high degree of generalization, but do not have a mathematical and purely logical, conceptual nature [1, 2].
2. Special mathematical models of simulation type, devoted to the description of specific historical events and phenomena (Y. N. Pavlovsky, L. I. Borodkin, D. Medouz, Forrester, et al.). These models focus on careful consideration and description of factors and processes that influence the phenomenon under consideration. Such models are usually applied in a rather narrow space-time interval: they are «tied» to a particular historical event and cannot be extrapolated over a period of time [1–3].
3. Mathematical models that are intermediate between the types mentioned above. Those models describe a certain category of social processes without a detailed description of the peculiarities

of each historical event. The task of such models is to find basic mechanisms characterizing the processes flow of the examined type. According to this, such models are called basic [2].

Modeling the dynamics of nonlinear systems in the classical models [3–10] is based on the use of multidimensional differential equations [7, 11], difference equations [12–14], mathematical apparatus of cellular automata [13, 15], mathematical apparatus of catastrophe theory [16, 17], mathematical apparatus of the theory of self-organized criticality [18, 19], stochastic differential equations of Langevin and Ito-Stratonovich [3, 8, 20, 21], analysis of systems with chaos and reconstruction of stable states (attractors) time series [13, 15].

These models are most often used to solve local partial problems and are poorly applicable to modeling complex social distributed systems. The reason for this lies in the complexity of social and historical processes modeling, as well as the weak formalization of many concepts and factors of social evolution [22].

The socio-energy approach is based on a systematic approach and view of the state system in terms of energy. This view allows us to represent the intra and extra-systematic processes such as modification or redistribution of energy within the system and between systems. Introduction of the concept of «social energy» or simply «energy» (E) is also possible. Here, this concept means the quantity that characterizes the potential of the social system to do the work. Attempts to introduce such a concept made before, but with no further use for the creation of a mathematical model, limited to generalities [22].

The practical application of such models is most effective in predicting socio-political processes in the context of sharp changes in the structure of the social system (conflicts, protests, crises, etc.)

1. Socio-energy approach

Social energy is a parameter similar in meaning to the physical concept of potential «energy» in physics. However, it includes not only the energy of resources (from the standpoint of their influence on the social system), but human labor, intellectual potential, etc. These concepts are hard to formalize and parameterized using classical physical quantities, but they have a significant impact on social systems. Therefore, the introduction of a new parameter was required. In addition, social energy will make it possible to represent social and political processes within the system as a redistribution of energy.

1.1. Basis of the mathematical model. This view allows you to represent the intra and extra-systematic processes such as modification or redistribution of energy within the system and between systems. The main principles of the system approach are also used [23]. Let's write down the energy of the system:

$$\sum_{i=1}^{n} E_i = E_\Sigma \,.$$

Of this, we get the model on the basis of a system of differential equations:

$$\sum_{i=1}^{n} \overrightarrow{P}_i = \overrightarrow{P_\Sigma} \,,$$

where

$$\overrightarrow{P_\Sigma} = \overrightarrow{\chi} \frac{dE_\Sigma}{dt} \,. \qquad (1)$$

Thus, we have obtained the equation for the change in the flow of energy in time. In fact, this is a physical quantity of «power», which is determined through the created work per unit of time. $\overrightarrow{\chi}$ – unit vector of the energy flow direction.

Next, we introduce two types of social energy, which include other subspecies. Material system energy

$$E_m = f\left(E_m^{sc},\ E_m^{\Sigma h}, K_d, K_{si}\right),$$

where:

E_m^{sc} – energy resources (if any) of the social system and its material property;
$E_m^{\Sigma h}$ – savings and ownership of material living (existing) energy in the social system of people;
$K_{sc} = (\overrightarrow{a}, I_I, K_d, K_s)$ – rate of scientific and technological progress and development of the system;
$\overrightarrow{a}$ – set of parameters that define the scientific and technological progress in the system;
I_I – transfer function of the inter-system data exchange;
K_d – coefficient of the manager in the system;
I_I – information permeability of the system (determines the speed of information transmission and its possible losses);
$K_s = (\overrightarrow{\beta}, I_I, K_d, N)$ – factor of social activity and the moral state of society;
K_s and K_{sc} – these coefficients can be determined for each individual and subsystem separately; the general coefficient for all systems can be obtained in different ways, for example, using a fractal transformation;
N – number of individuals in the social system;
$\overrightarrow{\beta}$ – set of parameters that define the spiritual and moral development and moral state of society.

And the energy of human labor composing the social system

$$E_h = f\left(E_h^{\Sigma}, K_d, K_{sc}\right),$$

where E_h^{Σ} – total energy labor of the system members depends on N.

As can be seen from above, we suggest the following formula:

$$\overrightarrow{P}_{\Sigma} = \overrightarrow{P}_{\Sigma}^{\,m} + \overrightarrow{P}_{\Sigma}^{\,h} + \overrightarrow{P}_{\Sigma}^{\,\text{out}}$$

$\overrightarrow{P}_{\Sigma}^{\,\text{out}}$ – external influence on the system;
$\overrightarrow{P}_{\Sigma}^{\,m}$ – the flow of material energy through the system;
$\overrightarrow{P}_{\Sigma}^{\,h}$ – the flow of human energy through the system.

Hence, using (1)

$$\overrightarrow{j}\frac{dE_m}{dt} + \overrightarrow{k}\frac{dE_h}{dt} + \overrightarrow{\gamma}\frac{dE_{\text{out}}}{dt}.$$

As a result, we get –

$$\overrightarrow{P}_{\Sigma} = \overrightarrow{j}\left(\frac{dE_m{}^{\Sigma h}}{dt}K_dK_{sc} + \frac{dE_m{}^{sc}}{dt}K_dK_{sc}\right) + \overrightarrow{k}\left(\frac{dE_h{}^{\Sigma}}{dt}K_dK_{sc}K_s\right) + \overrightarrow{\gamma}\left(\frac{dE^{\Sigma}{}_{\text{out}}}{dt}\,\xi(K_dK_{sc}K_sI_I)\right).$$

Or without external influence:

$$\overrightarrow{P}_{\Sigma} = \overrightarrow{j}\left(\frac{dE_m{}^{\Sigma h}}{dt}K_dK_{sc} + \frac{dE_m{}^{sc}}{dt}K_dK_{sc}\right) + \overrightarrow{k}\left(\frac{dE_h{}^{\Sigma}}{dt}K_dK_{sc}K_s\right).$$

The final equation is the main formula for calculating the flow of social energy through the system (without taking into account external influence or the equation above – taking into account external influence).

1.2. Calculation of coefficients. We now try to build a model basing on the socio-energy approach. Let us suppose that we have a system with n-number of components (or members). How the interactions inside such a system will be held and how we can calculate the sum coefficients K_s and K_{sc}, if we know the parameters of each component (and each component has its own set of K_s and K_{sc}, E_h and E_m)?

It's necessary to pay attention energy is additive, but rates (coefficients) are not. Those the coefficient of social activity of the system will not be equal to the sum of the coefficients of separate individuals.

In order to simplify we will take a special case for $n = 6$. Now we consider the creation of internal energy flow and the addition of the coefficients inside such a system. To facilitate the submission of such a system we will present it analog among real-life social systems, for example, a small business with 6 employees engaged in it (including the head or leader). See Fig. 1.

Let us assume that we have three employees working in one area, the other two – in the other, and the sixth member of the system – the general director of the business.

Accordingly, the labor energy of the first three persons will converge in a nodal point, as they operate in virtually the same direction, which means they have a common energy flow (bad or good quality of work is defined by their personal factors) is directed to one side (Fig. 1, the components 1, 2 and 3). The nodal point of the system is called a contingent intersection of interests or «directions» of the labor of members, as a result, the flow of social energy of their action is directed identically. That is, for example, the design team engaged in the resolution of one project problem, respectively, their total work is summed, their energy flux vectors are also added, as the work is carried out in one direction. The same is true for the other two members working in the same area.

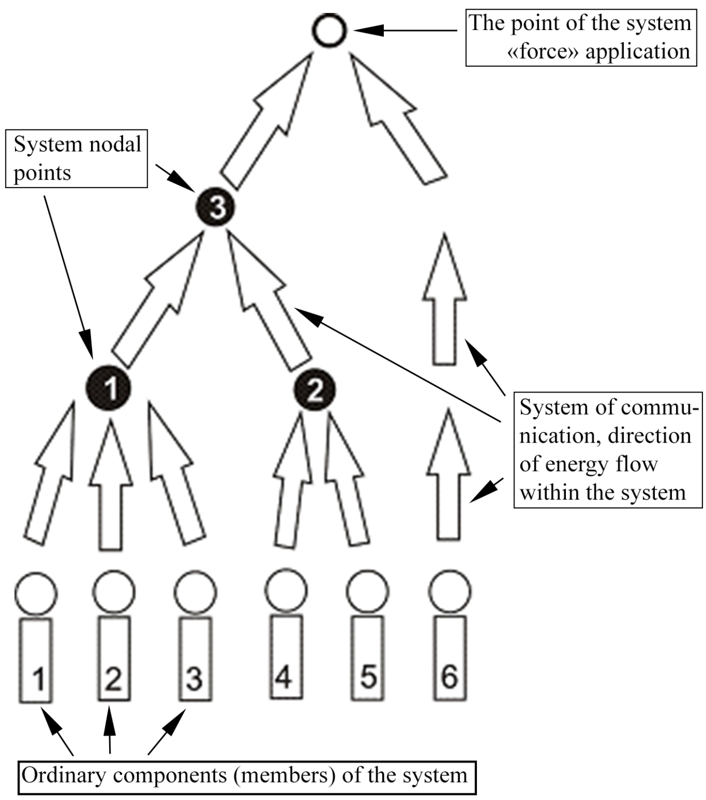

Fig. 1. Social system model from the socio-energy flow perspective (Special case for 6 components)

Since we are talking about the system, there is a hierarchy, in which smaller groups are engaged in narrow objectives, which are summarized in more general. As well as individual members of the system, they are at their level when matching tasks are summarized (Fig. 1 nodal points 1 and 2). For example, when a new car is being developed in a large enterprise the work of separate departments engaged in production of parts is added, respectively, to a common cause, which is the new car.

The leader is included in the total flow at a level higher than the part of the system responsible before him. Since points 1 and 2 do not have determined supervisors or leaders (simply because of the small magnitude of the system), then the member no. 6 is responsible for the overall system guidance. Accordingly, he/she is engaged in the process at the very last stage; the leader energy flow and his\her coefficients determine the final performance of the entire system (Fig. 1).

The last nodal point of the system is called the point of the system «force» application, as part of defining the final direction of the flow of energy distribution. The word «force» is in quotation marks because in this case, it is a relative concept that does not take into account its physical sense, but rather an intuitive one, to convey the meaning of this part of the system.

It is easy to note that even if you count the ratio of the total sum of the components as an arithmetic mean through the nodal points (i.e. first of all, the arithmetic mean of the nodal points, then among them and the components that are included at a higher level, and so on to the highest level), it will depend heavily on managers, especially of high level. High parameters of the system, in general, can be substantially reduced if its leader does not correspond to the level required on the last level (in this case, he/she is equivalent to the rest of the system when calculating the arithmetic mean). We can see some examples from our real life when unscrupulous heads of enterprises, governments, business or any other structure can spoil all the work of their subordinates with an unlettered decision or desire to get self-profit.

It is obvious that the system develops in clusters and its fractal structure is manifesting, where each cluster has its subsystem with its parameters, and by increasing the number of components, the self-similarity of the picture becomes obvious.

How we can demonstrate it mathematically? We calculate the rate for such a system for 6 components. Where:

n_i = 3 (these are the 1st, 2nd and 3rd components);
n_λ = 2 (the 4th and 5th components);
n_ζ = 1 (the 6th);
n_j = 2 (the 1st and 2nd nodal points);
n_l = 1 (the 3rd nodal point ant the head\leader, i.e. the 6th component).

Then through the rows, we can calculate the system arithmetic average taking into account the nodal points:

$$A = \frac{\left(\frac{\sum_{i=1}^{n_i} k}{n_i} + \frac{\sum_{\lambda=1}^{n_\lambda} k}{n_\lambda}\right)/n_j + \frac{\sum_{\zeta=1}^{n_\zeta} k}{n_\zeta}}{n_l}, \qquad (2)$$

where:

k is a variable, equal to the value of the corresponding component or nodal point, while calculation inside the cluster (each represented by a row);
n – the number of elements;
i, j, λ, ζ, l – element number;
k – individual coefficient of the individual.

For the calculation of systems with a large number of components a greater number of rows is required as well as an increase in the size of the formula, but due to self-similarity and monotony of the rows calculations, such formulas are calculated through the computer programs and does not pose a serious difficulty. A much more serious obstacle could be the determining factors of individual members of the system.

1.3. Model. Let us suppose that we have a social system A, with a determined distribution of coefficients K_c and K_{sc} (each individual i has a corresponding coefficient k_i). We will try to write down the equations describing the change in the parameters of the system in time.

Holyst J. A., Kasperski K., Schweiter F. [12] offered a suitable model of public opinion bases on the representation of interaction between individuals, in the form of Brownian motion. We have improved this model so that it can be effectively used for the selected case. The individuals are involved in this process by interacting through the communication field $h_k(x,t), x \in S \subset \mathbb{R}^2$.

This field simulates information processes in a public system. It takes into account that in the modern informational measure it is not necessarily physically close to the object of communication, that there is a certain parameter of «information permeability» in the system.

Thus, the distance between objects in this model will speak about their possibility for information communication, and not about the physical concept of distance (which leads us to the need for a new definition of classical coordinates, taking into account the social position of the individual).

Spatial and temporal variation of the communication field is considered by the equation (based on the heat conduction equation):

$$\frac{\delta}{\delta t}h_k(x,t) = \sum_{i=1}^{N} f(k_i,k_n)\delta(x-x_i) + D_h\Delta h_k(x,t) \tag{3}$$

$\delta(x-x_i)$ – Dirac δ-function;

$f(k_i,k_n)$ – function that determines the strength of influence of the individual on the other specific individual, depending on their coefficients;

N – the number of individuals;

D_h – the diffusion coefficient, characterizing the distribution of the communication field.

Each individual in the point x_i, constantly contributes to the development of the field $h_k(x,t)$ in accordance with the values of their coefficients (the level and radius of influence of the individual on others also depends on these coefficients).

The field $h_k(x,t)$ influences the individual i in the following way. While being in the point x_i, the individual gets into the influence of the communication filed of another individual (or several of them). Depending on its difference from the coefficients and the coefficients of individuals acting on it, i-individual may react in the following ways.

1. He/she can change the value of his/her coefficients while being under influence of other individuals.
2. Move towards the area where the difference of the coefficients is relatively minimal in the moment of action.

Let us suppose that $p_{ij}(k_i,k_j,t,x_i,x_j)$ – the probability of impact on the communication field of i-individual (or the whole cluster of individuals) on j in such a way, that can change the coefficients K_c and K_{sc} (together or separately) in the time period t. Then, the probability of the move of the individual-i to the field with relatively minimal coefficients difference in the moment of action is $1-p_{ij}(k_i,k_j,t,x_i,x_j)$.

Then the change of given probability is the following:

$$\begin{aligned}\frac{d}{dt}p_{ij}\left(k_i,k_j,t,x_i,x_j\right) = \sum_{k'_i}\upsilon\left(k_i \mid k'_i\right)p_{ij}\left(k'_i,k'_j,t,x_i,x_j\right)\vartheta\left(\Delta x_{ij}\Delta k_{ij}\right) - \\ - p_{ij}\left(k_i,k_j,t,x_i,x_j\right)\sum_{k'_i}\upsilon\left(k'_i \mid k_i\right)\vartheta\left(\Delta x_{ij}\Delta k'_{ij}\right),\end{aligned} \tag{4}$$

where $\vartheta(\Delta x_{ij}\Delta k_{ij})$ – parameter, characterizing induction influence of the communication field, $\upsilon\left(k'_i \mid k_i\right)$ is conditional probability of coefficient change per time unit

$$\upsilon\left(k'_i \mid k_i\right) = \begin{cases} k_i \neq k'_i \to \eta\exp\left\{\left[h_{k'_i}\left(x_i,t\right) - h_{k_i}\left(x_i t\right)\right]/Q\right\}, \\ k_i = k'_i \to 0, \end{cases} \tag{5}$$

Q – is parameter that proposes degree of freedom in chosen space, from the point of view of the possibility of transferring information from an individual to an individual.

By analogy with the specified model [12] the movements of individuals is described by the Langevin equation:

$$\frac{dx_i}{dt} = k_{i\vartheta}(\Delta x_{ij}\Delta k_{ij})\nabla_x h_{\sum}(x_i,t)\big|_{x_i} + \sqrt{2D_n}\zeta_i(x_i,t)\,. \quad (6)$$

Where D_n is the spatial diffusion coefficient of individuals, $h_{\sum}(x_i,t)$ is the resulting field of communication, that impacts on individual-i.

Random fluctuations and impact are modeled through the stochastic force $\zeta_i(x_i,t)$, in such ways that ζ_i is flat noise, which also depends on the location of the individual, (this function takes into account that the influence of various factors at different points in the system is different) with

$$\langle\zeta_i\left(x_i,t\right)\zeta_i\left(x_i,t'\right)\rangle = \delta_{ij}\delta\left(t-t'\right).$$

The developed model allows you to explore different states of the social system in different conditions, with different types of parameterization (based on statistical data), taking into account external influences [12].

2. Computer modelling

One of the first tasks set for the simulation is the assessment of the Russian society historical development based on the socio-energy approach.

Modeling was based on the fundamental equation of socio-energy flow 5 and reflects the change of the coefficients K_{sc} and K_s over time from 1910 to 2009. Evaluation factors were based on the statistics mentioned above and historical and political, historical, and social analysis conducted by the authors.

Fig. 2 represents the graph illustrating the development of society from 1910 to 2009.

Since the statistics cover the specific time periods, it was necessary to distinguish the 12 points in which either the data density was highest or this point was prominently important to demonstrate the general dependence.

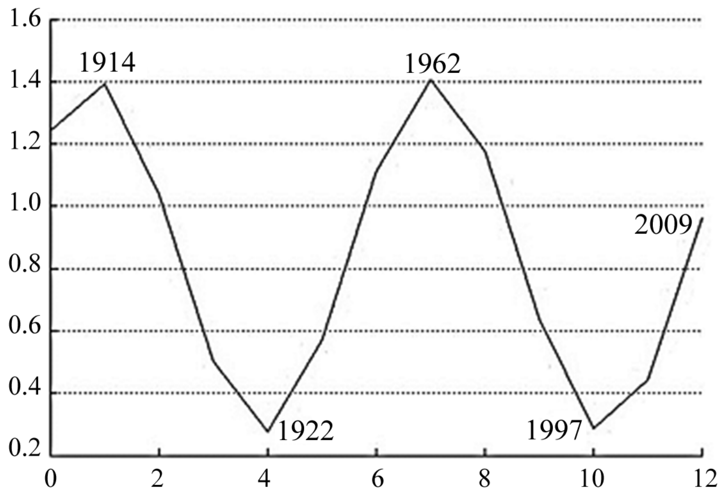

Fig. 2. The axis of graphic «y» – $K_{\sum}$ – the coefficients sum K_s+ K_{sc}. The axis of graphic «x» – T – time point reading scale

It is important to emphasize once again that the coefficients K_s and K_{sc} are values relative to its time period (i.e. 1 is arbitrarily maximum rate for the world at that time, 0 is the minimum).

Respectively if the value is, for example, $K_s = 0.5$ in the time point T_1 and $K_s = 0.5$ in the time point T_2, it does not mean that in the time point T_1 the country had the progress as if in the time point T_2. It demonstrates that in the time point T_1 the country was at the same relative level of development, as at the time point T_2.

Fig. 2 demonstrates certain peaks and valleys, showing moments of the highest and lowest total performance factors. It is important to note that these ratios reflect the state of society, but not the total energy power of the state. The state still can exceed the first (though, of course, the opposite is also possible).

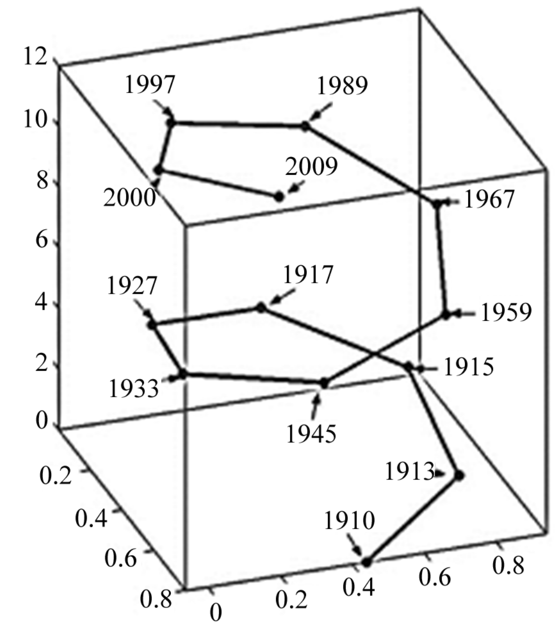

Fig. 3. The axis of graphic «y» – K_s, The axis of graphic «x» – K_{sc}, The axis of graphic «z» – T

These parameters, of course, have a direct impact, but a small state can also obtain high performance ratio, while a large low one, but the general flow of energy is significantly superior. Thus, the schedule reflects the relative for the time efficiency of the use of the state resource (social power) at any given moment in terms of its social and scientific development. Points marked with the years 1914 and 1962 are the peaks in terms of approach. So, in 1914, Russia, after the 1910–1913 industrial boom was quite high relative to the world rate K_s.

At the same time, the begin of World-War I caused the raise of patriotic enthusiasm in a large part of the society, which in turn briefly raised K_{sc}. In 1962, the Soviet Union was at the peak of scientific and military competition, as well as made a number of significant achievements in space sphere; the state had significant belief in the correctness of the chosen path to communism, confirmed by the victory in the Great Patriotic War.

Fig. 3 was built for better visualization of the same dependence, however with separate coefficients K_s and K_{sc}. Accordingly, the fall is in the point 1922 that comes right after the revolution, civil war, war communism politics, when the country was in ruins, and the 1997 pre-default year, when in the first few years of democratic reforms all the achievements of socialism were demolished, i.e. the majority of factories were closed and mass migration out of the country was triggered.

Conclusions

For this graph, we can notice the spiral pattern of the Russian society in the 20th century. Of course, the scale in this figure is not saved, because the time scale is the scale of conditional discrete time points, but the general trend is obvious. Depending on the density, scale and diversity of events «the flow» of history is accelerating, and the movement in the coil becomes «faster». For instance, within 7 years (1910–1918), half of the first turn was passed and the second lasted for 22 years (1918–1940).

Of course, statistics and the result of their analysis can both have significant errors (which depends primarily on the reliability and objectivity of statistical information), but the overall pattern is quite easily noticed and visible. On its basis, the further development of the Russian society and the state as a whole can be predicted.

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