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SYNCHRONIZATION: FROM SIMPLE TO COMPLEX

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Introduction

It would not be too much of an exaggeration to say that oscillations are one of the main forms of motion. They range from the periodic motion of planets to random openings of ion channels in cell membranes. They are observed at various levels of organization, have various origins and various properties. Since Newton's crack at the three-body problem and until just a few decades ago, the range of phenomena regarded as oscillations were limited to damped, periodic and quasiperiodic oscillations at best. A significant achievement of the second half of the 20th century is the admission of deterministic chaos and noise-induced rhythms as equals into the oscillation family.

Nature is not based on isolated individual systems. It is rich in connections, interactions and communications of different kinds that are complex beyond belief. With this, synchronization is the most fundamental phenomenon associated with oscillations. It is a direct and widely spread consequence of the interaction of different systems with each other. In most general terms, synchronization means that different systems adjust the time scales of their oscillations due to interaction, but there is a large variety of its manifestations and of the accompanying fascinating phenomena.

Anyone writing a book on synchronization is faced with two problems: on one hand, one has to deal with a huge amount of material on the particular aspects and effects; and on the other hand, there is a need to formulate a universal approach that would embrace all the particular cases. Fortunately, an essential contribution to the second problem has been made by Pikovsky, Rosenblum, and Kurths in their recent book (A. Pikovsky, M. Rosenblum, J. Kurths, Synchronization: A Universal Concept in Nonlinear Sciences, Cambridge University Press, Cambridge, 2001)*, that has provided a contemporary view on synchronization as a universal phenomenon that manifests itself in the entrainment of rhythms of interacting self-sustained systems. This viewpoint is in agreement with the approach developed since the time of Huygens, and is completely shared by ourselves. In writing the present book we were motivated by the following considerations:

• Recently, a large variety of new synchronization phenomena were discovered that are inherent in complex (chaotic) systems, but do not occur in simple periodic oscillators. With the modern fascination for the beauty and the complexity of the new effects, there is a tendency to forget about the basic phenomena and theoretical results associated with «simply» periodic oscillations. This is largely due to the fact that not all involved in the studies of these phenomena, and especially younger researchers and students, have the respective education. It turns out to be difficult to recommend a book, which would consistently present, equation after equation, the most fundamental theoretical results on synchronization. Without such background, it is problematic to analyze the synchronization of irregular oscillations from the general viewpoint, and to avoid discovering «new» effects that often appear to be merely manifestations of the general principles in a particular situation.

• There is a number of fascinating aspects of synchronization (phase multistability, dephasing, self-modulation, etc.), that are observed in a variety of systems and with various types of interaction, that have not been discussed yet in the framework of the general concept of synchronization.

In order to cover the above problems, our book contains two parts. The first part is a consistent and detailed description of the classical approach to forced and mutual synchronization that is based on frequency/phase locking and suppression of natural dynamics. It is oriented to the people not familiar with the fundamental results of synchronization theory obtained by a number of physicists and mathematicians, such as B. van der Pol, A.A. Andronov, A.A. Vitt, M.L. Cartwright, A.W. Gillies, P.J. Holmes, D.A. Rand, R.L. Stratonovich, V.I. Tikhonov, P.S. Landa, D.G. Aronson and co-authors, and published in their original works. It was our aim:

• To reproduce in every detail the derivations of the most fundamental results, which until now were given only schematically and presented a significant challenge for beginners because of the traditional brevity typical of the scientific works of the beginning and middle of the 20th century. We have made every effort to make the reading easy for non-experts, to reduce to the minimum the need to refer to other literature when following the calculations or the description of geometrical effects, and to exclude expressions like «It is easy to show». As a result, the lengths of the respective sections have increased substantially as compared to those in the original books and papers, but we believe it was worth doing this and hope that the readers will find this material helpful.

• To describe the same phenomena using different languages: the ones of physics and of mathematics. In the early experiments on synchronization, the latter was detected by means of listening to the volume of sound (organ pipes), visually observing the positions of pendulums (clocks), and later Lissajous figures and Fourier power spectra on the oscilloscopes (electric circuits). Thus, synchronization can be naturally understood in physical terms like power, frequency or phase. On the other hand, the systems that synchronize can be described by non-linear mathematical equations. Transitions that occur in coupled systems when their parameters change, can be

*А. Пиковский, М. Розенблюм, Ю. Куртц. Синхронизация. Фундаментальное нелинейное явление. Москва: Техносфера, 2003. 496 с. (*Прим ред.*)

described in mathematical terms of bifurcation and stability theory. In this book we will analyze the phenomena of synchronization and the associated effects using both languages and making a clear connection between these different means of description.

• To generalize theoretical results to complex oscillations. An important achievement of modern oscillations theory is the recognition of the role of irregular oscillations that can be either deterministic or stochastic. We start by considering synchronization in simple periodic oscillators. Then we move to chaotic and stochastic oscillations and show that in spite of their complexity, they can synchronize according to the same mechanisms as periodic ones.

We will deem to have achieved our goal, if after reading this part the reader will be convinced that very different types of oscillations obey the same mechanisms of synchronization, although the particular manifestations can be different.

The second part is devoted to the general mechanisms and principles of synchronization, describing them with regard to the non-linear properties of the particular classes of systems and couplings. We discuss synchronization of anisochronous oscillations, when fast and slow motions along the trajectory give arise to additional phase-shifted coexisting regimes and thus change the bifurcational structure of the synchronization region. A separate chapter is devoted to the concept of phase multistability and its development in the systems that oscillate with complex waveform (essential for period-doubling and self-modulated oscillations) and have a particular structure of their phase space. The latter might include regions of fast and slow motion, closeness of the trajectories to some singular points, etc. (essential for bursting behavior). The concept of synchronization is extended to the systems with several time scales of either deterministic, or stochastic origin. Finally, we consider cooperative behavior of systems with a particular type of coupling through the primary resource supply and discuss their applications.

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