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SMALL-WORLD NETWORKS: DYNAMICAL MODELS AND SYNCHRONIZATION

Vladimir N. Belykh, Igor V. Belykh, and Martin J. Hasler

This paper provides a short review of recent results on synchronization in small-world dynamical networks of coupled oscillators. We also propose a new model of small-world networks of cells with a time-varying coupling and study its synchronization properties. It is shown that such a time-varying structure of the network can ensure more reliable synchronization than the conventional small-worlds. The term «small world» refers to a network of locally connected nodes having a few additional long-range shortcuts chosen at random. The addition of the shortcuts sharply reduces the average distance between the nodes and therefore provides the so-called small-world effect. Discovered first in social networks, the small-world effect appeared to be a characteristic of many real-world structure both human-generated or of biological origin. For social networks, this property implies that almost any pair of people in the world can be connected to one another by a short chain of intermediate acquaintances, of typical length about six. However, the structure of social networks is not homogeneous, there are always key persons that provide distant out-local-world connections between people. This paper is written in honor of the 60th birthday of our friend and colleague, Wadim S. Anishchenko, who is one of such key persons in the Nonlinear Dynamics community.

1. Introduction

The study of networks pervades all of science, from physics and neurobiology to engineering and social sciences. From the perspective of nonlinear dynamics, we would like to understand how a huge network of interacting dynamical systems be they neurons, computers connected in Internet or power stations will behave collectively, given their individual dynamics and coupling structure [1]. This paper contributes to elucidate the relation between the network dynamics and graph theory and to apply mathematical theory of synchronization to networks of different nature. Ordinarily, the connection topology is assumed to be either completely regular or completely random. However, many biological, technological, and social networks lie somewhere between these extremes. In 1998, Watts and Strogatz found a simple model of networks that can be tuned through this middle ground: regular coupled networks with the addition of increasing amounts of disorder (a few additional randomly arising connections). These coupled systems were called «small-world» networks [2], by analogy with the small-world phenomenon.

This famous phenomenon was discovered in 1967 by the american social psychologist Milgram [3]. He performed a simple experiment as follows. He sent roughly 300 letters to randomly selected people in Omaha, Nebraska with the instruction to get the letter to a single «target» person in Boston using only personal contacts. Milgram gave each «sender» some information about the target including name, location, and occupation, so that if the sender did not know the target (and it was extremely unlikely that they would), they could send the letter to someone they did know who they thought would be «closer» to the target. Thus began a chain of senders, each member of the chain attempting to zero is on the target by sending the letter to someone else: a friend, family member, business associate, or casual acquaintance. Milgram's surprising finding was that for the 60 chains that eventually reached the target, the average number of steps in a chain was around six, a result that has entered folklore as the phrase «Six degrees of separation». From this experiment, Milgram concluded that six was the average number of acquaintances separating any two people in the world. Given the form of Milgram's experiment, one could be forgiven for supposing that the figure six is probably not a very accurate one. The experiment certainly contained many possible sources for errors. However, the general result that two randomly chosen human beings can be connected by only short chain of intermediate acquaintances has been subsequently verified, and is now widely accepted. This small-world property of social networks, that the average distance between the nodes is relatively short, has been shown to be widespread in many other real-world structures including the WWW connections [4], scientific networks [5], epidemiological models [6], electrical power grid [7], electronic circuits [8] and neural and biochemical networks [9,10].

The semi-random model of Watts and Strogatz, that reproduces remarkably well main characteristic of many real-world networks, is the following. It starts from a ring lattice with n vertices (the pristine, original, world), each node is connected to its $2k$ nearest neighbors (periodic boundary conditions are applied just for convenience and not strictly necessary). Then shortcuts links are added between random pairs of nodes with probability p per link. Watts and Strogatz conjectured that dynamical systems coupled in this way would display enhanced propagation speed, synchronizability and computation power, as compared with regular lattices of the same size [1]. The intuition is that the short path could provide high-speed communication channels between distant parts of the system, thereby facilitating any dynamical process (like synchronization or computation) that requires global coordination and information flow.

This model has been the subject of significant recent interest within the physics, mathematics, and engineering community. Most theoretical studies were concerned with statistical and combinatoric properties of small-world networks (graphs) where the cells do not have the individual temporal dynamics [1]. Dynamical processes on small-world networks were studied relatively little and mainly by means of computer simulation [7,10-12]. In particular, it was numerically shown that small-world connections may essentially improve synchronization properties of networks of limit-cycle and chaotic oscillators. In turn, synchronization in networks of periodic and chaotic oscillators with different regular and random coupling configurations has been intensively studied [13-21].

More recently, significant progress in the study of the relation between the addition of random shortcuts and the synchronization properties of networks was made by Barahona and Pecora [22]. They applied the Master Stability function approach [18] to the study of local synchronization in small-world networks and showed, through numerics and analysis, how the addition of random shortcuts improves network synchronizability. The connectivity matrices G were once chosen at random and then fixed forever. This is the usual approach of defining the small-world networks. Within this approach, statistics of the connectivity matrices G was translated into statistics of the synchronization thresholds.

In this paper, we propose a new model of dynamical small-world networks where the shortcuts change as a function of time [23]. Instead of randomly choosing the shortcuts and leaving them fixed, we randomly choose the shortcuts, leave them only for an interval of time τ fixed, then randomly choose another set of shortcuts, leave them again for a lapse of time τ fixed, etc. More precisely, our probabilistic model is the following. During each time interval of length τ, every possible shortcut is turned on with probability p, independently of the switching on and off of the other shortcuts, and independently of whether or not it has been turned on during the previous time interval. Furthermore, we assume that the switching time τ is small with respect to the intrinsic time constants of the dynamics of the individual cells.

This way of transforming a network with fixed couplings, the «pristine world», into a time-varying small-world network can always be applied. We call it the «blinking model» [23]. In this paper, we shall concentrate on global synchronization in the important example where the pristine world is a ring of $2k$-nearest neighbor coupled chaotic oscillators. The methods developed here, however, are more generally applicable.

The blinking model is actually of practical importance. In practice, often collections of subsystems that are organized into a network actually interact only sporadically. This is true in biology as well as in technology. Neurons in the brain send out electrical signals in the form of spikes and most of the interaction with the other neurons takes place during the arrival of the spikes at the connection points, the synapses. Since the spike duration is usually small with respect to the interspike intervals, this is an important example of «blinking» interaction. Of course, here the occurrence of spikes of different neurons and at different times are not just independent random variables, and the spike durations are actually caused by the dynamics of the individual neurons. Nevertheless, the distant node interaction is of intermittent nature.

In technology, practical systems exist that can be modelled rather precisely by the blinking model. Packet switched networks such as the Internet are an important example. Dynamical processes in the computers that are networked through Internet interact by sending messages that are subdivided into packets and sent over the network. Both in the network links as in the computers themselves, they have to share the available communication time slots with many other packets that belong to communications between different computers and/or different processes. The occurrence of the other packets can be considered as independent, and the timeslots available for the communication between specific processes can also often been considered independent due to the congestion of the links by the other packets. Thus, the blinking model may be appropriate in many different situations.

The model of small-world networks, that we propose, consists of the pristine world (the regular locally coupled lattice of oscillators) and time-dependent on-off coupling between any other pair of cells. Hence we consider the network

$$\dot{x}_i = F(x_i) + \Sigma_{j=1}^{n}\varepsilon_{i,j}(t/\mu)\mathrm{P}x_j, \quad i = 1,\dots,n, \tag{1}$$

where $x_i=(x_i^1,\dots,x_i^d)$ is the d-vector of the coordinates of the i-th oscillator, and μ is a scalar parameter. The matrix P determines by which variables the oscillators are coupled. The $n\times n$ connectivity matrix $\mathrm{G}=\varepsilon_{i,j}(t)$ is symmetric and has vanishing row-sums and nonnegative off-diagonal elements such that $\varepsilon_{i,j}=\varepsilon_{j,i}$, $\varepsilon_{i,j}\geq 0$ for $i\neq j$, and $\varepsilon_{i,i}=-\Sigma_{j=1;j\neq i}^{n}\varepsilon_{i,j}$, $i=1,\dots,n$. The number of non-zero off-diagonal elements of the matrix G equals m.

As the pristine world, we take a conventional network, a ring of $2k$-nearest neighbor coupled oscillators. In this case the connectivity matrix $\mathrm{G}_{\mathrm{blnk}}$, corresponding to the blinking model, has the $2k$ adjacent diagonals with the coupling constants ε and on-off time-dependent small-world connections parameters $\varepsilon_{i_r j_r}(t/\mu)$ standing in all remaining places of the matrix $\mathrm{G}_{\mathrm{blnk}}$, where $r=1,2,\dots,n(n-2k-1)/2$.

We assume the functions $\varepsilon_{i_r j_r}(t/\mu)$ to be binary signals that take the constant value ε

with probability p and the value 0 with probability q=1-p in time interval of length τ. Therefore the random variables $\varepsilon_{i_n j_r}(m\tau)$ are independent and identically distributed for different m and τ. We assume that $\mu=\tau/T<<1$, where T is a characteristic transient time of the individual oscillator, and τ can be also interpreted as a characteristic interval of the time-varying connectivity matrix G.

Typically, in networks of continuous time oscillators, synchronization becomes stable when the coupling strength between the oscillators exceeds a critical value. In this context, a central question is to know the bounds on the coupling strengths such that the stability of synchronization is guaranteed. In this paper, we obtain the conditions for the stability of the synchronous state in the blinking model and reveal their dependence on the coupling configuration, probability p of reswitchings, and properties of the individual oscillators.

2. Synchronization in the pristine world

We start off with the study of global synchronization in the blinking model by considering first synchronization in the pristine world.

To calculate analytical bounds for the synchronization threshold value of coupling in the ring of $2k$ nearest neighbor coupled oscillators, we apply our Connection Graph Stability method, developed in [24], to this network. This general synchronization method combines the Lyapunov function approach with graph theoretical reasonings and allows us to tackle the problem of global stability of synchronization in rather irregular complicated networks.

Hereafter, we omit the proofs [24] and describe only the main results. For the pristine world, sufficient conditions of global synchronization are:

$$\varepsilon > \varepsilon^* = (a/n)R(k,n), \qquad (2)$$

where a is a parameter defined by the individual node dynamics and introduced similar to [19,21], and $R(k,n) \simeq (n/2k)^3 = L^3(0)$, where $L(0)$ is the average path length of the pristine world [2]. Consequently, we obtain the following bounds on the synchronization thresholds of global synchronization in the pristine world with the $2k$ diagonals:

$$\varepsilon > \varepsilon^* = an^2/(8k^3). \qquad (3)$$

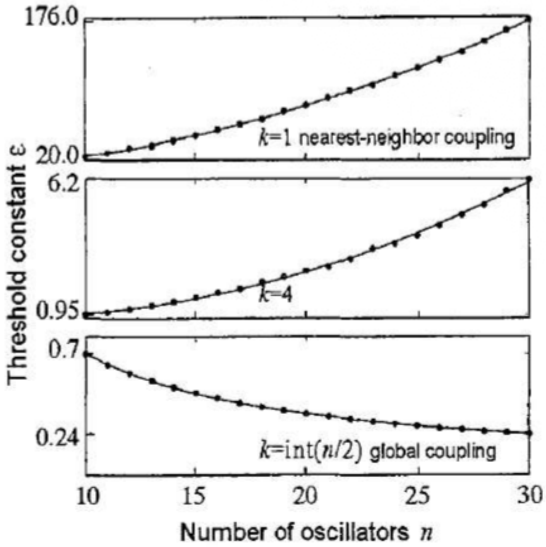
One can check the effectiveness and generality of the estimate (3) for different k. For one extreme case where k=1, the network is a ring of diffusively coupled oscillators and the estimate takes the form

$$\varepsilon^* = \tilde{a}n^2,$$

where $\tilde{a}=a/8$. This estimate presents a quadratic law of the dependence of the synchronization threshold of global synchronization on the number of oscillators. For another extreme case where k=int(n/2), all oscillators of the ensemble are globally coupled and the estimate presents the law $\varepsilon^*=b/n$ that is well-known for the oscillators with the mean field coupling. Here, b is a new constant. Note that between these extremes there is a case with $k_{\text{const}} \simeq n^{2/3}$, where the synchronization threshold is constant and does not depend on the number of oscillators.

We conjecture that the real threshold for complete synchronization follows closely the same law of dependence on n and k, but with a constant c lower than a which we obtained by stabilizing explicitly the individual oscillators. In support of this claim, we have determined numerically the thresholds for complete synchronization as functions of n for various values of k and we have fitted a curve of the form $cn^2/(8k^3)$ to the data, by

Fig. 1. Dependence of the synchronization thresholds ε^* on the number of oscillators n and on the depth of nearest-neighbor interaction k in the ring of $2k$-nearest neighbor coupled Lorenz systems. The analytical curves $cn^2/(8k^3)$ (solid lines) for different k fit the numerical data (small circles) in a least-squares sense.



letting vary c (Fig. 1). It can be seen that the deviation of the data from the fitted curve is very small, indeed. Note that we consider only the networks of oscillators admitting global synchronization with increasing coupling. In fact, most known chaotic dynamical systems belong to this class of networks.

3. Auxiliary regular coupling scheme

Let us now consider a regular configuration by adding to the pristine world (with the coupling matrix G) an additional global coupling such that the coupling coefficient $\nu\varepsilon$ is added to all free places of the matrix G, $0 \le \nu \le 1$. In this extended matrix G_e, the main diagonal elements are such that they preserve vanishing null row-sums. Thus we obtain the all-to-all regular coupling configuration with two different coupling strengths ε and $\nu\varepsilon$. The rigorous bound of global synchronization threshold in the network with the extended matrix G_e is calculated as follows [23]:

$$\varepsilon^* = (a/n)R(k,n)/[1+\nu(R(k,n)-1)], \tag{4}$$

where $R(k,n)=(n/(2k))^3$.

In the context of introducing additional small-world connections with sn edges added at random to the pristine world, where $s>0$ is rational, the parameter $\nu=2s/(n-1-2k)$ may be considered as the mean frequency of the appearance of shortcuts. The added coupling parameter $\nu\varepsilon$ may be thought of the averaged coupling strength of the sn connections. One can observe that the dependence (4) of the threshold ε^* on the mean frequency of the shortcuts appearance ν has a drastic diminution in the region of small ν (see Fig. 2,a).

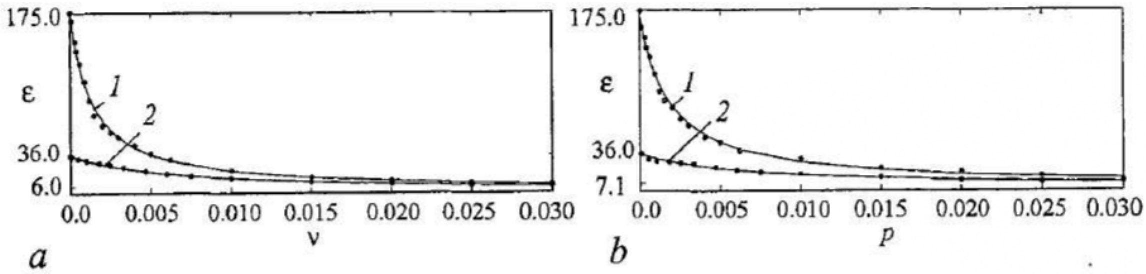


Fig. 2. Dependence of the synchronization thresholds on the parameter ν in the all-to-all coupled network with the coupling matrix G_e (a) and on the probability p of the shortcut appearance in the blinking model (b). The pristine world is a ring of 30 nearest-neighbor coupled Lorenz systems. The time step of switchings in the blinking model $\tau=0.1$. The analytical curves $\varepsilon^*=(a/n)L(0)/[1+\nu(L(0)-1)]$ and $\varepsilon^*=(a/n)L(0)/[1+p(L(0)-1)]$ fit the data remarkable well. $k=1$ (*1*); $k=2$ (*2*)

4. Synchronization in the blinking model

Let us now return to our blinking model of the small-world shortcut addition with the time-dependent connectivity matrix G_{blnk}. Figure 3 shows the time-varying structure of shortcut connections in the blinking model of 30 coupled oscillators. Here, the pristine world is a ring of locally coupled systems (k=1).

Recall that the switching time τ is fast with respect to the characteristic transient time T of the individual oscillator such that the parameter μ in Eq. (1) is small. Under this assumption, the blinking model becomes a slow-fast system. Thus, applying the Averaging Theorem [25] to the slow-fast system (1) with the time dependent coupling matrix G_{blnk}, we obtain the system (1) with the averaged graph matrix G_{mean} with the constant link strengths $\varepsilon_{i_r j_r}(t)=p\varepsilon=\text{const}$, where p is the probability of shortcut switchings in this blinking model.

Therefore the synchronization problem within the small-world network with blinking on-off shortcuts is reduced by averaging to the network with the constant matrix G_0 that is similar to the matrix G_e, where the probability p stands for the additional all-to-all coupling multiplicative parameter ν.

Hence for this case, the rigorous bound of global synchronization is calculated as follows:

$$\varepsilon^* = (a/n)L^3(0)/[1+p(L^3(0)-1)], \qquad (5)$$

where $L^3(0)=(n/(2k))^3$. For p=0, the estimate (5) becomes the synchronization threshold for the pristine world, and for p=1, it gives the synchronization threshold for all-to-all coupling. For $0<p<1$, the dependence (5) of the synchronization threshold on p reveals the sharp reduction of the synchronization threshold such that the addition of a few small-world connections (p is small) significantly improves the synchronization properties of the network (see Fig. 2, *b*).

Let us now present the effective path length for our blinking model and its dependence on the probability p. Recall that for p=0, the threshold (5) becomes the threshold (5) for the pristine world. Rewriting the dependence (5) in the form similar to Eq. (2), we introduce the effective path length of the blinking model as follows: $L^3(p)=L^3(0)/(1+p(L^3(0)-1))$. Therefore the normalized effective path length has the following dependence on the probability p:

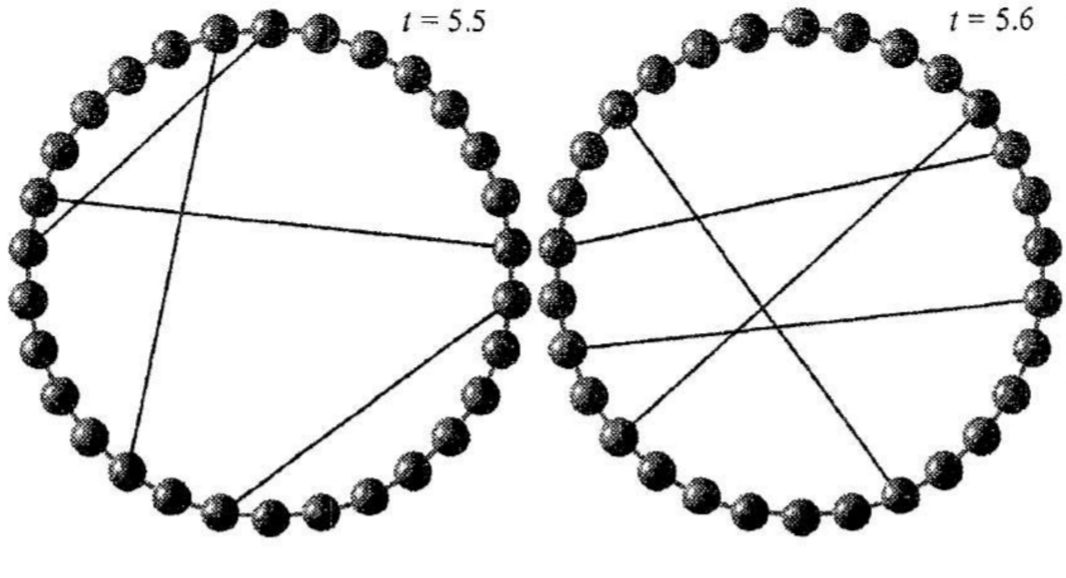


Fig. 3. The blinking model of shortcuts connections. Probability of switchings p=0.01, the time step of switchings τ=0.1

$$L(p)/L(0) = 1/(1+(n^3/(8k)^3-1)p)^{1/3}.$$

This formula clearly manifests the sharp decrease of the effective path length under a small increase of p from 0 in the blinking model.

5. Conclusions

A new type of dynamical small-world networks of chaotic cells has been proposed. For the first time for such networks with a time-varying coupling configuration, mathematically rigorous and tight bounds on the strength of coupling between the cells have been established that are necessary to achieve complete synchronization independently of the initial conditions. The synchronization thresholds have been explicitly linked with the average path length of the coupling graph and with the probability p.

In previous papers on synchronization in small-world networks a fraction of shortcuts are chosen at random at the beginning and they remain fixed for the rest of the time. In such an approach the synchronization threshold is the mean value of the thresholds for all possible shortcut combinations. However, these thresholds strongly depend on the particular choice of the shortcuts such that the addition of fixed in time small-world links does not necessarily guarantee synchronizability. It was stated in [11,12] that a sufficient amount of randomly chosen shortcuts will cause total synchronization. In other words, there exists a critical value for the probability p for which the small-world network, obtained by adding any given shortcuts, will synchronize completely. This statement is, in general, incorrect. In fact, the addition of fixed in time small-world links *does not* necessarily guarantee synchronizability. The addition of links filling out an entire row in the coupling matrix G does produce a tremendous increase of connectivity and a sharp reduction of the synchronization threshold. At the same time, the addition of coupling coefficients, located in the matrix G as a dense small «spot» and forming an all-to-all coupling within a small subgroup, does not reduce substantially the synchronization threshold. The latter case is not very likely to happen when there are many cells but it has nontheless a nonzero probability for a finite number of cells.

On the contrary, when the critical probability p is reached in the blinking model, then almost surely the system will synchronize. In other words, the set of on-off shortcut switching sequences that fail to force total synchronization has probability zero. For this property to be true, necessarily the switching time τ must be much smaller than T, the typical time constant for the individual cell dynamics. In this context, for many technical applications and, probably, for the coordinating brain functioning, the blinking effect of the shortcut appearance provides more reliable synchronization and global coordinating properties than the networks with the small-world but fixed coupling structure.

Let us end this paper devoted to Wadim's anniversary by a somewhat frivolous conclusion. In the context of the blinking model of a scientific collaborating network, the distant short-time connections can be considered as telephone calls, personal visits to friends and colleagues, etc. The characteristic time of reswitching τ in the blinking model is a time interval between these desirable events, T is the time when the network studies a particular problem. As we have learned from the above study of the blinking model, to improve synchronization properties of the network, the time τ of coordinating phone calls and visits should be small with respect to T, but these desirable events should be frequent. Consequently, we wish Wadim at the occasion of his 60-th birthday a bright scientific future with many short and intensive interactions with his scientific friends to whom we belong.

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Mathematics Department, Volga State Academy, Nizhny Novgorod, Russia
Laboratory of Nonlinear Systems, Swiss Federal Institute of Technology Lausanne (EPFL), Switzerland

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СВЯЗАННЫЕ СИСТЕМЫ ТИПА «МИР ТЕСЕН»: ДИНАМИЧЕСКИЕ МОДЕЛИ И СИНХРОНИЗАЦИЯ

В.Н. Белых, И.В. Белых, Martin J. Hasler

В работе представлен краткий обзор результатов исследования синхронизации взаимосвязанных динамических систем типа «мир тесен» (small-world). Предложена новая модель сетей типа «мир тесен» с изменяющейся во времени структурой связи. Показано, что такая структура связи обеспечивает более надежную синхронизацию, чем традиционные системы типа «мир тесен» с фиксированными связями. Термин «small world» (в прямом русском переводе «маленький мир» или, правильней, «мир тесен») относится к связанной системе, состоящей из локально связанных элементов и имеющей, в то же время, небольшое количество дальних вероятностных связей (shortcuts). Действительно, добавление нескольких дальних связей может существенно уменьшить среднее характеристическое расстояние между элементами даже очень большой локально связанной сети. Эффект типа «мир тесен», обнаруженный впервые социологами при исследовании структуры общества, является важной характеристикой многих других взаимодействующих систем, например, таких как ансамбли связанных нейронов в мозге, компьютерные сети и Интернет, взаимодействующие популяции и т.д. В применении к структуре общества это свойство означает, что два любых человека в мире связаны между собой через небольшое количество промежуточных знакомств. Считается, что среднее число звеньев такой цепи равно шести. Однако структура таких связей неоднородна, и всегда в обществе есть ключевые люди, обеспечивающие реальное взаимодействие между различными группами людей. Эта статья написана в честь 60-летия нашего друга и коллеги, Вадима Семеновича Анищенко, который является именно таким ключевым человеком в научном сообществе людей, занимающихся нелинейной динамикой.

Vladimir N. Belykh was born in N. Novgorod region (1943). He received the Diploma in 1966, the first PhD degree in 1972 from the University of Nizhny Novgorod, and the second Doctoral degree in 1985 from the University of St.-Petersburg, all in physics and mathematics. He became full professor in 1986. Nowadays he is currently the Head of Mathematics Department of Volga State Academy (Nizhny Novgorod and Professor at the Advanced School of General and Applied Physics of the University of Nizhny Novgorod). He received numerous Russian and international prizes, including Lenin's Komsomol prize (1974), Medal of Honored Scientist of the Russian Federation (1993), Prominent Scientist distinction of the Russian Federation (1994), Soros Professorship grant (1995), Distinguished Professor Scholarship (1996-2003). He held visiting Professor positions and gave lectures in different overseas universities including University of California, Berkeley; the Technical University of Denmark; University of Potsdam, Swiss Federal Institute of Technology; Eindhoven University of Technology, etc. His research interests include nonlinear dynamics and chaos, qualitative theory and bifurcation theory of concrete dynamical systems, mathematical modeling in electrical engineering, physics, biology and economics. A chaotic attractor of a two-dimensional map proposed by him in 1981, called the Belykh attractor, is now recognized as a classical object of ergodic theory.

E-mail: belykh@unn.ac.ru

Igor Belykh was born in Nizhny Novgorod (1973). He received the Diploma in 1995 and the PhD degree in 1999 from the University of Nizhny Novgorod, both in physics and mathematics. Nowadays he is a postdoctoral fellow at the Laboratory of Nonlinear Systems and a lecturer at the School of Computer and Communication Sciences of Swiss Federal Institute of Technology Lausanne (EPFL). He holds also a researcher position at the Department of Differential Equations, headed by L.P. Shilnikov, at the Institute of Applied Mathematics and Cybernetics, Nizhny Novgorod (on leave). He received several Russian and international prizes, including the Soros Student Award (1996), Razuvaev Award (1998), Soros PhD student Award (1997,1998), and Research Scholarship from Swiss Government (1998). He held visiting researcher positions at the Department of Physics of the Technical University of Denmark in 1997, 1999, 2000, and at the Department of Electrical Engineering of Swiss Federal Institute of Technology Lausanne (1998-1999). His present interests include nonlinear dynamics of biological neurons and neural systems, coupled dynamical systems, synchronization and cooperative behavior, modeling of nonlinear circuits and systems, and applications of nonlinear dynamics in physics, biology, and engineering.

Martin Hasler was born in Wetzikon canton Zurich (1945). He received the Diploma in 1969 and the PhD degree in 1973 from the Swiss Federal Institute of Technology, Zurich, both in physics. He continued research in mathematical physics at Bedford College, University of London, from 1973 to 1974. At the end of 1974 he joined the Circuits and Systems group of the Swiss Federal Institute of Technology Lausanne (EPFL), where he was given the title of a Professor in 1984. He became associate and full professor, respectively, in 1996 and 1999. In 2002, he was acting Dean of the newly created School of Computer and Communication Sciences of EPFL. During the 70's, his research was concentrated on filter theory and design, in particular active and switched capacitor filters. Since 1980 his research is centered on nonlinear circuits and systems, including the qualitative analysis of resistive and dynamic circuits, the modeling and identification of nonlinear circuits and systems, neural networks and the engineering applications of complicated nonlinear dynamics, in particular chaos. Chaos is applied to the transmission of information and to signal processing. Among the applications of the modeling and identification of nonlinear systems is the modeling of high-temperature superconductors for energy applications. Very recently, he is interested in the information processing in the brain.

He is a Fellow of the IEEE. He was the chairman of the Technical Committee on Nonlinear Circuits and Systems IEEE CAS Society from 1990 to 1993. From 1993 to 1995 he was the Editor of the IEEE Transactions on Circuits and Systems, Part I. He was the Chairman of ISCAS 2000, Geneva, Switzerland. He was a member of the Board of Governors of the IEEE CAS Society and currently he is its Vice-President for Technical Activities.

He is a member of the Scientific Council of the Swiss National Science Foundation since Jan. 2000.