Izv. VUZ «AND», vol.11, № 3, 2003

INVESTIGATING THE TRANSITION TO PHASE SYNCHRONIZATION IN EXPERIMENTAL NONLINEAR OPTICS

S. Boccaletti, E. Allaria, R. Meucci, and F.T. Arecchi

The transition route to phase synchronization in a chaotic laser with external modulation is investigated. We show evidence of the presence of a regime of periodic phase synchronization, where phase slips occur with maximal coherence in the phase difference between output signal and external modulation. We demonstrate that such a regime occurs at the crossover point between two different scaling laws of the intermittent type behavior of phase slips.

Synchronization phenomena in coupled chaotic systems has been a topic of great interest over the past decade [1-5], insofar as its relevance and ubiquitousness have been demonstrated in laboratory experiments [6], as well as in natural phenomena [7]. These processes happen when several coupled of forced chaotic units correlate with each other a given property of their motion, and they encompass behaviors ranging from a perfect hooking of the chaotic trajectories [1], to the emergence of a functional relationship relating the chaotic outputs [8], to a weaker form of correlation consisting in the locking of the phases of the systems [3,4].

This latter behavior has been called phase synchronization (PS). There, a coupling or a forcing induce a phase locked regime, where the amplitudes remain chaotic and almost uncorrelated, whereas the difference between the two free running [3] phases $\Phi_{1,2}$ evolves in a bounded manner and obeys the synchronization relation

$$\Delta = |\Phi_1 - \Phi_2| < \text{const.} \qquad (1)$$

PS was first demonstrated in mutually coupled [3] or periodically forced chaotic oscillators [4], and later observed in theoretical models [8] and experiments [9,10]. In particular, PS has been shown to play a crucial role in many physiological systems, such as human heartbeat and respiration [11], magnetoencephalography and electromyography of Parkinsonian patients [12], and electroencephalograms during visual stimulations [13].

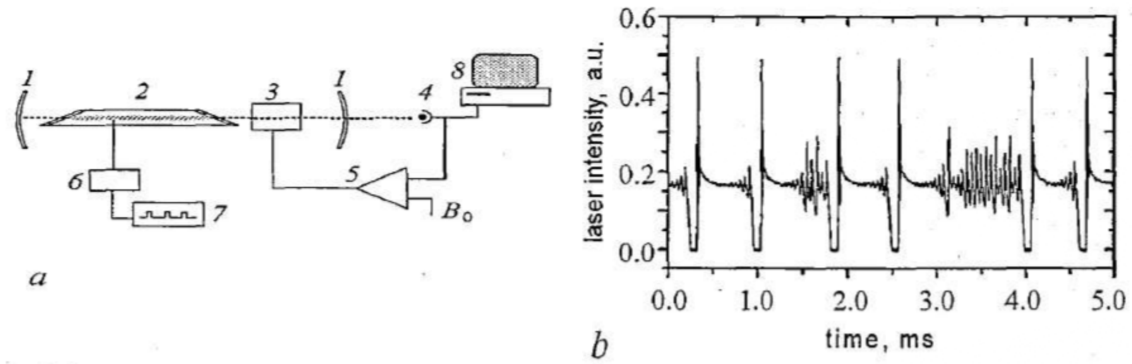
Being PS the weakest stage of synchronization, a relevant issue is to understand the transition route to such a behavior from unsynchronized motion. On the border of PS, the dynamical evolution of the system is characterized by epochs of almost constant phase difference intermittently interrupted by sudden 2π jumps in Δ, which are called *phase slips*.

For coupled or forced periodic oscillators, the transition to PS corresponds to a saddle-node bifurcation, and the average duration τ between successive phase slips obeys a type-I intermittency scaling law [14] $\tau \sim |P-P_c|^{-1/2}$, P being the relevant parameter of the transition (either the coupling strength or the external frequency), and P_c denoting its transitional value to PS.

A different scenario emerges for chaotic systems. If one considers a forced chaotic oscillator, and supposes that the system is phase synchronized for $\nu<\nu_c$, at forcing frequencies $\nu \geq \nu_c$, one observes another transition point $\nu_t > \nu_c$ such that for $\nu > \nu_t$, the scaling law for τ is the same as the classical case ($\tau \sim |\nu-\nu_c|^{-1/2}$), while for $\nu_c < \nu < \nu_t$, the intermittency shifts from type-I to that of superlong laminar periods described by $\ln(1/\tau) \sim -|\nu-\nu_c|^{-1/2}$. The theoretical picture of this transition has been described as a boundary crisis mediated by an unstable-unstable pair bifurcation [15], and the two above scaling behaviors have been numerically reported for coupled chaotic model systems [16].

Another important feature of such transition is that it can be identified by inspection of the Lyapunov spectrum. Precisely, PS is set around the passage to a negative value of a Lyapunov exponent that was zero in the uncoupled or unforced regime [3]. More recently, it has been demonstrated that the transition from no synchronization to PS is mediated by a regime, called *periodic phase synchronization* (PPS), where the time intervals between successive phase slips are almost equal to one another [17].

In this communication, we review the main features of the entire transition route from no synchronization to PS, that were firstly observed by us in Ref. [18]. The experimental setup is sketched in Fig. 1, *a* and consists of a CO_2 laser tube, pumped by an electric discharge current of 6 mA and inserted within an optical cavity closed by a totally reflecting mirror and a partially reflecting one. The detected laser output intensity suitably amplified drives an intracavity electro-optic modulator that controls the cavity losses. The feedback loop is realized by the voltage exiting a HgCdTe fast infrared diode detector, conveyed into an amplifier together with a bias voltage B_0, and driving the electro-optic modulating crystal. Under these conditions, and in the absence of any further modulation, the output intensity consists of a train of homoclinic spikes repeating at chaotic times and interconnected by minor oscillations [10] (see Fig. 1, *b*). This sequence of homoclinic



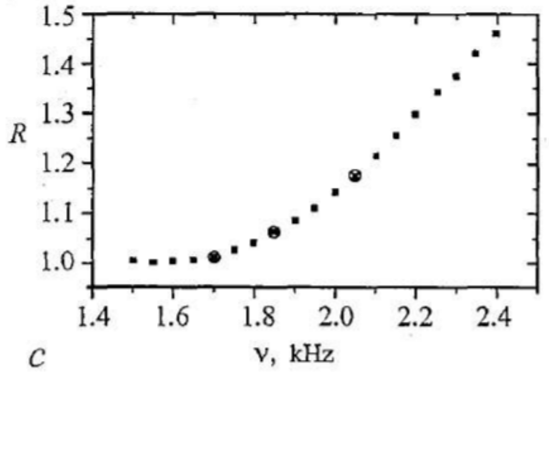


Fig. 1. (*a*) The experimental setup. *1* - Mirrors delimiting the optical cavity; *2* - CO_2 laser tube; *3* - intracavity electro-optic modulator; *4* - HgCdTe fast infrared diode detector; *5* - amplifier; *6* - generator for the pumping discharge; *7* - external modulation; *8* - PC based classification of phase slips. (*b*) Laser intensity (in arbitrary units) vs time for zero amplitude modulation. (*c*) Synchronization parameter R (see text for definition) vs ν. The three circles highlight the values for which phase slips are reported in Fig. 2

spikes can be phase entrained by an external sinusoidal modulation [10]. By adding a square signal modulation in the pumping discharge whose amplitude provides a 2% perturbation in the electric discharge current, one enters a regime of PS. The modulation is applied on a control unit of the generator (element 6 in Fig. 1, *a*). As for phases, the phase Φ_e of the external modulation evolves linearly in time ($\Phi_e=2\pi\nu t$), and the phase Φ_s of the chaotic signal is calculated by linear interpolation between successive spiking times following the rule $\Phi_s=2\pi k+2\pi(t-T_k)/(T_{k+1}-T_k)$, $T_k\le t<T_{k+1}$, where T_k denotes the time at which the k^{th} spike is produced. We call R the ratio between the number of maxima in the input modulation and the number of output spikes, and report in Fig. 1, *c* the route toward PS (R=1) as ν approaches $\nu_c\approx1.62$ kHz.

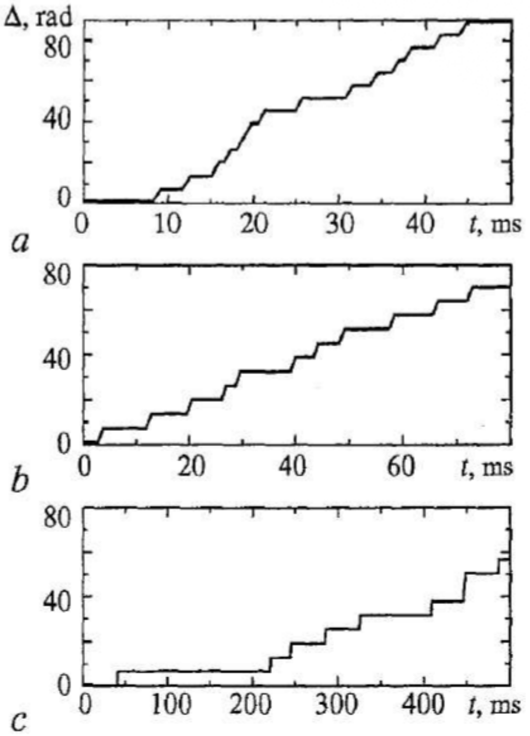


Fig. 2. Temporal evolution of $\Delta=|\Phi_e-\Phi_s|$ for: (*a*) ν=2.05 kHz, (*b*) ν=1.85 kHz, and (*c*) ν=1.70 kHz

We record sequences of more than 150 000 interspike intervals and study the occurrence of phase slips in the proximity of the transition to PS. Figure 2 reports the temporal evolution of $\Delta=|\Phi_e-\Phi_s|$ for (a) ν=2.05 kHz, (b) ν=1.85 kHz, and (c) ν=1.70 kHz. A sequence of 2π phase slips characterizes the evolution of Δ, whose occurrence becomes rarer and rarer as ν approaches ν_c. We furthermore calculate the distribution of interslip time intervals (ITI) and monitor its coherence factor $C=\tau/\sigma^2$ as a function of ν, where τ represents the average interslip time interval, and σ the standard deviation of the ITI distribution. According to [17], one should expect a value $\nu_{PPS}>\nu_c$ where phase slips occur periodically in Δ, reflected by a maximum in the coherence factor C close to the transition point for PS.

This is reported in Fig. 3, where a maximum of $C(\nu)$ is apparent at $\nu_{PPS}\approx1.84$ kHz. The further growth of C beyond ν~2.1 kHz is due to the approaching of a new locking regime, namely, 2:1 rather than 1:1. The temporal evolution of Δ at ν_{PPS} is shown in Fig. 2, *b*, where one can see that phase slips are almost equispaced in time.

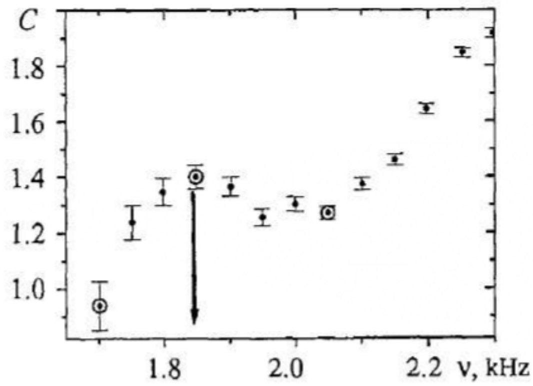


Fig. 3. Coherence factor C (see text for definition) vs ν. The arrow at ν_{PPS}=1.84 kHz indicates the frequency value for which phase slips are maximally coherent. The circles surround the three points for which measurements of $\Delta(t)$ are reported in Fig. 2

A further question concerns how the occurrence of PPS is related to the cross-over between the two scaling behaviors of phase slips. These scaling properties can be explained as follows. The type-I intermittency behavior describes the classical case of periodic systems, and it characterizes the intermittent phase slip duration just outside the border of PS.

For chaotic systems, the PS region corresponds to the overlap of all the phase-locking regions of the unstable periodic orbits (UPO) embedded in the chaotic attractor [19]. Each locked UPO is associated with an attractor and a repeller in the

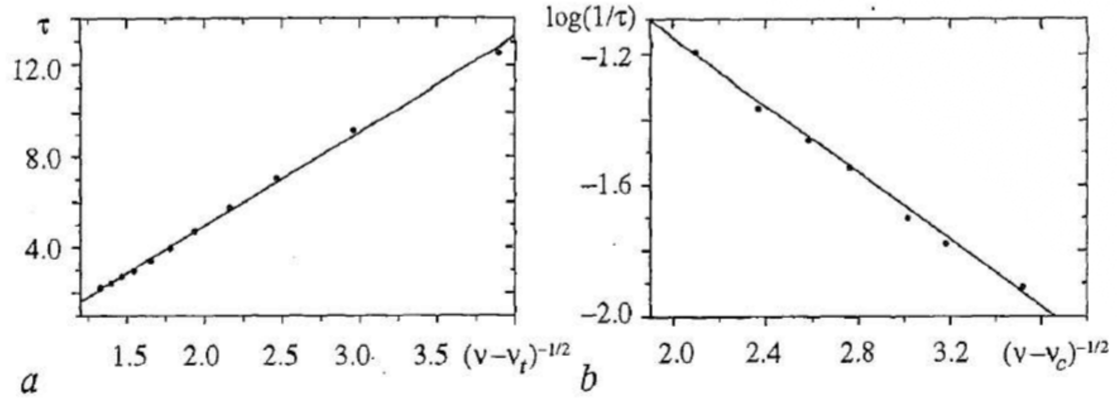


Fig. 4. Type-I intermittency scaling behavior (*a*) and superlong laminar scaling behavior (*b*) of interslip time intervals. Dots indicate the experimental measurements. Lines are the best fits: (*a*) $\tau=-3.4+4.2|\nu-\nu_t|^{-1/2}$, $\nu_t=1.84$ kHz; (*b*) $\log(1/\tau)=-0.13-0.51|\nu-\nu_c|^{-1/2}$, $\nu_c=1.62$ kHz. The crossover point for the two scaling laws is located at $\nu=\nu_t=1.84$ kHz, that corresponds exactly to the value ν_{PPS} of maximal coherence (periodic phase synchronization) in the phase slip occurrence

direction of the phase. The repellers are periodic orbits on the basin boundary of the attractors. As we approach the PS bifurcation point, the attractor and the repeller of each of a few UPOs approach, coalesce, and annihilate through a saddle-node bifurcation [15]. As a consequence, these UPOs are unlocked to the external force and phase slips occur. Just beyond the transition point, most UPOs are still attractive, and phase slips can develop only when the trajectory of the system stays for a sufficiently long time τ_1 in a close vicinity of the unlocked UPO. Due to ergodicity, the probability for a trajectory to visit a particular UPO for a duration τ_1 is proportional to $e^{-\lambda\tau_1}$ (λ being the largest Lyapunov exponent). The average interslip interval (the inverse of this probability) will be given by $\tau\sim e^{(k|\nu-\nu_c|^{-1/2})}$, where τ_1 has been substituted with its type-I intermittent scaling behavior, hence the expression for the superlong laminar behavior. Such a scaling behavior was also verified by numerical simulation of maps [20], and by direct simulation of the chaotic Roessler oscillator driven by external forcing [15,19].

In summary, one expects a type-I intermittent scaling law only for frequencies $\nu>\nu_t$, where ν_t denotes the value for which all UPOs are in the unlocked regime, and a superlong laminar behavior for $\nu_c<\nu<\nu_t$.

These expectations have been demonstrated by us, by making use of a series of measurements at different values of ν, obtaining the results shown in Fig. 4. The best fits yield $\nu_c=1.62$ kHz and $\nu_t=1.84$ kHz. Fig. 4 confirms the existence of two different scaling behaviors, and shows that the crossover point for the two scaling laws coincides with ν_{PPS} of Fig. 3, thus indicating that the coherence between successive phase slips mediates the transition from the two scaling behaviors.

We take the occasion to gratefully acknowledge the many illuminating discussions with V. Anishchenko on the whole issue of phase synchronization of chaotic oscillators, which have stimulated and inspired the present research, and continue stimulating our main experimental efforts in the field. Work supported by EU Contract HPRN-CT-2000-00158 (COSYC of SENS).

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Instituto Nazionale di Ottica Applicata,
Florence,Italy
Department of Physics, University of Florence,Italy

Received 28.08.03

УДК 621.373

ИССЛЕДОВАНИЕ ПЕРЕХОДОВ К ФАЗОВОЙ СИНХРОНИЗАЦИИ В ЭКСПЕРИМЕНТАЛЬНОЙ НЕЛИНЕЙНОЙ ОПТИКЕ

S. Boccaletti, E. Allaria, R. Meucci, F.T. Arecchi

Исследуется путь перехода к фазовой синхронизации в лазере с хаотической динамикой при наличии внешней модуляции. Приводится доказательство наличия режима периодической фазовой синхронизации, когда фазовые сбои случаются с

максимальной когерентностью, проявляющейся в фазовом различии между выходным сигналом и внешней модуляцией. Показано, что такой режим возникает в переходной точке между двумя различными скейлинговыми законами, описывающими режим перемежаемости в поведении фазовых сбоев.

Stefano Boccaletti graduated in Physics summa cum laude in 1992 and was awarded the Ph.D. in Physics in 1995. His major studies concerned the theoretical modeling of pattern formation and competition and the study of chaos recognition, control and synchronization.

Boccaletti is Referee of major physical journals, Member of the Advisory board of the AIP Journal CHAOS, Member of the Editorial Board of the Journal «Dynamical Systems: Chaos and Complexity Letters», and Associate Editor of the «Journal of Mathematical Biosciences and Engineering» of the American Institute of Mathematical Sciences. Boccaletti coauthored 97 research papers published on the major peer-review physical Journals. He was member elected of the Florence City Council from 1995 to 1999.

Enrico Allaria graduated in Physics at the University of Florence in 2000. He received its Specialization on Optics at the University of Florence with the support of the European Contract «Control Synchronization and Characterization of Spatially Extended Nonlinear Systems» on 2002. His research activity is focused mainly on nonlinear dynamics effects in optics with experiments on «Control and synchronization of chaos», «Models for the nonlinear dynamics in isotropic lasers» and «Optical measurements in the infrared». Allaria coauthored 20 research papers published on the major peer-review physical and optical Journals. Actually he is collaborating with the University of Florence and he is searching for a permanent position.

Riccardo Meucci graduated in Physics on December 20 1982 and was awarded the Ph.D. He received its Specialization on Optics at the University of Florence on March 13 1987. He received a position of researcher at Istituto di Cibernetica of the Italian National Research Council (CNR) on March 1984. He joined the Istituto Nazionale di Ottica (INO) now transformed into Istituto Nazionale di Ottica Applicata (INOA). At the present time he is a senior research charged with the division of Experimental Quantum Optics at INOA. His main scientific contribution are in the field of Chaotic instabilities in single mode lasers, Laser transients, Dynamical models for the CO_2 laser, Control and Synchronization of chaos, Spatio-temporal instabilities and delayed systems. Actually he curries out didactic activity as contract professor at the University of Florence. Meucci is referee of major physical and optical journals and is coauthor of about 100 papers.

Fortunato Tito Arecchi started his scientific activity in 1957 as Researcher at CISE, Milano. Since 1970 he is Chair of Physics in the Italian University (Pavia 1970-77, Firenze since 1977). Between 1975 and 2000 he was President of Istituto Nazionale di Ottica (INO), later transformed into Istituto Nazionale di Ottica Applicata (INOA). Since 2001 he is Scientific Responsible of INOA.

He is member of the Italian Physical Society, the Academie Internationale de Philosophie des Sciences, the Academia Europea and Fellow of Optical Society of America (OSA). He was Max Born Medal of OSA in 1995.

He is Editor of many scientific journals. His main scientific contributions are in the following areas: Cooperative effects in quantum optics, Photon statistics and laser fluctuations, Deterministic chaos in optics, Pattern formation in extended media, Complex phenomena and cognitive processes.

He is coauthor of more than 350 scientific papers and several books.