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OPTIMAL CONTROL OF FLUCTUATIONS APPLIED TO THE SUPPRESSION OF NOISE-INDUCED FAILURES OF CHAOS STABILIZATION

I.A. Khovanov, N.A. Khovanova and P.V.E. McClintock

Double strategy of chaos and fluctuation controls is developed. Noise-induced failures in the stabilization of an unstable orbit in the one-dimensional logistic map are considered as large fluctuations from a stable state. The properties of the large fluctuations are examined by determination and analysis of the optimal path and the optimal fluctuational force corresponding to the stabilization failure. The problem of controlling noise-induced large fluctuations is discussed, and methods of control have been developed.

Introduction

The control of chaos represents a very real and important problem in a wide variety of applications, ranging from neuron assemblies to lasers and hydrodynamic systems [1]. The procedure used consists of stabilizing an unstable periodic orbit by the application of precisely designed small perturbations to a parameter and/or a trajectory of the chaotic system. Different methods of chaos control have been suggested and applied in many different physical contexts, as well as numerically to model systems [1]. For practical applications of these control methods, it is important to understand how noise influences the stabilization process, because fluctuations are inherent and inevitably present in dissipative systems. The problem has not been well studied. Typically, a method is developed for stabilization of the orbit without initially taking any account of fluctuations. Only then the authors do check the robustness of their method by introducing weak noise into the system [1]. Thus, in the celebrated pioneering work of Ott, Grebogi and York, «Controlling chaos» [2], the authors just noted that noise can induce failures of stabilization.

In this work we consider noise-induced failures in the stabilization of an unstable orbit and the problem of controlling these failures. The method of Ott, Grebogi and Yorke (OGY) [2] and a modification of the adaptive method (ADP) [1] are used to stabilize an unstable point of the logistic map. We consider the small noise limit where stabilization failures are very rare and therefore they can be considered as large fluctuations (deviations) from a stable state. We study the properties of large deviations by determining the optimal paths and the optimal fluctuational forces corresponding to the failures. We employ two methods to determine the optimal paths and forces. The first of these builds and analyzes the prehistory probability distribution to determine the optimal path and optimal force [3]. The second method considers an extended map (relative to the initial one) which defines fluctuational paths and forces in the zero-noise limit [4,5]. Furthermore we use the optimal paths and forces to develop methods of controlling the large deviations, i.e. the noise-induced failures of stabilization. In the literature, methods for stabilization are often referred to as a control methods too. To differentiate controlling large fluctuations from controlling chaos, we therefore use the term «stabilization» to indicate the control of chaos.

In section 1 we describe the procedures for stabilization of an unstable orbit of the logistic map. The general approach to the control of a large deviation is presented in section 2. Noise-induced failures of stabilization are considered in section 3. The results obtained are discussed in the conclusion.

1.Chaos stabilization

For simplicity we will stabilize an unstable fixed point x^* of the logistic map:

$$x_{n+1} = rx_n(1-x_n), \tag{1}$$

where x_n is a coordinate, n is discrete time and r is the control parameter that determines different regimes of the map's behavior (1). The coordinate of the fixed point x^* is defined by the condition: $x_{n+1}=x_n$, and consequently its location depends on the parameter r:

$$x^* = 1 - 1/r. \tag{2}$$

We set the parameter r=3.8, a value for which an aperiodic (chaotic) regime is observed in (1), and the point x^* is embedded in the chaotic attractor.

From the range of existing stabilization methods, we chose to work with just two: the OGY and ADP methods mentioned above.

To stabilize a fixed point by the OGY method, perturbations Δr are applied to the parameter r, leading to the map being modified (1) in the following manner:

$$x_{n+1} = (r+\Delta r_n)x_n(1-x_n), \quad \Delta r_n = r(2x^*-1)(x_n-x^*)/[x^*(1-x^*)]. \tag{3}$$

To stabilize a fixed point by the ADP method, perturbations Δx are applied to the map's coordinate. The value of the perturbation Δx is defined by the distance between the current system coordinate and the coordinate of the stabilized state:

$$x_{n+1} = rx_n(1-x_n) + \Delta x_n, \quad \Delta x_n = (x_n-x^*). \tag{4}$$

We consider local stabilization procedure. During local stabilization, the perturbations Δr and Δx differ from zero only if the following condition is satisfied:

$$|x_n-x^*| < \varepsilon. \tag{5}$$

Here ε is a small value: we fixed ε=0.01. If the condition (5) is not satisfied then stabilization is absent, i.e. Δr=0 or Δx=0.

So, stabilization involve modifications of the initial map (1), and thus we use another map in the form (3) or (4). The fixed point x^* is an attractor of the new map. After the stabilization is switched on, a trajectory of the map tends to the fixed point x^*, and subsequently remains there.

In the presence of noise the trajectory fluctuates in the vicinity of the stabilized state, i.e. noise-induced dynamics appears. In addition, noise can induce stabilization failures, i.e. breakdown in the condition (5).

Our aim is to study these noise-induced stabilization failures and analyze the problem of how to suppress them. We therefore consider the maps (3) and (4) in the presence of additive Gaussian fluctuations:

$$x_{n+1} = (r+\Delta r_n)x_n(1-x_n) + D\xi_n, \quad \Delta r_n = r(2x^*-1)(x_n-x^*)/[x^*(1-x^*)], \tag{6}$$

$$x_{n+1} = rx_n(1-x_n) + \Delta x_n + D\xi_n, \quad \Delta x_n = (x_n-x^*). \tag{7}$$

Here D is the noise intensity; ξ_n is a Gaussian random process with zero-average $\langle\xi\rangle=0$, δ-correlation function $\langle\xi_n\xi_{n+k}\rangle=\delta(k)$ and dispersion $\langle\xi^2\rangle=1$.

2. Control of large fluctuations

Large fluctuations manifest themselves as large deviations from the stable state of the system under the action of fluctuational forces. Large fluctuations play a key role in many phenomena, ranging from mutations in DNA to failures of electrical devices. In recent years significant progress has been achieved both in understanding the physical nature of large fluctuations and in developing approaches for describing them. The latter are based on the concept of optimal paths - the paths along which the system moves during large fluctuations. Large fluctuations are very rare events during which the system moves from the vicinity of a stable state to a state remote from it, at a distance significantly larger than the amplitude of the noise. Such deviations can correspond to a transition of the system to another state, or to an excursion along some trajectory away from the stable state and then back again. During such deviations the system is moved with overwhelming probability along the optimal path under the action of a specific (optimal) fluctuational force. The probability of motion along any other (non-optimal) path is exponentially smaller. In practice, therefore large fluctuations must of necessity occur along deterministic trajectories. The problem of controlling large fluctuations can thus be reduced to the task of controlling motion along a deterministic trajectory. Consequently, the control problem can be solved through application of the control methods developed for deterministic systems [6].

Let us consider the control problem. We will follow the work [7] and consider the control of large fluctuations by a weak additive deterministic control force. Weakness means here that the energy of the control force is comparable with the energy (dispersion) of the fluctuations (see [8] for details). In this case, the extremal value of the functional R for optimal control, which moves the system from an initial state x^i to a target state x^f, takes the form [7]:

$$R_{\text{opt}}(x^f,F) = S^{(0)}(x^f) \pm \Delta S, \quad \Delta S = (2F)^{1/2}[\Sigma_{k=N^i}^{N^f} (\xi_k^{\text{opt}})^2]^{1/2}, \tag{8}$$

where ξ_k^{opt} is the optimal fluctuational force that induces the transition from x^i to x^f in the absence of the control force; $S^{(0)}$ is an energy of the transition, N_i and N_f are the times at which the fluctuational force ξ_k^{opt} starts and stops, and F is a parameter defining the energy of the control force.

The optimal control force u_n^{opt} for the given functional (8) is defined [7] by:

$$u_n^{\text{opt}} = \mp(2F)^{1/2}\xi_n^{\text{opt}}[\Sigma_{k=N^i}^{k=N^f} (\xi_k^{\text{opt}})^2]^{1/2} \times \delta(x_n-x_n^{(0)\text{opt}}), \tag{9}$$

where $x_n^{(0)\text{opt}}$ is the optimal fluctuational path in the absence of the control force. The minus sign in the expression (9) decreases the probability of a transition to the state x^f, and the plus sign increases the probability. It can be seen that the optimal control force u_n^{opt} is completely defined by the optimal fluctuational force ξ_k^{opt}, and the optimal fluctuational path $x_n^{(0)\text{opt}}$, corresponds to the large fluctuation. Therefore to solve the control problem it is necessary, first, to determine the optimal path $x_n^{(0)\text{opt}}$ leading from the state x^i to the state x^f under the action of the optimal fluctuational force ξ_k^{opt}. Thus, a solution of the control problem depends on the existence of an optimal path: it is obvious that the approach described should be straightforward to apply, provided that the optimal path exists and is unique.

We consider below an application of the approach described to suppress large fluctuations in the one-dimensional map. The large fluctuations in question are considered here to correspond to failures in the stabilization of an unstable orbit.

The control procedure depends on the determination of the optimal path and optimal fluctuational force and, to define them, we will use two different methods. The first method is based on an analysis of the prehistory probability distribution (PPD) and the second one consists of solving a boundary problem for an extended map which defines fluctuational trajectories.

The PPD was introduced in [3] to analyze optimal paths experimentally in flow systems. We will use the distribution to analyze fluctuational paths in maps. Note, that in [9,10] it was shown that analysis of the PPD allows one to determine both the optimal path and the optimal fluctuational force. The essence of this first method consists of a determination of the fluctuational trajectories corresponding to large fluctuations for extremely small (but finite) noise intensity, followed by a statistical analysis of the trajectories. In this experimental method the behaviour of the dynamical variables x_n and of the random force ξ_n are tracked continuously until the system makes its transition from an initial state x^i to a small vicinity of the target state x^f. Escape trajectories x_n^{esc} reaching this state, and the corresponding noise realizations ξ_n^{esc} of the same duration, are then stored. The system is then reset to the initial state x^i and the procedure is repeated. Thus, an ensemble of trajectories is collected and then the fluctuational PPD p_n^h is constructed for the time interval during which the system is monitored. This distribution contains all information about the temporal evolution of the system immediately before the trajectory arrives at the final state x^f. The existence of an optimal escape path is diagnosed by the form of the PPD p_n^h: if there is an optimal escape trajectory, then the distribution p_n^h at a given time n has a sharp peak at optimal trajectory x_n^{opt}. Therefore, to find an optimal path it is necessary to build the PPD and, for each moment of time n, to check for the presence of a distinct narrow peak in the PPD. The width of the peak defines the dispersion σ_n^h of the distribution and it has to be of the order of the mean-square noise amplitude $D^{1/2}$ [3]. The optimal fluctuational force that moves the system trajectory along the optimal path can be estimated by averaging the corresponding noise realizations ξ_n^{esc} over the ensemble. Note, that investigation of the fluctuational prehistory also allows us to determine the range of system parameters for which optimal paths exist.

To determine the optimal path and force by means of the second method we analyze extended maps [4,5] using the principle of least action [5]. Such extended maps are analogous to the Hamilton-Jacobi equation in the theory of large fluctuations for flow systems. For the one-dimensional map $x_{n+1}=f(x_n)+D\xi_n$, the corresponding extended map in the zero-noise limit takes the form:

$$\begin{aligned} x_{n+1} &= f(x_n) + y_n/g(x_n), \\ y_{n+1} &= y_n/g(x_n), \\ g(x_n) &= \partial f(x_n)/\partial x_n. \end{aligned} \qquad (10)$$

The map is area-preserving, and it defines the dynamics of the noise-free map $x_{n+1}=f(x_n)$, if $y_n=0$. If $y_n \neq 0$ then the coordinate x_n corresponds to a fluctuational path, and the coordinate y_n to a fluctuational force. Stable and unstable states of the initial map become saddle states of the extended map. So, the fixed point x^* of the ADP (7) and OGY (6) maps becomes a saddle point of the corresponding extended map. Fluctuational trajectories (including the optimal one) starting from x^* belong to unstable manifolds of the fixed point $(x^*,0)$ of the extended map.

The procedure for determination of the optimal paths consists of solving the *boundary problem for the extended map* (10):

$$x_{-\infty} = x^*, \quad y_{-\infty} = 0, \tag{11}$$

$$x_{\infty} = x^f, \quad y_{\infty} = 0, \tag{12}$$

where x^* is the initial state and x^f is a target state.

To solve the boundary problem different methods can be used. For the one-dimensional maps under consideration, a simple shooting method is enough [11]. We choose an initial perturbation l along the linearized unstable manifolds in a vicinity of the point $(x^*,0)$ of the map (10). The procedure to determine a solution can be as follows: looking over all possible values l, we determine a trajectory which tends to the point $(x^f,0)$. Note that, because these maps are irreversible there exits, in general, an infinite number of solutions of the boundary problem. The optimal trajectory (path) has minimal action (energy) $S=\Sigma_{n=-\infty}^{\infty} y_n^2$; here y_n is calculated along the trajectory, corresponding a solution of the boundary task.

3. Noise-induced failures in stabilization

A breakdown of the condition (5) corresponds to a failure of stabilization, i.e. to the noise-induced escape of the trajectory from an ε-vicinity of the fixed point x^*. The target state x^f corresponds to the boundaries of the stabilization region: $x^f = x^* \pm \varepsilon$.

Instead of analyzing the maps (6) and (7) in the ε-vicinity of the fixed point x^* we can investigate linearized maps of the following form: (path) has minimal

$$x_{n+1} = ax_n + D\xi_n, \tag{13}$$

here a is a value of derivative $\partial f(x_n)/\partial x_n$ in the fixed point x^*. For the map (6) the derivative is equal to zero $a_{\mathrm{OGY}}=0$, and for the map (7) $a_{\mathrm{ADP}}=-0.8$.

Let us investigate stabilization failure by considering the most probable (optimal) fluctuational paths, which lead from the point x^* to boundaries $x^* \pm \varepsilon$. For linearized maps (13) the extended map (10) can be reduced to the form:

$$\begin{aligned} x_{n+1} &= ax_n + y_n/a, \\ y_{n+1} &= y_n/a \end{aligned} \tag{14}$$

with the initial condition ($x_0=x^*$, $y_0=0$) and the final condition $x^f=x^*\pm\varepsilon$. It can be seen that a solution of the map (14) increases proportionally to $y_n=\mathrm{const}/a^n$ [12]. This means that, for the ADP map (7), the amplitude of the fluctuational force increases slowly but that, for the OGY map (6), the failure arises as the result of only one fluctuation (iteration). Because equation (14) is linear, the boundary problem will have a unique solution [11]. Thus, analysis of the linearized extended map (14) shows that there is an optimal path, and it gives a qualitative picture of exit through the boundary $x^*\pm\varepsilon$.

Let us check the existence of the optimal paths through an analysis of the prehistory of fluctuations. To obtain exit trajectories and noise realizations we use the following procedure. At the initial moment of time, a trajectory of the map is located at point x^*. The subsequent behaviour of the trajectory is monitored until the moment at which it exits from the ε-region of the point x^*. The relevant parts of the trajectory, just before and after its exit, are stored. The time at which the exit occurs is set to zero. Thus ensembles of exit trajectories and of the corresponding noise realizations are collected and PPDs are built.

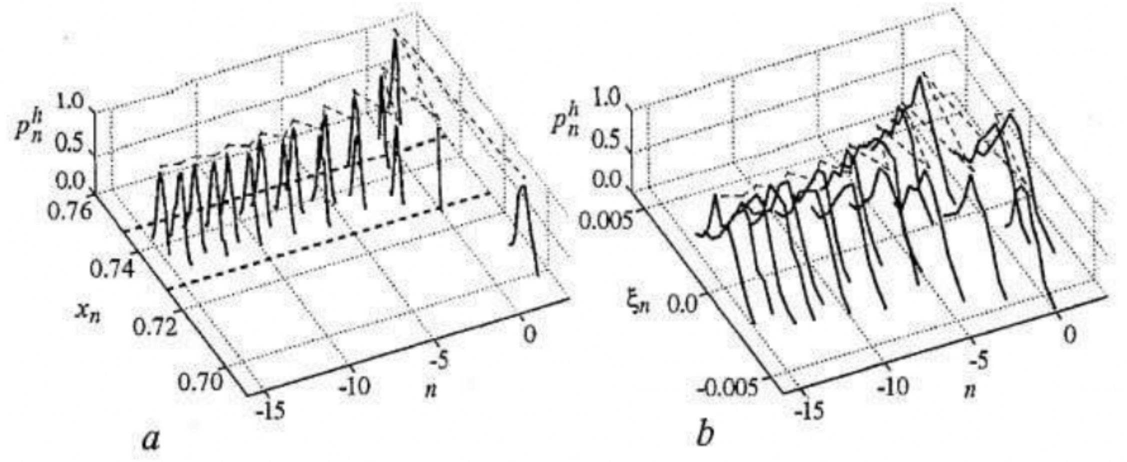

Fig. 1. PPDs p_h^n of the exit trajectories (*a*) and noise realizations (*b*) of ADP map for the boundary $(x^*-\varepsilon)$. The thick dashed lines indicate ε-region of stabilization. The thin dashed lines connect maxima of PPDs. The noise indensity is D=0.0011

To start with, we will discuss these ideas in the context of the ADP map. Fig. 1, *a* shows PPDs of the escape trajectories of the ADP map, and the corresponding noise realizations for the exit through the boundary $(x^*-\varepsilon)$ are shown in Fig. 1, *b*. The picture of exit through the other boundary $(x^*+\varepsilon)$ is symmetrical, so we present results for one boundary only. It is evident (Fig. 1) that there is the only one exit path. Note, that the path to the boundary $(x^* - \varepsilon)$ is approximately 2.8 more probable than the path to the boundary $(x^*+\varepsilon)$. This difference arises from an asymmetry of the map in respect of the boundaries.

Because for each boundary there is the only one exit path, the optimal path and the optimal fluctuational force can be determined by simple averaging of escape trajectories and noise realizations respectively. In Fig. 2 the optimal exit paths and the optimal fluctuational forces are shown for the boundaries $(x^*-\varepsilon)$ and $(x^*+\varepsilon)$. The paths and the forces coincide with a solution of the boundary problem (circles in the Fig. 2) of the extended linear map (14). As can be seen the optimal path is long, and the amplitude of the fluctuational force increases slowly, in agreement with analysis of the linearized map (14).

The optimal fluctuational force obtained (Fig. 2, *b*) must correspond [10] to the energy-optimal deterministic force that induced the stabilization failure. We have

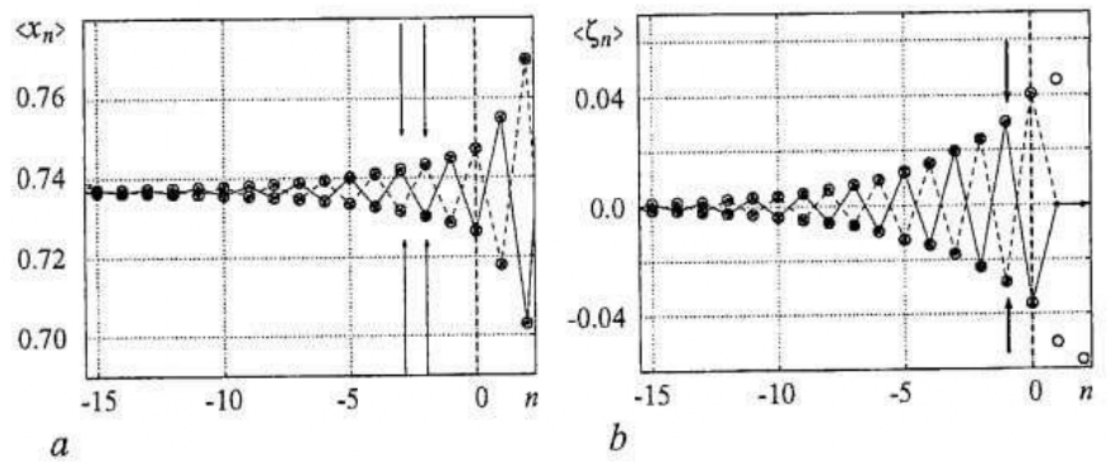

Fig. 2. The optimal paths (*a*) and the optimal forces (*b*) for exit through the boundary $(x^*-\varepsilon)$ (solid line) and the boundary $(x^*+\varepsilon)$ (dashed line) for ADP map. Circles indicate the optimal paths and forces obtained by solving the boundary problem for the linearized extended map (14). The optimal paths and forces used in the control procedure are marked by arrows

checked this prediction and found that the optimal force induces the exit from an ε-region of the point x^*: we selected an initial condition at the point x^* and included the optimal fluctuational force additively; as a result we observed the stabilization failure. If we decrease the amplitude of the force by 5-10%, then the failure does not occur. It appears, therefore, the deduced force allows us to induce the stabilization failure with minimal energy (see [10] for details).

Using the optimal path and the force we can solve the opposite task [7,8] - to decrease the probability of the stabilization failures. Indeed, if during the motion along the optimal path we will apply a control force with the same amplitude but with the opposite sign as the optimal fluctuational force has, then, obviously, the failure will not occur. Because we know the optimal force then, in accordance with the algorithm [7] described above, it is necessary to determine the time moment when system is moving along the optimal path. For the ADP method the optimal path is long enough to identify that a trajectory is moving along the optimal path, and then to apply a control force.

In the presence of control the map (7) is modified:

$$x_{n+1} = rx_n(1-x_n) + \Delta x_n + D\xi_n + u_n, \quad \Delta x_n = (x_n - x^*), \tag{15}$$

here u_n is the deterministic control force.

We use the following scheme to suppress the stabilization failures. Initially the control force is equal to zero (u_n=0) and the map is located in the point x^*; we continuously monitor a trajectory of the map (15) and define the time moment when the system starts motion along the optimal path $\langle x_n \rangle$. We assume that the system moves along the optimal path $\langle x_n \rangle$ if it passes within a small vicinity of the coordinate $\langle x_{-2} \rangle$ and then within a small vicinity of $\langle x_{-3} \rangle$ (see arrows in Fig. 2, *a*). Then on the following iteration we add the control force u_n=-sign(ξ_n)$\langle \xi_n \rangle$, n=-1 (see Fig. 2, *b*).

In Fig. 3, *a* dependences of the mean time $\langle \tau \rangle$ between the failures on the noise intensity D are plotted in the absence, and in the presence, of the control procedure. It is clear that the mean time $\langle \tau \rangle$ is substantially increased by the addition of the control, i.e. stability in the face of fluctuations is significantly improved by the addition of the control scheme. The efficiency of the control procedure depends exponentially [7] on the amplitude of the control force (Fig. 3, *b*), and there is an optimal value of the control force, which is very close to the value (arrow in Fig. 3, *b*) of the optimal fluctuational force.

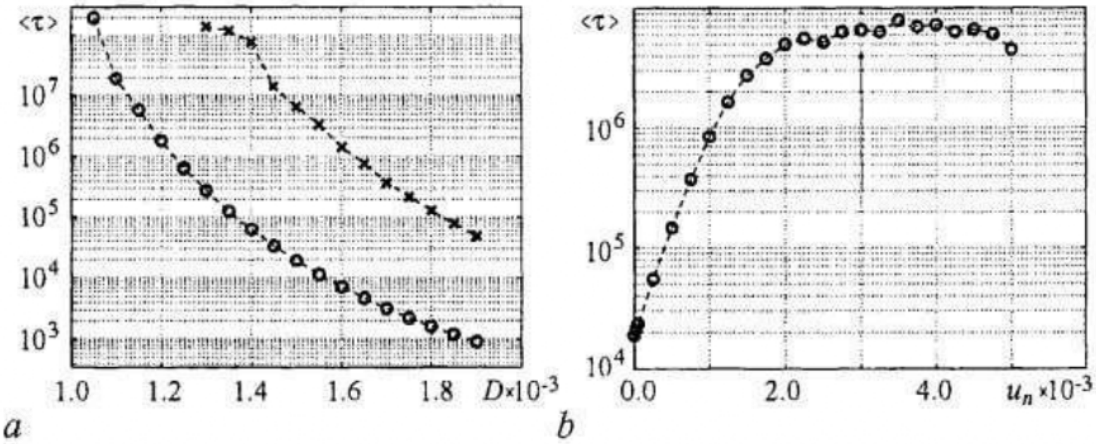

Fig. 3. (*a*) The dependences of mean time $\langle \tau \rangle$ between stabilization failures on noise intensity D in the absence (circles) and in the presence (crosses) of the control. The size of the stabilization region is ε=0.01. (*b*) The dependence of the mean time $\langle \tau \rangle$ on the amplitude of the control force u_n is presented for the ADP method. The value of $\langle \tau \rangle$ corresponding to the optimal fluctuational force is marked by the arrow

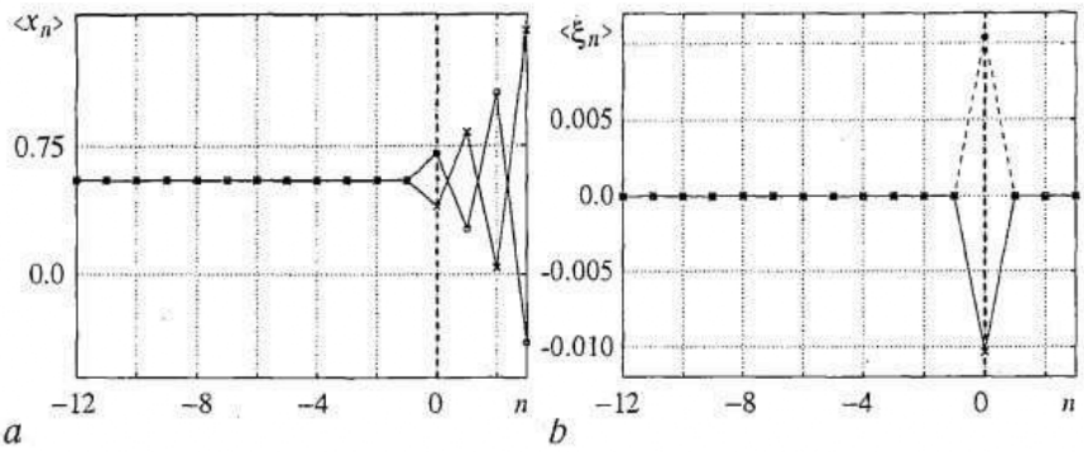

Fig. 4. For the OGY map, the optimal path (a) and the optimal force (b) are shown for exit through the boundary (x^*-ε) (crosses) and the boundary (x^*+ε) (circles)

Now consider noise-induced stabilization failures for OGY map (6). An analysis of the linearized map has shown that the failure occurs as the result of a single fluctuation. We have checked the conclusion by an analysis of the fluctuational trajectories of the map (6), much as we did for the ADP map. The optimal path and optimal force are shown in Fig. 4 for both boundaries, (x^*+ε) and (x^*-ε). An exit occurs during one iteration and there is no a prehistory before this iteration. It means that we cannot determine the moment at which the large fluctuation starts and, consequently, that we cannot control the stabilization failures. The existence of a long prehistory is thus a key requirement in the control the large fluctuations.

Conclusion

We have considered noise-induced failures in the stabilization of an unstable orbit, and the problem of how to control such failures. In our investigations, they correspond to large deviations from stable points. We have shown that noise-induced failures can be analyzed effectively in terms of linearized noisy maps.

Large noise-induced deviations from the fixed point in one-dimensional maps have been analyzed within the framework of the theory of large fluctuations. The key point of our consideration is that the dynamics of the optimal path, and the optimal fluctuational force, correspond directly to stabilization failures. We have applied two approaches - experimental analysis of the prehistory probability distribution and the solution of the boundary problem for extended maps - to determine the optimal path and the optimal fluctuational force, and we have compared their results. The two approaches give the same results.

A procedure for the control of large fluctuations in one-dimensional maps has been demonstrated. It is based on the control concept developed in [7] for continuous systems. We have introduced an additional control scheme which significantly improves the stabilization of an unstable orbit in the presence of noise. It was successful for the ADP method of stabilization, and problematic for the OGY method. We have shown that the control procedure has limitations connected with the presence of long time prehistory of large fluctuation.

Our consideration of the control problem is relevant to a continuous system which has a one-dimensional curve in its Poincaré section, e.g. the Rössler system. For such systems we can formulate the control task as that of control at discrete moments of time (the moments of intersection of the Poincaré section) by using impulsive forces. The intervals between these moments were used to calculate and to form the necessary control force. Note that a similar approach is widely used in control technology.

The main limitation of our present control approach lies in the necessity of studying the fluctuational dynamics of a given system prior to consideration of its control. Such a study can be carried out by use of an extended map of the system, if model equations are known, and/or experimentally by analysis of the fluctuational prehistory distribution. A system model can be easily written down by determination of the eigenvalue of a stabilized unstable point: there are many effective methods of doing so [13].

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ОПТИМАЛЬНЫЙ КОНТРОЛЬ ФЛУКТУАЦИЙ В ПРИМЕНЕНИИ К ПОДАВЛЕНИЮ ИНДУЦИРОВАННЫХ ШУМОМ НАРУШЕНИЙ СТАБИЛИЗАЦИИ ХАОСА

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Разрабатывается двойная стратегия управления хаосом и флуктуациями. Индуцированные шумом нарушения стабилизации неустойчивых орбит в одномерном логистическом отображении рассматриваются как большие флуктуации

от устойчивого состояния. Свойства больших флуктуаций проверяются путем определения и анализа оптимального пути и оптимального флуктуационного воздействия, соответствующего нарушению стабилизации. Обсуждается проблема управления индуцированных шумом больших флуктуаций, и разрабатываются методы управления.

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