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RECOVERY OF DYNAMICAL MODELS OF TIME-DELAY SYSTEMS FROM TIME SERIES

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We develop the method for the estimation of the parameters of time-delay systems from time series. The method is based on the statistical analysis of time intervals between extrema in the time series and the projection of the infinite-dimensional phase space of a time-delay system to suitably chosen low-dimensional subspaces. We verify our method by using it for the reconstruction of different time-delay differential equations from their chaotic solutions.

Introduction

The present paper deals with the problem of reconstruction of nonlinear dynamical models of time-delay systems from time series. The importance of this problem is determined by the fact that time-delay systems are wide spread in nature. The behavior of such systems is affected not only by the present state, but also by past states. These systems are usually modeled by delay-differential equations. Such models are successfully used in many scientific disciplines, such as physics, physiology, biology, economic, and cognitive sciences. Typical examples include population dynamics [1], where individuals participate in the reproduction of a species only after maturation, or spatially extended systems, where signals have to cover distances with finite velocities. Within this rather broad class of systems, one can find the Ikeda equation [2] modeling the passive optical resonator system, the Lang-Kobayashi equations [3] describing semiconductor lasers with optical feedback, the Mackey-Glass equation [4] modeling the production of red blood cells, and various models describing different phenomena from glucose metabolism to infectious diseases. The advantage of methods proposed in the paper is that they can be applied to the systems of different nature if these systems have similar structure of model equations.

In the most general case the time-delay systems are described by the following equation

$$x^{(n)}(t) + \varepsilon_{n-1}x^{(n-1)}(t) + \ldots + \varepsilon_1 x^{(1)}(t) = F(x(t), x(t-\tau_1), \ldots x(t-\tau_k)), \tag{1}$$

where $x^{(n)}(t)$ is the derivative of order n; $\varepsilon_1,\ldots,\varepsilon_{n-1}$ are the coefficients; and $\tau_1,\ldots,\tau_k$ are the delay times. To uniquely define the system (1) state it is necessary to prescribe the initial conditions in the entire time interval $[-\tau_k,0]$. Therefore, the phase space of the system has to be considered as infinite-dimensional. In fact, for large delay times even scalar delay-differential equations can possess high-dimensional chaotic dynamics. Thus, the direct reconstruction of the system by the time-delay embedding techniques runs into severe problems. For a successful recovery of the time-delay systems one has to use special methods. The most of them are based on the projection of the infinite-dimensional phase space of time-delay systems onto low-dimensional subspaces. These methods use different criteria of quality for the reconstructed equations, for example, the minimal forecast error of constructed model [5-8], the minimal value of information entropy [9], or various measures of complexity of the projected time series [10-14]. Several methods of time-delay system analysis exploit regression analysis [15,16] and correlation function construction [17,18]. In this paper we further develop the methods proposed by us recently [19,20] for the estimation of the parameters of time-delay systems from time series for a more wide class of time-delay systems.

Reconstruction of scalar time-delay systems

Let us consider one of the most popular first-order delay-differential equation

$$\varepsilon_0 \dot{x}(t) = -x(t) + f(x(t-\tau_0)), \tag{2}$$

where $x(t)$ is the system state at time t, function f defines nonlocal correlations in time, τ_0 is the delay time, and parameter ε_0 characterizes the inertial properties of the system. In general case Eq. (2) is a mathematical model of an oscillating system composed of a ring with three ideal elements: nonlinear, delay, and inertial ones (Fig. 1). In the present paper we develop a technique for estimating τ_0, f, and ε_0 from the time series.

It should be noted that available for measurement dynamical variable could be obtained from different points of the time-delay system (2), indicated in Fig. 1 by the numerals *1*-*3*. Let us consider first the case when the observed dynamical variable is $x(t)$ measured at the point *1*. To estimate the delay time τ_0 we exploits the features of extrema shape and location in the system (2) temporal realization $x(t)$. The peculiarities of extrema location in time are clearly illustrated by $N(\tau)$ plot in Fig. 2. To construct it one has to define for different τ values the number N of pairs of extrema in $x(t)$, that are separated in time by τ. If N is normalized to the total number of extrema, then for sufficiently large extrema number it can be used as an estimation of probability to find a pair of extrema in $x(t)$ separated by the interval τ. Let us explain the qualitative features of $N(\tau)$ for various values of parameter ε_0.

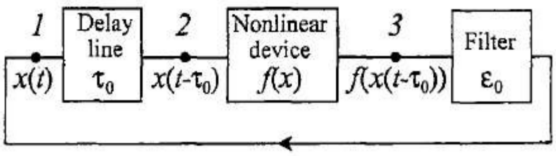

Fig. 1. Delayed nonlinear feedback system. Arabic numerals designate points where a dynamical variable is measured

In the absence of inertial properties (ε_0=0) time differentiation of Eq. (2) gives

$$\dot{x}(t) = \dot{x}(t-\tau_0) df(x(t-\tau_0))/dx(t-\tau_0). \tag{3}$$

From Eq. (3) it follows that if $\dot{x}(t-\tau_0)=0$, then $\dot{x}(t)=0$. Thus, for $\varepsilon_0=0$ every extremum of $x(t)$ is followed within the time τ_0 by the extremum[1]. As the result, $N(\tau)$ shows a maximum for $\tau=\tau_0$ in Fig. 2, *a*.

In the presence of inertial properties ($\varepsilon_0>0$), which corresponds to real situations,

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[1] For chaotic temporal realizations of the systems under investigation practically all critical points with $\dot{x}(t)=0$ are the extremal ones, and therefore we call the points with $\dot{x}(t)=0$ the extremal points throughout this paper.

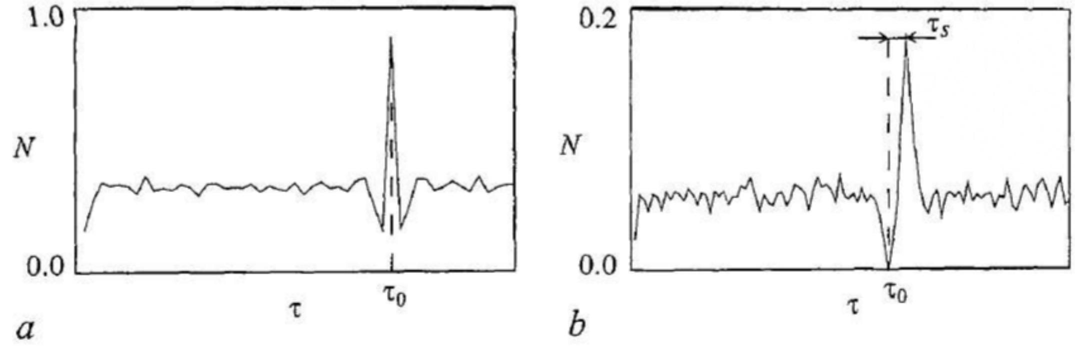

Fig. 2. Number N of pairs of extrema in a realization of Eq. (2) separated in time by τ, as a function of τ. $N(\tau)$ is normalized to the total number of extrema in time series. (a) $\varepsilon_0=0$. $N(\tau)$ has a sharp maximum at the level of the delay time of the system. (b) $\varepsilon_0>0$. $N(\tau)$ has a pronounced minimum at the level of the delay time of the system. The location of maximum is determined by the parameter ε_0

the most probable value of the time interval between extrema in $x(t)$ shifts from τ_0 to larger values. This effect can be explained using the ring system shown in Fig. 1: the filter introduces a certain additional delay in the system. As the result, the extrema in $x(t)$ can be found most often at the distance $\tau_0+\tau_s$ apart (Fig. 2, b). For instance, the computational investigation of Eq. (2) with quadratic nonlinear function $f(x)=\lambda-x^2$ allows us to obtain an estimation $\tau_s\approx\varepsilon_0/2$ for large values of the parameter of nonlinearity λ.

For $\varepsilon_0>0$ the extrema in $x(t)$ are close to quadratic ones and therefore $\dot{x}(t)=0$ and $\ddot{x}(t)\neq0$ at the extremal points. It can be shown that in this case there are practically no extrema in $x(t)$ separated in time by τ_0. To prove this let us differentiate Eq. (2) with respect to t:

$$\varepsilon_0\ddot{x}(t) = -\dot{x}(t) + \dot{x}(t-\tau_0)df(x(t-\tau_0))/dx(t-\tau_0). \tag{4}$$

If for $\dot{x}(t)=0$ in a typical case $\ddot{x}(t)\neq0$, then, as it can be seen from Eq. (4), for $\varepsilon_0\neq0$ the condition $\dot{x}(t-\tau_0)\neq0$ must be fulfilled. Thus, there must be no extremum separated in time by τ_0 from a quadratic extremum and hence $N(\tau_0)\to0$. For $\tau\neq\tau_0$, the derivatives $\dot{x}(t)$ and $\dot{x}(t-\tau)$ can be simultaneously equal to zero, i.e., it is possible to find extrema separated in time by τ. The specific configuration presented in Fig. 2, b in the neighborhood of $\tau=\tau_0$ is duplicated at larger τ in the neighborhood of $\tau=2\tau_0,3\tau_0,\ldots$

The procedure of the delay time estimation from the $N(\tau)$ plot considered with systems like (2) can be successfully applied to time series gained from a more general class of time-delay systems

$$\dot{x}(t) = F(x(t),x(t-\tau_0)). \tag{5}$$

Time differentiation of Eq. (5) gives

$$\ddot{x}(t) = \dot{x}(t)\partial F(x(t),x(t-\tau_0))/\partial x(t) + \dot{x}(t-\tau_0)\partial F(x(t),x(t-\tau_0))/\partial x(t-\tau_0). \tag{6}$$

Similarly to Eq. (4), Eq. (6) implies that in the case of quadratic extrema derivatives $\dot{x}(t)$ and $\dot{x}(t-\tau_0)$ do not vanish simultaneously, i.e., if $\dot{x}(t)=0$, then $\dot{x}(t-\tau_0)\neq0$.

Thus, for τ_0 definition one has to determine the extrema in the time series and after that to define for different values of time τ the number N of pairs of extrema separated in time by τ and to construct the $N(\tau)$ plot. The absolute minimum of $N(\tau)$ is observed at the delay time τ_0.

To recover the parameter ε_0 and the nonlinear function f of system (2) from the chaotic time series let us rewrite Eq. (2) as

$$\varepsilon_0\dot{x}(t) + x(t) = f(x(t-\tau_0)). \tag{7}$$

Thus, it is possible to reconstruct the nonlinear function by plotting in a plane a set of points with coordinates $(x(t-\tau_0), \varepsilon_0\dot{x}(t)+x(t))$. According to Eq. (7), the constructed set of points reproduces the function f. Since the parameter ε_0 is a priori unknown, one needs to plot $\varepsilon\dot{x}(t)+x(t)$ versus $x(t-\tau_0)$ under variation of ε, searching for a single-valued dependence in the plane $(x(t-\tau_0),\varepsilon\dot{x}(t)+x(t))$, which is possible only for $\varepsilon=\varepsilon_0$. As a quantitative criterion of single-valuedness in searching for ε_0 we use the minimal length of a line $L(\varepsilon)$, connecting all points ordered with respect to $x(t-\tau_0)$ in the plane $(x(t-\tau_0), \varepsilon\dot{x}(t)+x(t))$. The minimum of $L(\varepsilon)$ is observed at $\varepsilon=\varepsilon_0$. The set of points constructed for the defined ε_0 in the plane $(x(t-\tau_0), \varepsilon_0\dot{x}(t)+x(t))$ reproduces the nonlinear function, which can be approximated if necessary. In contrast to methods presented in [11,12] which use only extremal points or points selected according to a certain rule for the nonlinear function recovery, the proposed technique uses all points of the time series. It allows one to estimate the parameter ε_0 and to reconstruct the nonlinear function from short time series even in the regimes of weakly developed chaos.

To test the efficiency of the proposed technique we apply it to a time series produced by numerical integration of the passive optical resonator system of Ikeda [2]

$$\dot{x}(t) = -x(t) + \mu\sin(x(t-\tau_0)-x_0) \tag{8}$$

with $\mu=20$, $\tau_0=2$, $x_0=\pi/3$, $\varepsilon_0=1$. Note that the nonlinear function in the Ikeda equation is multimodal one. Part of the time series is shown in Fig. 3, *a*. The time series is sampled in such a way that 200 points in time series cover a period of time equal to the delay time

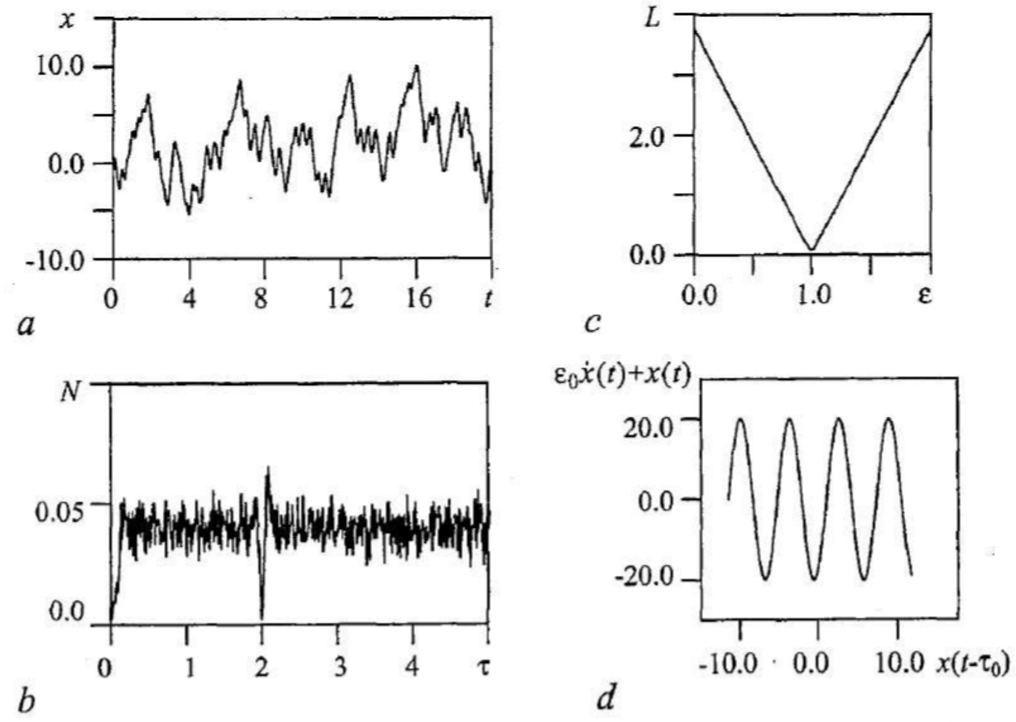

Fig. 3. (*a*) Time series of the Ikeda equation (8); (*b*) number N of pairs of extrema in the time series separated in time by τ, as a function of τ. $N(\tau)$ is normalized to the total number of extrema in the time series. $N_{min}(\tau)=N(2.00)$; (*c*) - length L of a line connecting points ordered with respect to $x(t-\tau_0)$ in the plane $(x(t-\tau_0), \varepsilon_0\dot{x}(t)+x(t))$ as a function of ε. $L(\varepsilon)$ is normalized to the number of points. $L_{min}(\varepsilon)=L(1.00)$; (*d*) - the recovered nonlinear function

τ_0=2. The data set consists of 25000 points and exhibits about 1000 extrema. Figure 3, *b* illustrates the τ-dependence of the number N of pairs of extrema separated in time by τ. The time derivatives $\dot{x}(t)$ are estimated from the time series by applying a local parabolic approximation. The step of τ variation in Fig. 3, *a* is equal to the integration step h=0.01. The absolute minimum of $N(\tau)$ takes place exactly at $\tau=\tau_0$=2.00. To construct the $L(\varepsilon)$ plot (Fig. 3, *c*) the step of ε variation was also set by 0.01. The minimum of $L(\varepsilon)$ takes place accurately at $\varepsilon=\varepsilon_0$=1.00. In Fig. 3, *d* the nonlinear function is shown. This recovered function coincides practically with the true function of Eq. (8).

To investigate the robustness of the method to additional noise we analyze the data produced by adding to the time series of Eq. (8) zero-mean Gaussian white noise. The presence of noise in time series brings into existence spurious extrema. These extrema are not caused by the intrinsic dynamics of a system and temporal distances between them are random. With the extrema number increasing, a probability to find a pair of extrema in time series separated in time by τ has to increase in general. The extrema number increasing induced by noise is also followed by the increase of probability to find a pair of extrema separated by the interval τ_0. However, for moderate noise levels this probability is still less than the probability to find a pair of extrema separated in time by $\tau\neq\tau_0$. Since the absolute minimum of $N(\tau)$ is very well pronounced in the absence of noise, it can be clearly distinguished even in the noise presence if the noise level is not very high. Hence, the qualitative features of the $N(\tau)$ plot specified by the delay-induced dynamics are retained for a moderate noise level. The presence of noise is more critical for the parameter ε_0 estimation and the nonlinear function recovery.

Figure 4 illustrates the results of the Ikeda equation reconstruction from the time series corrupted with zero-mean Gaussian white noise with a standard deviation of 20% of the standard deviation of the data without noise. The location of the absolute minimum of $N(\tau)$ (Fig. 4, *a*) allows one to estimate the delay time accurately, τ_0'=2.00. The minimum of $L(\varepsilon)$ (Fig. 4, *b*) takes place at ε_0'=0.98. The nonlinear function recovered using the estimated τ_0' and ε_0' is shown in Fig. 4, *c*. In spite of sufficiently high noise level and inaccuracy of ε_0 estimation the recovery of the nonlinear function has a good quality which is significantly higher than that reported in [21] for the same parameter values of the Ikeda equation with noise.

In the second case, when the observed dynamical variable is $x(t-\tau_0)$ measured at the point *2* (Fig. 1), one can use the described above procedure for estimation of the system parameters since the observable is simply shifted in time by the delay time τ_0. For the third possible case, when the observed variable is $f(x(t-\tau_0))$ which is measured at the point *3* (Fig. 1), one needs another technique for reconstruction of the time-delay system.

As well as in the time series of $x(t)$, there are also practically no extrema separated in time by τ_0 in the time series of the system (2) variable $f(x(t-\tau_0))$, since, $df(x(t-\tau_0))/dt=$

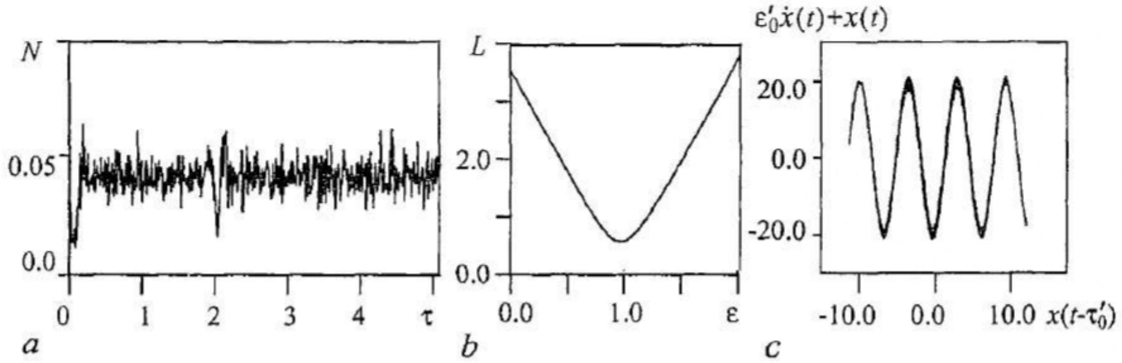

Fig. 4. Reconstruction of the Ikeda equation from its time series $x(t)$ with additive Gaussian white noise for noise level of 20%. (*a*) The $N(\tau)$ plot. $N_{min}(\tau)=N(2.00)$. (*b*) The $L(\varepsilon)$ plot. $L_{min}(\varepsilon)=L(0.98)$. (*c*) The recovered nonlinear function

$=\dot{x}(t-\tau_0)df(x(t-\tau_0))/dx$. Then, the delay time τ_0 can be estimated by the location of the absolute minimum in the $N(\tau)$ plot constructed from the variable $f(x(t-\tau_0))$.

To recover the parameter ε_0 and the function f we filter the chaotic signal $f(x(t-\tau_0))$ with a first-order low-pass filter and plot $f(x(t-\tau_0))$ versus $u(t-\tau_0)$, where $u(t-\tau_0)$ is the signal at the filter output, shifted by the time τ_0 defined earlier. If the filter inertial properties are characterized by the parameter $\varepsilon=\varepsilon_0$, then $u(t-\tau_0)=x(t-\tau_0)$ and the set of points constructed in the plane $(x(t-\tau_0)), f(x(t-\tau_0)))$ reproduces the nonlinear function f. Since the parameter ε_0 is a priori unknown, one needs to plot $f(x(t-\tau_0))$ versus $u(t-\tau_0)$, under variation of the filter parameter ε, searching for a single-valued dependence in the plane $(u(t-\tau_0), f(x(t-\tau_0)))$, which is possible only for $\varepsilon=\varepsilon_0$. As a quantitative criterion of single-valuedness in searching for ε_0 we use the minimal length of a line $L(\varepsilon)$, connecting all points ordered with respect to $u(t-\tau_0)$ in the plane $(u(t-\tau_0), f(x(t-\tau_0)))$. The minimum of $L(\varepsilon)$ is observed at $\varepsilon=\varepsilon_0$. The set of points constructed for the defined ε_0 in the plane $(u(t-\tau_0), f(x(t-\tau_0)))$ reproduces the nonlinear function, which can be approximated if necessary.

We apply the method to a time series of the variable $f(x(t-\tau_0))$ of the Mackey-Glass equation [4]

$$\dot{x}(t) = -bx(t) + ax(t-\tau_0)/(1+x^c(t-\tau_0)), \tag{9}$$

which can be converted to Eq. (2) with $\varepsilon_0=1/b$ and the function

$$f(x(t-\tau_0)) = ax(t-\tau_0)/(b(1+x^c(t-\tau_0))). \tag{10}$$

The parameters of the system (10) are chosen to be $a=0.2$, $b=0.1$, $c=10$, $\tau_0=300$ to produce a dynamics on a high-dimensional chaotic attractor. The sampling time is set by 1.

Figure 5 illustrates the reconstruction of the Mackey-Glass system parameters. Figure 5, *a* shows the number N of pairs of extrema in the time series of $f(x(t-\tau_0))$, separated in time by τ. The step of τ variation in Fig. 5, *a* is equal to the integration step $h=1$. The location of the absolute minimum of $N(\tau)$ allows us to estimate the delay time, $\tau_0'=300$. To construct the $L(\varepsilon)$ plot (Fig. 5, *b*) we use the step of ε variation equal to 0.1. The minimum of $L(\varepsilon)$ takes place at $\varepsilon_0'=10.0$ ($\varepsilon_0=1/b=10$). The nonlinear function recovered using the estimated τ_0' and ε_0' is shown in Fig. 5, *c*. This recovered function coincides practically with the true function (10).

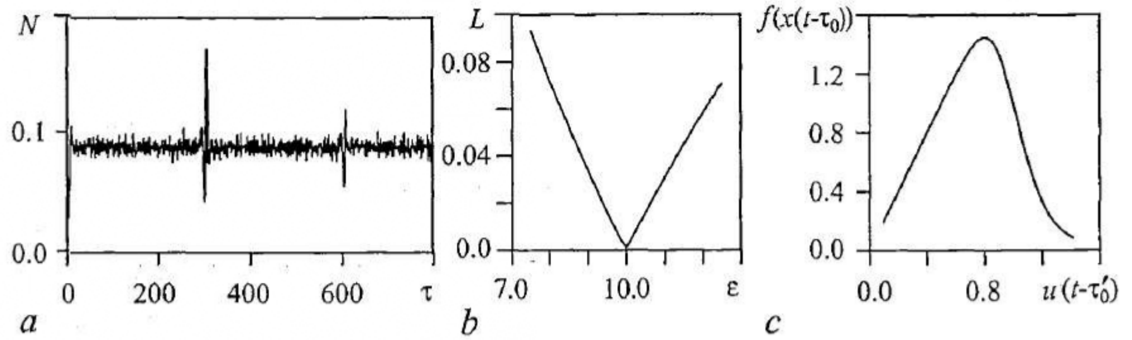

Fig. 5. Reconstruction of the Mackey-Glass system from the variable $f(x(t-\tau_0))$. (*a*) The $N(\tau)$ plot. $N_{min}(\tau)=N(300)$. (*b*) The $L(\varepsilon)$ plot. $L_{min}(\varepsilon)=L(10.0)$. (*c*) The recovered nonlinear function

Reconstruction of nonscalar time-delay systems

The method of τ_0 definition from time series described above for scalar time-delay systems can be extended to high-dimensional time-delay systems having the following form

$$x^{(n)}(t) + \varepsilon_{n-1}x^{(n-1)}(t) + \ldots + \varepsilon_1\dot{x}(t) = F(x(t), x(t-\tau_0)), \tag{11}$$

Differentiation of Eq. (11) with respect to t gives

$$\begin{aligned} &x^{(n+1)}(t) + \varepsilon_{n-1}x^{(n)}(t) + \ldots + \varepsilon_1\ddot{x}(t) = \\ &= \dot{x}(t)\partial F(x(t), x(t-\tau_0))/\partial x(t) + \dot{x}(t-\tau_0)\partial F(x(t), x(t-\tau_0))/\partial x(t-\tau_0). \end{aligned} \tag{12}$$

The condition $\dot{x}(t-\tau_0)\neq 0$ for $\dot{x}(t)=0$ will be satisfied if the left-hand side of Eq. (12) does not vanish. In general, a probability to obtain zero in the left-hand side of Eq. (12) is very small and therefore, the $N(\tau)$ plot qualitatively must have a shape similar to that inherent in the case of first-order delay-differential equations like (2) and (5).

The proposed method of estimation of the parameter ε_0 and the nonlinear function can be also applied to a variety of time-delay systems of order higher than that of (2). For instance, if the dynamics of a time-delay system is governed by the second-order delay-differential equation

$$\varepsilon_2\ddot{x}(t) + \varepsilon_1\dot{x}(t) = -x(t) + f(x(t-\tau_0)), \tag{13}$$

the nonlinear function can be reconstructed by plotting in a plane a set of points with coordinates $(x(t-\tau_0),\ \varepsilon_2\ddot{x}(t)+\varepsilon_1\dot{x}(t)+x(t))$. The constructed set of points reproduces the function f. Since the parameters ε_1 and ε_2 are a priori unknown, one needs to plot $\hat{\varepsilon}_2\ddot{x}(t)+\hat{\varepsilon}_1\dot{x}(t)+x(t)$ versus $x(t-\tau_0)$ under variation of $\hat{\varepsilon}_1$ and $\hat{\varepsilon}_2$, searching for a single-valued dependence in the plane $(x(t-\tau_0),\ \hat{\varepsilon}_2\ddot{x}(t)+\hat{\varepsilon}_1\dot{x}(t)+x(t))$, which is possible only for $\hat{\varepsilon}_1=\varepsilon_1$, $\hat{\varepsilon}_2=\varepsilon_2$. As a quantitative criterion of single-valuedness in searching for ε_1 and ε_2 we use the minimal length of a line $L(\hat{\varepsilon}_1,\hat{\varepsilon}_2)$ connecting all points ordered with respect to $x(t-\tau_0)$ in this plane. The minimum of $L(\hat{\varepsilon}_1,\hat{\varepsilon}_2)$ is observed at $\hat{\varepsilon}_1=\varepsilon_1$, $\hat{\varepsilon}_2=\varepsilon_2$. The set of points constructed for the defined ε_1 and ε_2 in the plane $(x(t-\tau_0),\ \varepsilon_2\ddot{x}(t)+\varepsilon_1\dot{x}(t)+x(t))$ reproduces the nonlinear function. However, the quality of reconstruction deteriorates, since the procedure involves numerical calculation of the second derivative.

Recovery of the delay times for time-delay systems with two coexisting delays

Let us consider now a time-delay system with two different delay times τ_1 and τ_2

$$\dot{x}(t) = F(x(t), x(t-\tau_1), x(t-\tau_2)). \tag{14}$$

Time differentiation of Eq. (14) gives

$$\ddot{x}(t) = \dot{x}(t)\partial F/\partial x(t) + \dot{x}(t-\tau_1)\partial F/\partial x(t-\tau_1) + \dot{x}(t-\tau_2)\partial F/\partial x(t-\tau_2). \tag{15}$$

Similarly to temporal realization of Eq. (5), the realization $x(t)$ of Eq. (15) has mainly quadratic extrema and therefore $\dot{x}(t)=0$ and $\ddot{x}(t)\neq 0$ at the extremal points. Hence, if $\dot{x}(t)=0$, the condition must be fulfilled,

$$a\dot{x}(t-\tau_1) + b\dot{x}(t-\tau_2) \neq 0 \tag{16}$$

where $a=\partial F(x(t), x(t-\tau_1), x(t-\tau_2))/\partial x(t-\tau_1)$ and $b=\partial F(x(t), x(t-\tau_1), x(t-\tau_2))/\partial x(t-\tau_2)$. The condition (16) can be satisfied if $\dot{x}(t-\tau_1)\neq 0$ or/and $\dot{x}(t-\tau_2)\neq 0$. By this is meant that in the case of quadratic extrema derivatives $\dot{x}(t)$ and $\dot{x}(t-\tau_1)$, or $\dot{x}(t)$ and $\dot{x}(t-\tau_2)$ do not vanish simultaneously. As the result, the number of extrema separated in time by τ_1 and τ_2 from a quadratic extremum must be appreciably less than the number of extrema separated in time by other values of τ and hence the $N(\tau)$ plot will demonstrate minima at $\tau=\tau_1$ and $\tau=\tau_2$. But these minima are not so pronounced as in the case of a single delay time, since only one of the terms of Eq. (16) is necessary not equal to zero.

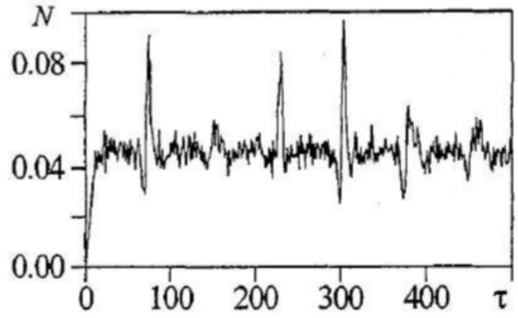

Fig. 6. Number N of pairs of extrema in a realization of Eq. (17) separated in time by τ, as a function of τ. $N(\tau)$ is normalized to the total number of extrema in time series. $N_{min1}(\tau)=N(70)$, $N_{min2}(\tau)=N(299)$

As an example, we demonstrate the method efficiency with a generalization of the Mackey-Glass equation by introducing a further delay,

$$\dot{x}(t) = -bx(t) + {}^1/_2\, ax(t-\tau_1)/(1+x^c(t-\tau_1)) + {}^1/_2 ax(t-\tau_2)/(1+x^c(t-\tau_2)) \qquad (17)$$

with a=0.2, b=0.1, c=10, τ_1=70, and τ_2=300. The $N(\tau)$ plot is presented in Fig. 6. The most pronounced minima of $N(\tau)$ are observed at τ=70 and τ=299 providing a good estimation of both the delay times τ_1 and τ_2.

Conclusion

We have proposed the methods for reconstructing different time-delay systems from time series. These methods are based on the statistical analysis of time intervals between extrema in the time series and the projection of the infinite-dimensional phase space of a time-delay system to suitably chosen low-dimensional subspaces. The methods allow one to estimate the delay time, the parameter characterizing the inertial properties of the system, and the nonlinear function even in the presence of sufficiently high noise. The method of the delay time definition uses only operations of comparing and adding. It needs neither ordering of data, nor calculation of approximation error or certain measure of complexity of the trajectory and therefore it does not need significant time of computation. The proposed techniques of the nonlinear function recovery and estimation of the parameter characterizing the system inertial properties use all points of the time series what allows one to apply the method to short time series even in the regimes of weakly developed chaos.

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ВОССТАНОВЛЕНИЕ ДИНАМИЧЕСКИХ МОДЕЛЕЙ СИСТЕМ С ЗАПАЗДЫВАНИЕМ ПО ВРЕМЕННЫМ РЯДАМ

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Работа посвящена развитию метода оценки параметров систем с запаздыванием по временным рядам. Метод основан на статистическом анализе временных интервалов между экстремумами временного ряда и проецировании бесконечномерного фазового пространства системы с запаздыванием в соответствующим образом выбранные подпространства малой размерности. Работоспособность метода продемонстрирована при реконструкции различных дифференциальных уравнений с запаздыванием по их хаотическим решениям.

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