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EVOLUTION OF RUNNING WAVES TO SPATIO-TEMPORAL CHAOS: INTERACTION OF TEMPORAL AND SPATIAL DYNAMICS IN A RING OF PERIOD-DOUBLING SELF-OSCILLATORS

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In the work we consider transition from regular running waves to developed spatio-temporal chaos in a chain of period-doubling oscillators. We investigate typical bifurcations which take place on the base of the chosen running wave regime from the period-one cycle to developed temporal chaos. We found that oscillations remain spatially periodic until transition to temporal chaos. The exact spatial periodicity is changed by the periodicity in the average in the chaotic region. Destroying of the averaged spatio-periodic structure is connected with losing of coherence on main frequencies in the temporal spectra of neighbor oscillators in the chain.

To V. Anishchenko, on the occasion of his sixtieth birthday

In recent years a problems of collective dynamics of interacting oscillators attracts a great interests. A series of works was devoted to consideration of coupled maps arrays modeling different physical phenomena [1-4]. The maps with chaotic behavior have rich dynamics and they allow to research formation of regular and chaotic spatio-temporal structures resulted from synchronization of oscillations. Another base models are arrays of phase oscillators which can be applied for investigation of phenomena of phase synchronization and of formation of phase structures. Most of studies was devoted to the global mean-field coupling systems [5-10]. It was demonstrated that the very simple periodic oscillators can demonstrate complex macroscopic behavior: periodic, quasi-periodic and even chaotic through quasi-periodic and period-doubling routes [6]. The mean-field approach allowing to consider the behavior of the system as a whole, does not take into account local connections between elements, which can lead to formation of local spatial structures. Locally coupled limit-cycle oscillators was intensively investigated in the works [11-13]. The phase regularities in nearest-neighbor coupled oscillators were also considered on the example of the circle maps [14]. The work [15] investigated spatial synchronization in the chain of unidirectionally coupled period-doubling self-oscillators and developing of the dynamics along the array.

It has been known that chains of the simplest limit-cycle oscillators with periodic boundary conditions exhibit running waves regimes when oscillations in nearest sites differ from each other on constant phase shifts. In the work [11] a more complex oscillators chain was considered. It was demonstrated that taking into account the second harmonics in the spectrum of oscillations can lead to spatially chaotic behavior. Hence the transition to more realistic models lead to dynamics which can't be realized in the simplest phase oscillators arrays. The works [16-19] demonstrate that running waves regimes are possible for rings of chaotic oscillators. Nevertheless, until now a lot of questions about the developing and destroying of the chaotic running waves remain unresolved. How does complicating of the temporal dynamics influence on the spatial structures? How the destroying of spatial structures is connected with the synchronization between nearest-neighbor oscillators? In our investigation we are focusing on these questions. We have chosen a chain of period-doubling self-oscillators (Chua's oscillators) with diffusion symmetric coupling

$$\dot{x}_i = \alpha(y_i - x_i - f(x_i)),$$

$$\dot{y}_i = x_i - y_i + z_i + \gamma(y_{i-1} + y_{i+1} - 2y_i), \qquad (1)$$

$$\dot{z}_i = -\beta y_i,$$

where

$$f(x) = \begin{cases} bx + a - b & \text{if } x > 1 \\ ax & \text{if } |x| \le 1 \\ bx - a + b & \text{if } x < -1, \end{cases} \qquad i=1,2,\dots,N,$$

with periodic boundary conditions:

$$x_1 = x_N, \;\; y_1 = y_N, \;\; z_1 = z_N.$$

All oscillators are identical. The behavior of the single oscillator is widely described in the literature (see, for example, [20]). It is characterized by period-doubling bifurcations cascade and bistability, when two symmetric attractors formed near two non-trivial equilibria P_1 and P_2 coexist in the phase space. With parameter α increasing these attractors merge and as a result double scroll chaotic attractor appears.

We investigated the system (1) with changing of the parameter α and of coupling coefficient γ. Other parameters were fixed in the values: a=-8/7, b=-5/7, β=-22. The number of oscillators in the chain was N=30 and N=1024. At the value of α is more than α=8.78 period-one cycle temporal regimes with different spatial structures coexist in the system. Choosing spatially-periodic initial values one can obtain attractors characterized by exact space periodicity. These attractors can be considered as running waves rotating along the ring with constant phase velocity because oscillations in the every site has equal amplitude and equal phase shift relatively to the neighbor oscillator. We investigated the waves with spatial periods of 6, 10, 15 oscillators (for the 30-sites chain). With increasing of the parameter α these waves undergo bifurcations which lead to complicating of their temporal behavior. At small coupling we observed period-doubling bifurcations cascades of finite length. The number of the bifurcations increases with decreasing of the coupling and tends to infinity at zero coupling. The every cascade is ended by the tori birth bifurcation which is followed by the destroying of the torus and the transition to chaos. At larger coupling the period-doubling bifurcations do not take place and the torus appears on the base of the period-one cycle. The fig. 1 demonstrates a diagram of typical regimes on the plane of parameters γ-α for the family of regimes originated from the running wave with spatial period Λ=15. The region of stability of this family is bounded by lines marked by «○» (lines *1*, *5*). The line 1 bounds this region from the right. With crossing the line the regimes with wavelength Λ=15 lose their stability by sudden way and the system transits to waves with larger spatial periods. The line *5* marks destroying of periodic spatial structure by soft way. Near this line the spatial structure begins to change its form and over it the regime awfully «forgets» its original spatial structure. Over the line *1* and before the line *2* the stable period-one cycle is observed. On the line *2* the system transits to quasi-periodic behavior. This is the line of the torus 1T

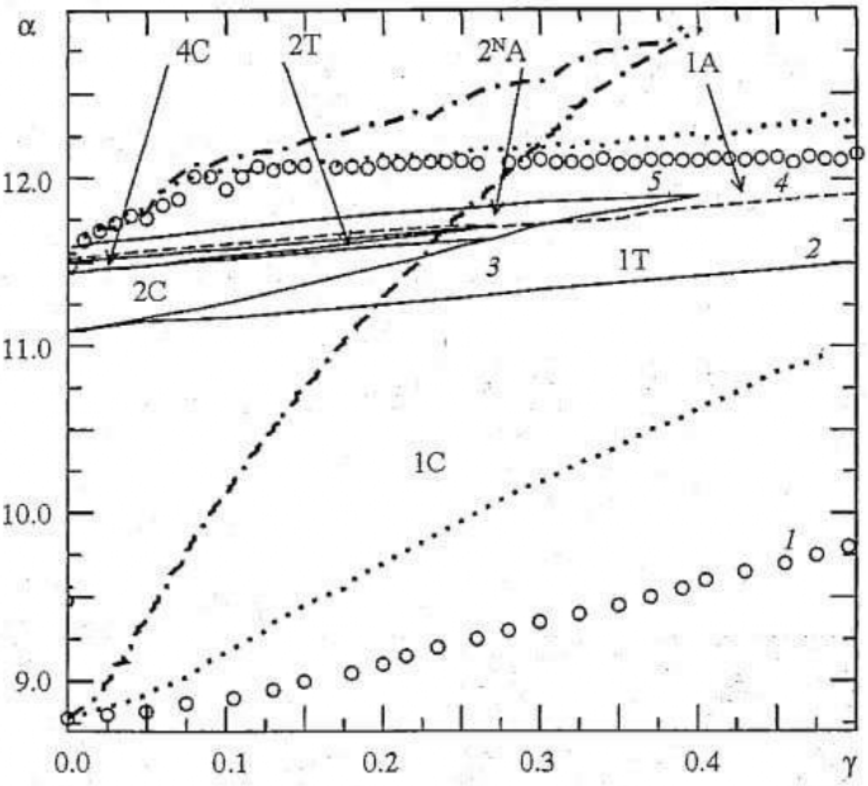

Fig. 1. Diagram of regular and chaotic regimes originated from the wave with spatial period Λ=15 (○); Λ=10 (dot lines); Λ=6 (dot-dashed line)

birth. At large coupling the system evaluates on the base of this torus and demonstrates transition to chaos through the torus breaking (dashed line *4* in the fig.1). Over this line a one-band chaotic attractor exists. At small coupling the transition to chaos occurs through the period-doubling bifurcations and quasi-periodic behavior originated from cycles with double periods. This region of smaller coupling is bounded by the line *3*. In the fig. 1 we also built lines which bound regions of existence of regimes originated from running waves with spatial periods $\Lambda=10$ (dot lines) and $\Lambda=6$ (dot-dashed lines). It is seen that regimes with longer wavelengths occupy larger regions on the parameters plane. The short-lengths waves exist only at rather small coupling. The waves with minimal possible spatial period $\Lambda=2$ (π-waves) were not found in the system possibly because of very narrow region of existence. The bottom boundaries for regimes with different wavelengths coincide at the values of parameters $\alpha=8.78$, $\gamma=0$. Hence, the all mentioned period-one running waves originated from the same equilibrium. The upper boundaries coincide at the values $\alpha\approx11.65$, $\gamma=0$ that corresponds to transition to one-band chaotic attractor in the uncoupled oscillator.

All bifurcations of regular regimes do not change their space periodicity. Oscillations remain exactly spatially periodic with the same periods until the transition to temporal chaos. In the chaotic region the spatial behavior changes its character. The exact space-periodicity destroys immediately after the transition to temporal chaos, but chaotic regimes preserve the space-periodicity in the average. Spatio-temporal diagrams and spatial spectra for these regimes are presented in the fig. 2. They are built for the one-band chaotic regime with averaged spatial period of $\Lambda=16$ for the chain of 1024 elements. The abscissa axis of the spatio-temporal diagram denotes the sites in the chain, the ordinate axis denotes the Poincare section of the variable x_i in the every oscillators observed for the long interval of time. Three parts of the figure demonstrate serial destroying of spatial structure with decreasing of coupling: at $\gamma=0.15$ (*a*), $\gamma=0.05$ (*b*), and

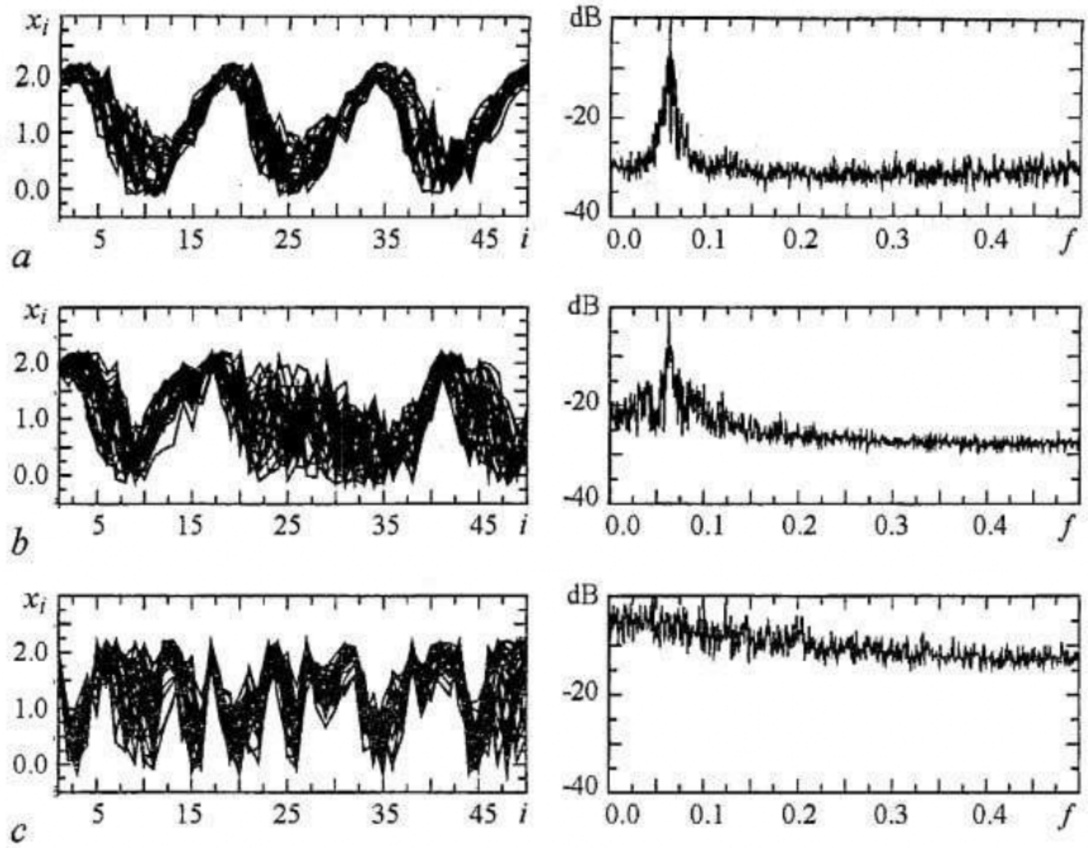

Fig. 2. Destroying of the averaged spatial structure of the wave at α=11.78, with decrease of coupling: γ=0.15 (*a*); 0.05 (*b*); 0.02 (*c*)

γ=0.02 (*c*). The first case corresponds to averaged spatial periodicity with sharp peak in the spatial spectrum (fig. 2, *a*). Then, with decreasing of coupling the spatial diagram begins to lose its periodic structure that is accompany by widening of the peak in the spatial spectrum (fig. 2, *b*) and as a result at small coupling the periodic spatial structure awfully disappears and the spectrum becomes plate (fig. 2, *c*). It is interesting that the destroying of periodic spatial structure with decreasing of coupling takes place only for one-band temporal chaos. If we chose α correspondent to many-band attractor regimes the averaged space-periodicity exists until neglect small couplings.

The periodic spatial structure in the chain is connected with coherence of oscillations on main peaks in the temporal spectra of neighbor oscillators. The structure is preserved until the coherence function on the main peaks is equal to 1. The fig. 3 demonstrates changing in the power spectrum and in the correspondent coherent function for the cases described in the fig. 2. When main harmonics in the spectra are coherent the chaotic regime is almost spatially periodic. Decreasing of coupling leads to decreasing of the coherence function except main frequencies (fig. 3, *b*). This is accompany by gradual destroying of spatial periodicity. Then if the coherence for main peaks becomes smaller than 1 (fig. 3, *c*) the periodic spatial structure awfully breaks.

Summarizing the contents of our research we can conclude that the developing of temporal dynamics in the ring of identical period-doubling oscillators with diffusing coupling does not lead to changing of spatial periodicity until the transition to temporal chaos. In the chaotic region exact spatial periodicity is changed by the periodicity in the averaged. The destroying of the averaged periodic structure takes place only for developed one-band chaotic attractor. It occurs both with developing of chaos and with decreasing of coupling coefficient and is accompanied by the loss of coherence on main

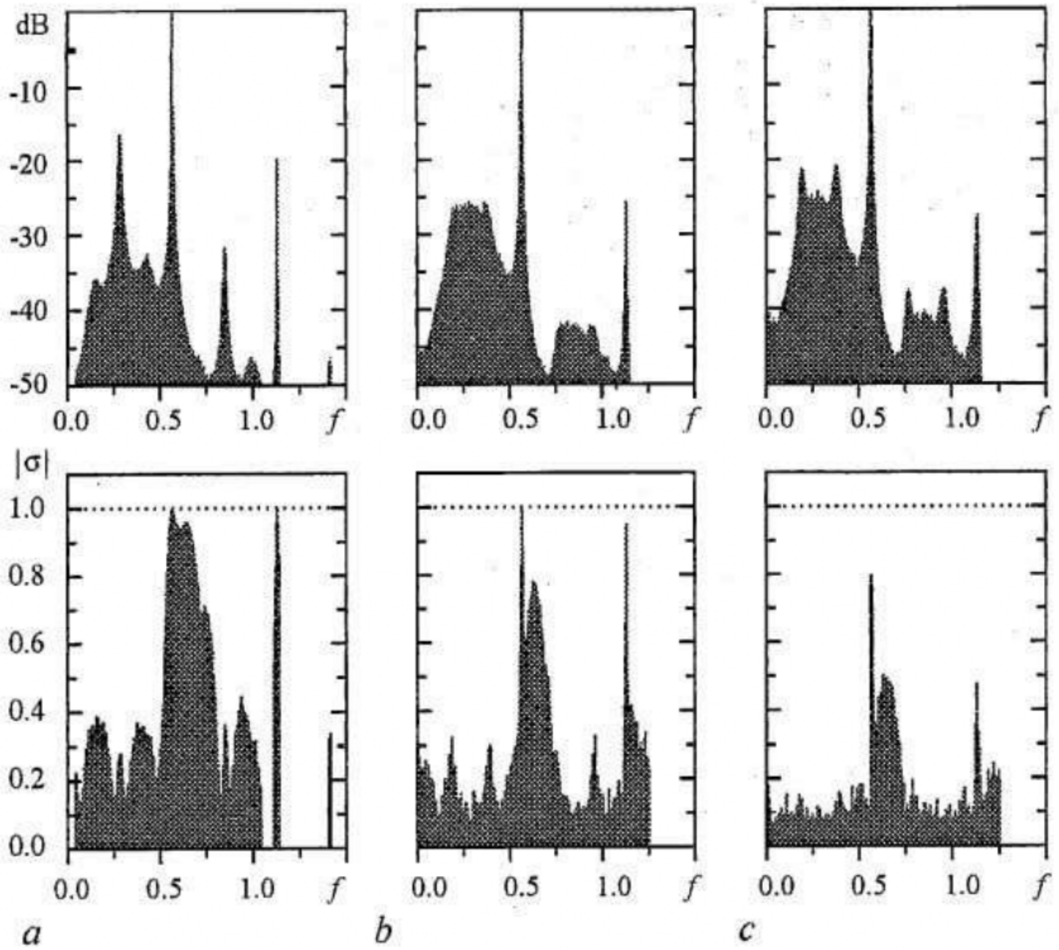

Fig. 3. Loss of the coherence between dynamics of the neighbor oscillators at α=11.78, with the coupling decrease γ=0.15 (*a*); 0.05 (*b*); 0.02 (*c*)

peaks in the temporal spectra of neighbor oscillators. The family of regimes with determine spatial period exists in the limited range of the coupling. If the coupling is increased over the determine maximal values the system transits to a regime with larger spatial period.

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ЭВОЛЮЦИЯ БЕГУЩИХ ВОЛН К ПРОСТРАНСТВЕННО-ВРЕМЕННОМУ ХАОСУ: ВЗАИМОДЕЙСТВИЕ ВРЕМЕННОЙ И ПРОСТРАНСТВЕННОЙ ДИНАМИКИ В КОЛЬЦЕ ГЕНЕРАТОРОВ С УДВОЕНИЕМ ПЕРИОДА

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В данной работе рассматривается переход от регулярных бегущих волн к развитому пространственно-временному хаосу в цепочке осцилляторов с

удвоением периода. Исследуются типичные бифуркации, которые происходят на основе выбранного режима бегущей волны от цикла периода один до развитого временного хаоса. Обнаружено, что до перехода к временному хаосу колебания остаются пространственно периодическими. В области хаоса точная пространственная периодичность сменяется периодичностью в среднем. Разрушение усредненной пространственно-периодичной структуры связано с потерей когерентности на основных частотах во временных спектрах соседних генераторов в цепочке.

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