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SELF-CONSISTENT PARTICLE DYNAMICS IN THE GEOTAIL MAGNETIC FIELD REVERSAL

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Dynamics of ions in the geotail magnetic field reversal plasmas is modelled with a hybrid code. Poincaré maps are calculated for stationary and for adiabatically changing field configurations starting from an anisotropic pressure self-consistent equilibrium. It is shown that the essential dynamics as found previously for single particle in prescribed fields persists in the hybrid code simulations of self-consistent fields. The possible interplay of dynamical processes in the Earth's magnetosphere and in the solar wind is discussed.

The Earth magnetosphere (Fig. 1) has the long magnetotail directed outwards of the sun. The magnetotail is thought to operate as a storage of the energy accumulated from the solar wind, i.e. from the inflow of the space plasma coming from the Sun. The consequent releases of the stored energy can cause magnetic substorms which have impact on the Earth inhabitants. One of the mechanisms connected with the origin of the energy releases is thought to be the changes in the complex dynamics of charged particles in the magnetosphere. We simulate particle dynamics with a hybrid code and calculate Poincaré maps for the particle trajectories in a self-consistent, adiabatically changing field.

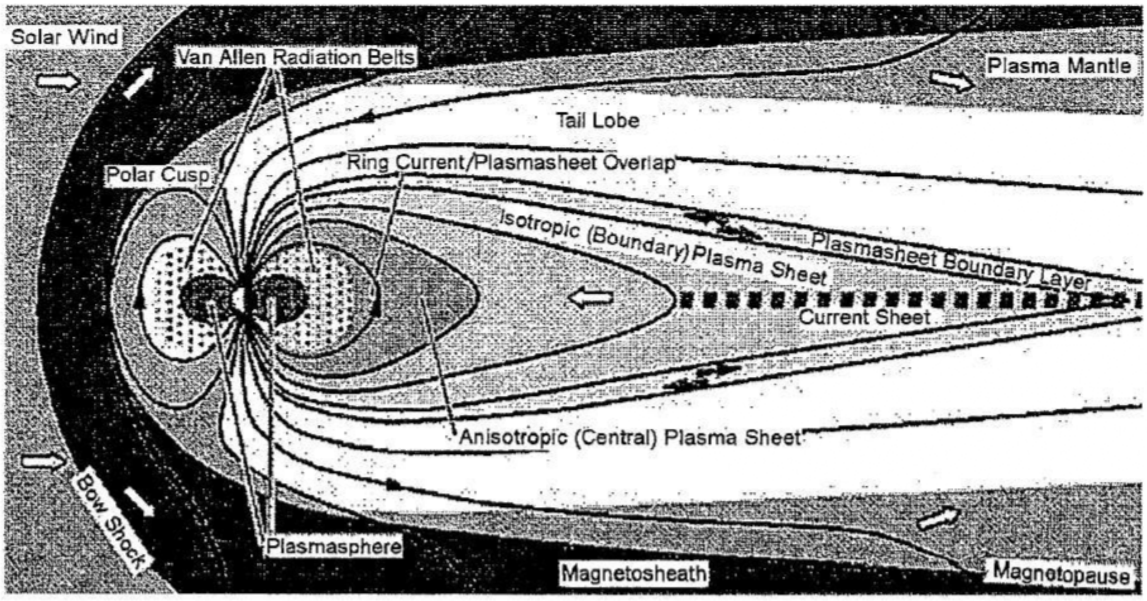

Fig. 1. Earth magnetosphere [1]

1. Method

In the magnetohydrodynamics (MHD) the plasma particle distribution function is Maxwellian and it defines its density n, and the first and the second moments (bulk velocity and pressure), $\mathbf{U}$, p. The MHD equations can be obtained by integration of the Vlasov equation [2,3]. But the MHD approach can not be used if the plasma distribution function can be essentially non-Maxwellian. Then other approaches allowing for an arbitrary distribution function could be used, such as the hybrid, particles-in-cell and Vlasov codes. The MHD approach also cannot be used if the time and space scales of the problem are of the order of the ion-gyromotion scales, because of appearing time dependence in the distribution function on the scales of averaging.

In the hybrid code [4-6] the ions are considered as particles with an arbitrary distribution function while the electrons are still considered as a massless fluid with the Maxwell distribution function. This approach is computationally efficient, as it allows us do not resolve the small time scales of the electron gyromotion. The particles (ions) move accordingly to the equation of motion:

$$d\mathbf{v}_j/dt = q/m(\mathbf{E}+\mathbf{v}_j\times\mathbf{B}), \tag{1}$$

and the field can be found from the Maxwell equations in the low frequency limit,

$$\partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E}, \quad \nabla\times\mathbf{B} = \mathbf{J}, \ \nabla\mathbf{B} = 0 \tag{2}$$

$$\mathbf{E} = -\mathbf{U}\times\mathbf{B} + (\nabla\times\mathbf{B}\times\mathbf{B}-\nabla P_e)/nq \tag{3}$$

$$\mathbf{U}_e = \mathbf{J}/nq - \mathbf{U}, \tag{4}$$

where $\mathbf{U}$ is the ion bulk velocity and $\mathbf{U}_e$, is the electron bulk velocity. The electron density is equal to the ion density, n, because of the plasma quasi-neutrality, and the electron pressure, P_e, can be found from the electron fluid equation,

$$\partial P_e/\partial t = -\mathbf{U}_e\nabla P_e - \gamma P_e\nabla\,\mathbf{U}_e.$$

The moments of the ion distribution function: the density n and the bulk velocity $\mathbf{U}$, can be obtained by averaging:

$$n(\mathbf{x},t) = \int_{\mathbf{v}} f(\mathbf{v},\mathbf{x},t)d\mathbf{v} + \xi_n, \qquad \mathbf{U}(\mathbf{x},t) = (1/n\,(\mathbf{x},t))\int_{\mathbf{v}}\mathbf{v}f(\mathbf{v},\mathbf{x},t)d\mathbf{v} + \xi_{\mathrm{U}}. \tag{5}$$

The limited number of ions, N, on the computational grid leads to appearing of statistical errors of averaging, ξ_n, ξ_{U}, which makes the code noisy. The noise intensity is inversely proportional to $N^{1/2}$, and proportional to the temperature. The statistical noise sources ξ_n, and ξ_{U} can be considered as internal noise sources of the plasma. Because the considered space plasma is collisionless with zero diffusion, it is necessary to introduce the numerical diffusion to stabilize the code.

2. A self-consistent magnetic field reversal

A simple one-dimensional ($\partial/\partial x=\partial/\partial y=0$) model for the magnetotail is the modified Harris field reversal (Fig. 2):

$$B(z) = B_0\tanh(z/L)\hat{\mathbf{x}} + B_n\hat{\mathbf{z}}. \tag{6}$$

We will study the self-consistent field of form (6) created by the motion of charged particles in that field. In order to get the reversal varying on timescale much slower than that of the particles we want to set the reversal into initial equilibrium: $\partial p/\partial t=0$, $\partial\mathbf{U}/\partial t=0$, $\partial\mathbf{B}/\partial t=0$. Using (2), (3), and (6), we obtain from the condition $\partial\mathbf{B}/\partial t=0$ that

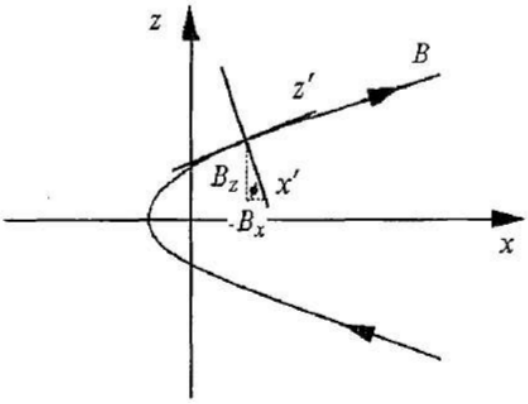

Fig. 2. Harris field reversal and axes of rotated distribution function

$$\mathbf{U} = ((1/\rho)\partial B_x/\partial z + C_{\mathrm{U}})\hat{\mathbf{y}}, \qquad (7)$$

and $\partial\rho/\partial t$=-$\nabla(\rho,\mathbf{U})$=0. From $\partial\mathbf{U}/\partial t$=0 using (7) and the MHD equation of conservation of the momentum one gets the *pressure balance* equation:

$$\mathbf{J}\times\mathbf{B} = \nabla\mathbf{P} + \nabla P_e \qquad (8)$$

which in the one-dimensional case considered is: $\nabla\mathbf{P}=\hat{\mathbf{x}}\partial P_{zx}/\partial z+\hat{\mathbf{y}}\partial P_{zy}/\partial z+\hat{\mathbf{z}}\partial P_{zz}/\partial z$ [7]. It is known that in the one-dimensional case (6) the pressure balance can not be satisfied with the isotropic pressure tensor [8]. In order to obtain an anisotropic pressure tensor one can use the bi-Maxwellian distribution function with different temperatures in the directions perpendicular and parallel to the magnetic field [3]

$$\begin{gathered} f(\mathbf{x},\mathbf{v},t) = n/(T_\perp T_\parallel^{1/2})[m/(2\pi k_B)^{3/2}] \times \\ \times \exp[2(mv_\perp - U_\perp)^2/(2k_B T_\perp)-(mv_\parallel-U_\parallel)^2/(2k_B T_\parallel)]. \end{gathered} \qquad (9)$$

The pressure tensor corresponding to the bi-Maxwellian distribution function is diagonal in the local field-aligned coordinates,

$$p_\parallel = n(\mathbf{x},t)k_B T_\parallel, \quad p_\perp = n(\mathbf{x},t)k_B T_\perp. \qquad (10)$$

It is convenient to use the cartesian coordinates rotated to the angle $\varphi(z)$=arctan($-B_x(z)/B_z$) (Fig. 2) as the local coordinates. The tensor components in these Cartesian coordinates are:

$$P_{ij}(z) = mn\int(v'_i-U_i)(v'_j-U_j)f(z,\mathbf{v}')dv_i\,dv_j, \qquad (11)$$

where $f(z,\mathbf{v}',t)$ is the local (rotated) ion distribution function,

$$\begin{gathered} f(z,\mathbf{v}',t) = n/(2\pi k_B)^{3/2}[m/(T_{x'}T_{y'}T_{z'})^{1/2}]\exp(-m/(2k_B)) \times \\ \times \exp[(v_{x'}-U_{x'})^2/T_{x'}-(v_{y'}-U_{y'})^2/T_{y'}-(v_{z'}-U_{z'})^2/T_{z'}], \end{gathered} \qquad (12)$$

where v_x' and v_z' are the velocity vector components in the local coordinates. Substituting (12) into (11) and integrating, one can obtain:

$$\begin{gathered} P_{zx}(z) = \sin 2\varphi(z)/(2m)(n(z)k_B T_\parallel(z)-n(z)k_B T_\perp(z)), \quad P_{zy}(z) = 0, \\ P_{zz}(z) = 1/m(n(z)k_B T_\parallel(z)\cos^2\varphi(z)+n(z)k_B T_\perp(z)\sin^2\varphi(z)). \end{gathered} \qquad (13)$$

Substituting (13) into (8), integrating and using (10), one can get the components of the equilibrium anisotropic pressure tensor:

$$p_\perp(z) = P_{m0} - P_m + \text{const}, \quad p_\parallel(z) = P_{m0} + P_m + \text{const}, \qquad (14)$$

where P_m=$B^2/(2\mu_0)$ is the magnetic pressure and P_{m0}=$(B_0^2+B_n^2)/(2\mu_0)$ is the maximum magnetic pressure.

The equation of state for the plasma with anisotropic pressure tensor aligned with directions parallel and perpendicular to the **B** field may be obtained in the CGL theory [8]:

$$(\partial/\partial t + \mathbf{U}\nabla)(p^2_{\perp} p_{\parallel}/\rho^5) = 0. \tag{15}$$

From (15) the equilibrium number density is:

$$n(z) = (p_{\perp}^2(z) p_{\parallel}(z))^{1/5}. \tag{16}$$

The obtained self-consistent equilibrium given by the functions (6), (7), (10), (14), and (16) is stable to the local mirror-mode perturbation: $(P_{\perp}/P_{\parallel})(P_{\perp}-P_{\parallel})<B^2/(2\mu_0)$, and neutrally stable to the local firehose-mode perturbations: $P_{\parallel}-P_{\perp}=B^2/\mu_0$.

To study the long-time stability of this magnetic reversal, we simulated its dynamics with the hybrid code discribed above. The simulation showed that the self-consistent equilibrium is stable for a relatively long time, $t>100P_g$, where P_g is the ion gyroperiod, and it slowly diffuses later due to the numerical diffusion introduced into the code. The numerical diffusion can be taken into account in the pressure balance equation (8)

$$\mathbf{J}\times\mathbf{B} = \nabla\mathbf{P} + \nabla P_e - D_n \nabla^2 \mathbf{B}. \tag{17}$$

It leads to the appearance of an additional component of the bulk velocity,

$$U_x = -D_n/B_z(\partial B_x/\partial z), \tag{18}$$

where the diffusion coefficient is $D_n=\Delta x^2/(4\Delta t)$, where Δt and Δx are the time and space steps of the grid. Then the diffusion of the equilibrium gradually decreases.

3. The Poincaré maps

The Poincaré surface of section can be used to study the nonlinear particle dynamics [9,10]. In the map, each crossing of a chosen surface of section in the phase space by the phase trajectory in a chosen direction corresponds to a point. The ensemble of these points defines a Poincaré map. In this approach, a periodical trajectory corresponds to a finite number of points, ergodic tori correspond to a closed curves and chaotic motion corresponds to chaotic set of points.

In general, the equation of motion for a single particle defines a six-dimensional phase space. In the one-dimensional case (6), the equations are:

$$dX/dt = v, \quad dv/dt = f(z,v), \quad X, v \in \mathrm{R}^3. \tag{19}$$

One could see that in (19) there is a partial subsystem of four equations which are independent of the other two. That subsystem is four-dimensional. For a chosen constant energy, $H=mv^2/2$ and a chosen direction of the crossings, e.g. $dv_z/dt>0$, set (19) corresponds to a two-dimensional map in the surface of section. There is also a transformation of variables based on the existence of constants of the motion P_y, and C_x [11]:

$$\begin{gathered} x' = (x-P_y/(m\omega_n))/(B_n L), \\ y' = (y+C_x/(m\omega_n))/(B_n L), \quad z' = z/(B_n L), \end{gathered} \tag{20}$$

where B_n and L are as in (6), and $\omega_n=qB_n/m$. The transformation (20) reduces the system (19) to a four-dimensional system with the two-dimensional map on the surface of section z=0. If the magnetic field is prescribed, the Poincaré map can be calculated from the particle equation of motion (19) [12]. The Poincaré maps for different energies corresponding to different layers in the distribution function are presented in Fig. 3. In the figure, one can observe periodic and ergodic tori, chaotic trajectories and transient (Speiser) trajectories carrying the currents (empty areas).

In order to calculate the Poincaré maps for the self-consistent field one should take into account the following problems. The fields created by the particles are affected by the statistical noise (5), that makes the maps noisy and gives them some width in the appearing third dimension (since the energy is no longer conserved). Affected by the noise, the particles can move into different areas of the phase space changing the behaviour of their motion. The diffusion of the field reversals increases the particles energy and makes the maps time-dependent. The noise and diffusion lead to development of a small B_y component that rotates and transforms [12] the Poincaré maps. The noise intensity can be reduced by the increase of number of particles in the grid. The Poincaré maps which were calculated for the self-consistent reversal are presented in Fig. 3. The Poincaré maps shown that the essential dynamics of single particles in prescribed fields [11] persists in the hybrid code simulations of self-consistent fields.

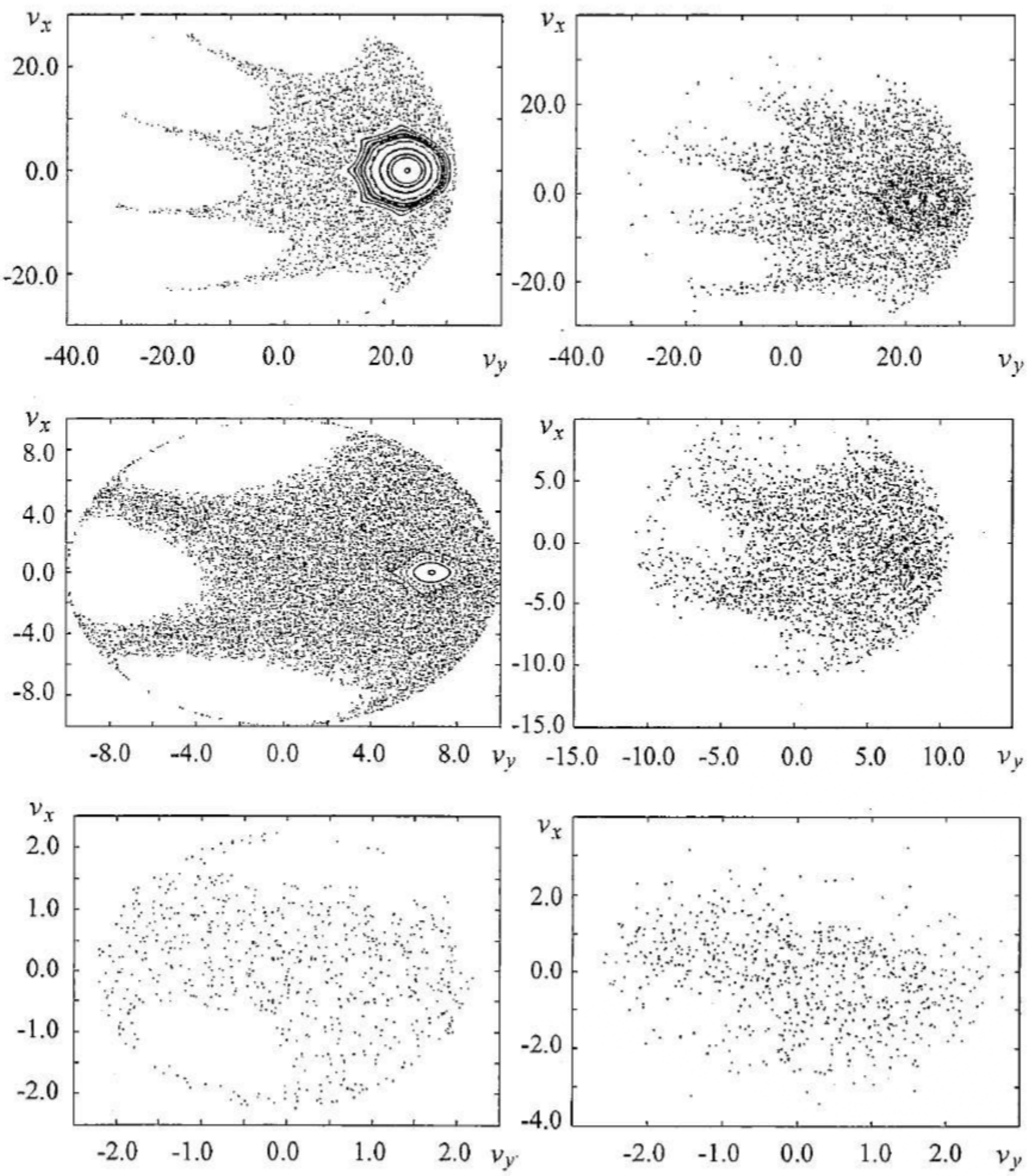

Fig. 3. Poincaré maps for prescribed magnetic field (left column) and for self-consistent magnetic field (right column). Dimensionless energy H is 500; 50; 2.5 from top to bottom

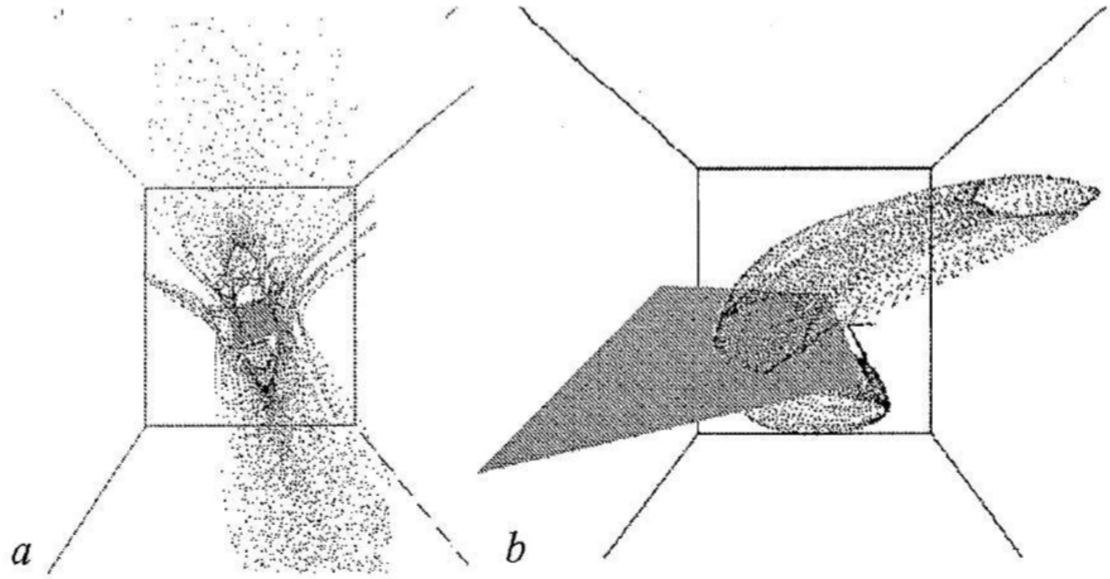

Fig. 4. Screen-shots of the virtual reality representation of particle trajectories in the phase space (v_y, v_x, z). *a*) Chaotic particle trajectory, *b*) quasiperiodic trajectory lying on a three-dimensional projection of a four-dimensional ergodic torus and its surface of section

The virtual reality set at Warwick space and Astrophysics group provides us with a useful tool for studying complex particle trajectories. The 3D semi-immersive environment creates the effect of presence of studied objects flying in the air in the dark room with the observers. The objects could be interactively stirred (arbitrary scaled, moved or rotated) which allows us to analyse the geometry of the complex particles trajectories and corresponding Poincaré maps in details as shown in Fig. 4.

4. Time dependent field reversal

During magnetic substorms the geometry of the Earth field reversal changes in time, interacting with the charged particles. As a simple model of a time dependent field reversal, the time dependence appearing in the reversal due to instability of the equilibrium caused by noise and numerical diffusion was chosen. Hybrid simulations showed that the time dependence could be well approximated by the following expressions:

$$B_x = B_0 \tanh(z/L(t)), \quad L(t) = L_0 + t/\tau, \tag{21}$$

where the value of τ depends on the initial energy of particles. Chapman et al., [13,14] showed that it is possible to introduce two dimensionless parameters: the parameter of adiabaticity of the system α, and the phase of the process, αt [13],

$$\alpha = (B_x/B_z)(\rho_z/L)(\Omega_f/\Omega_z),$$

where ρ_z is the gyroradius, and $\Omega_f = 1/\tau$. The changes in the behaviour of particles motion and time of the changes are defined by parameters α and αt. These parameters define the dynamics of the particles moving in the slowly changing magnetic field reversal. The detailed comparison of the self-consistent simulations with the predictions of the prescribed field theory [13] is the aim of the further studies.

5. Discussion

A sudden destruction of the geotail magnetic reversal because of the field slow

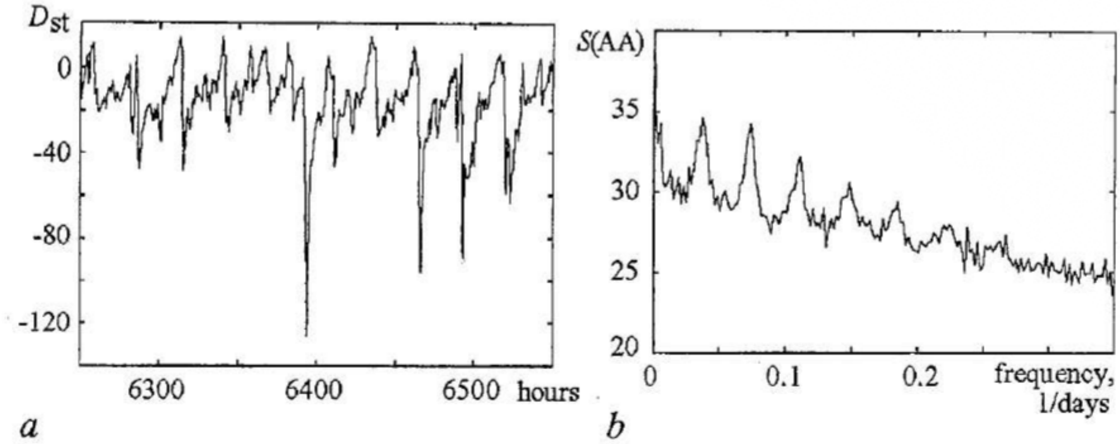

Fig. 5. *a*) D_{st} geomagnetic index, large negative drops correspond to magnetic substorms; b) power spectrum density of AA geomagnetic index averaged for 30 years

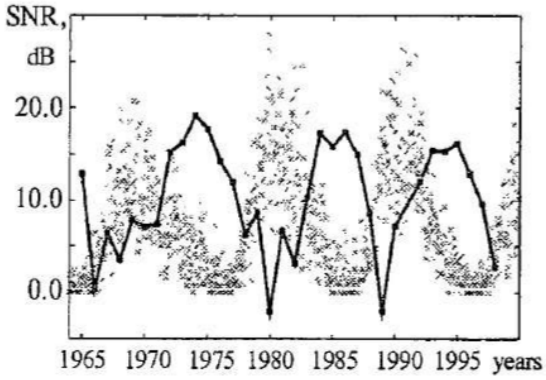

Fig. 6. Signal-to-noise ratio (SNR) in AA power spectrum (bold line), and Sun spot number, SSN (dots) divided by 10

evolution leads to the release of the energy accumulated from the solar wind, causing the events of storms and substorms. The understanding of the reversal destruction process would allow us to link statistical properties of geomagnetic activity with statistical properties of the solar wind. For example, the time dependence of the Earth geomagnetic index D_{st} shown in Fig. 5, *a* demonstrates the behaviour similar to the behaviour of simple integrate and fire systems affected by noise. This suggests the idea to try to use the simple stochastic integrate-and-fire models for the magnetospheric substorm studies. There is also periodicity in the solar wind inflow caused by the rotation of the sun around its axes with the period of approximately 27 days [15]. That periodicity can also be seen as the first peak in the power spectra which we calculated for the Earth geomagnetic index as shown in Fig. 5, *b*. The strength of the periodic component is not constant but correlates with the phase of the solar cycle (the number of the sun spots or solar activity) as presented in Fig. 6. Such a behaviour may be associated with the phenomenon of stochastic resonance [17,16] and needs to be studied.

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САМОСОГЛАСОВАННАЯ ДИНАМИКА ЧАСТИЦ В РАЗВОРОТЕ ПОЛЯ ГЕОМАГНИТНОГО ХВОСТА

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Динамика ионов в плазме геомагнитного хвоста моделируется с помощью гибридного кода. В случаях заданного и самосогласованного равновесия с медленно меняющимся магнитным полем вычисляются отображения Пуанкаре. Показано, что в рассматриваемом случае самосогласованного равновесия сохраняются основные свойства динамики заряженных частиц в заданных полях. Обсуждается возможная связь между рассматриваемыми динамическими процессами в Земной магнитосфере и в солнечном ветре.

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