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RIDDLING IN THE PRESENCE OF SMALL PARAMETER MISMATCH

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Riddling bifurcation leads to the loss of chaos synchronization in coupled identical systems. We discuss here the manifestation of the riddling bifurcation for the case of a small parameter mismatch between coupled systems. We show that for slightly nonidentical coupled systems, the transverse growth of the synchronous attractor is mediated by transverse bifurcations of unstable periodic orbits embedded into the attractor.

Introduction

Consider two symmetrically coupled identical systems $dx/dt=f(x)$ and $dy/dt=f(y)$ and $x, y \in R^n$ which evolve on an asymptotically stable bounded chaotic attractor A,

$$dx/dt = f(x) + C(y-x), \quad dy/dt = f(y) + C(x-y). \tag{1}$$

Complete synchronization occurs when the coupled systems asymptotically exhibit identical behaviour, i.e., $|x(t)-y(t)|\to 0$ as $t\to\infty$. The synchronous behaviour takes place on the synchronization manifold $x=y$, which is invariant in the phase space of the coupled system (1) and has half the dimension of the full system. The synchronization loss in system (1) is initiated with the riddling bifurcation [1] when the first unstable periodic orbit (UPO) embedded into chaotic attractor A loses its transverse stability. In this paper we discuss the manifestation of the riddling bifurcation for the case of a small parameter mismatch between coupled systems. We give evidence that for slightly nonidentical coupled systems, the transverse growth of the synchronous attractor is mediated by transverse bifurcations of unstable periodic orbits embedded into the attractor. The desynchronization mechanism is shown to be similar to the bifurcation of chaos-hyperchaos transition [2]. We also note that the parameter mismatch leads to the increase of transverse instabilities after the riddling bifurcation.

Model

Without loss of generality, a small difference between coupled systems can be incorporated in (1) as

$$dx/dt = f(x) + \alpha(x) + C(y-x), \quad dy/dt = f(y) + C(x-y), \tag{2}$$

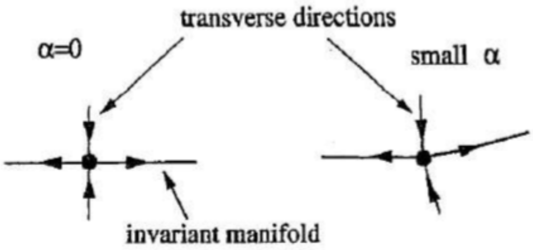

Fig. 1. Small parameter mismatch implies small perturbation of stable and unstable manifolds of saddle periodic orbits embedded in an attractor

where $\alpha(x)$ describes parameter mismatch. For sufficiently small α the evolution of the system (2) can be considered as the perturbed evolution of the system (1), so the motion of the system can be asymptotically close to the synchronization manifold $|x-y|<\varepsilon$ with small ε.[1] In these cases, the attractor of the system (2) is located in the neighborhood of the invariant manifold $x=y$ of system (1). For sufficiently small α, transverse stability of orbits embedded in A is also preserved in system (2).

It is also meaningful to speak about transverse and longitudinal stability of saddle periodic orbits embedded in the attractor A since a sufficiently small mismatch will cause only small perturbation of the local unstable and stable manifolds and will not affect stability properties of the UPOs, as sketched in Fig. 1.

Therefore, the moment of riddling bifurcation will correspond to the loss of transverse stability of some orbit embedded in the attractor. Here, of course, the situation may arise when the above mentioned orbit leaves the attractor before its transverse destabilization as it was described in [5]. In this situation, we may consider the remaining orbits that lose transverse stability with decrease of a coupling coefficient. In general, for nonidentical systems, we are dealing with a chaotic attractor which is no longer located in low-dimensional synchronization manifold but remains in the neighborhood of it. Moreover, periodic orbits embedded into this attractor are proved to lose transverse stability with the decrease of coupling [6]. Therefore, we have the same situation as for chaos-hyperchaos transition [2,4] where the growth of the attractor is mediated by doubly unstable orbits embedded in it. It was shown in [2] that this growth can be either smooth or abrupt depending on the type of «riddling» bifurcation.

In the following as the numerical example, we consider two coupled Rössler systems

$$dx/dt = f(x) + \bar{\alpha} + C(d)(y-x),$$
$$dy/dt = f(y) + C(d)(x-y), \tag{3}$$

where $C(d) = \mathrm{diag}\{d-0.6, 1.0, -3.1d+0.7\}$, $\bar{\alpha} = (0,0,\alpha)$,

$$f(x) = (-x_2 - x_3, x_1 + 0.42x_2, 2+x_3(x_1-4))^T.$$

The mismatch is introduced via parameter α.

It was shown in [8] that the corresponding system of identical coupled oscillators, i.e. for $\alpha=0$ loses complete synchronization with the decrease of parameter d. In particular, the riddling bifurcation occurs at $d=0.241$ when the embedded period-1 cycle becomes transversely unstable via supercritical transverse period-doubling bifurcation. At $d\approx0.192$ the blowout bifurcation takes place when transverse Lyapunov exponent of the synchronous attractor becomes negative. Note also, that using numerical simulation of coupled identical systems we were unable to detect bursts from the synchronization manifold for the parameter values $d\in(0.22,0.24)$, i.e. where synchronous attractor has already lost its transverse stability but is still weakly stable.

[1] This is the case, for example, when the synchronous object in system (1) is normally a hyperbolic torus or a saddle periodic orbit embedded into the attractor. Some generic cases where such estimation holds are also described in [3].

In the case for systems with the mismatch the above mentioned transverse period-doubling bifurcation persists and for α=0.003 it takes place at d=0.24. Numerically computed Lyapunov exponents for the system (3) are shown in Fig. 2. In the interval I the chaotic attractor A is located in the neighborhood of the manifold x=y. We observe the growth of the second Lyapunov exponent what is connected with the riddling bifurcation at d=0.24 and initiation of the chaos-hyperchaos transition. As it was shown in [2], this transition is mediated by the transverse destabilization of UPOs embedded in the chaotic attractor A. In the interval II, the system (3) has the stable hyperchaotic attractor with two positive Lyapunov exponents. At $d\approx$0.21 chaotic attractor A becomes unstable and disappears. The evolution of the system (3) switches to the limit cycle (interval IIIa) and torus (interval (IIIb). Fig. 3 shows the behavior of the synchronization error $x_1(t)$-$y_1(t)$ for different values of d. We can observe transverse bursts for the parameter values after the moment of riddling bifurcation (Fig. 3, b, c). More detailed information about the transverse size of the attractor can be seen in Fig. 4, where the maximum amplitude of bursts detected during time interval T=200000 versus coupling coefficient d is shown. It

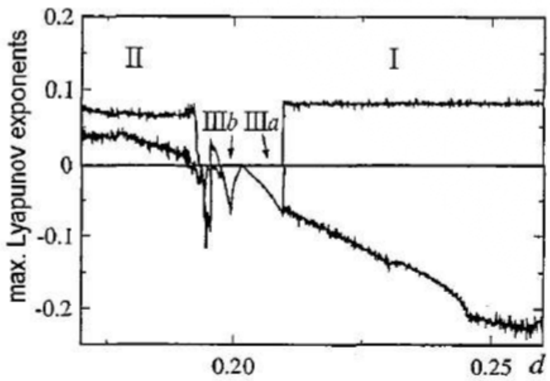

Fig. 2. Lyapunov exponents of system (3) versus d; α=0.003: I - interval in which chaotic attractor A is located in the neighborhood of the manifold x=y, II - interval in which hyperchaotic attractor exists, III - interval where the chaotic attractor A loses stability and solution switches into stable limit cycle (IIIa) and torus (IIIb)

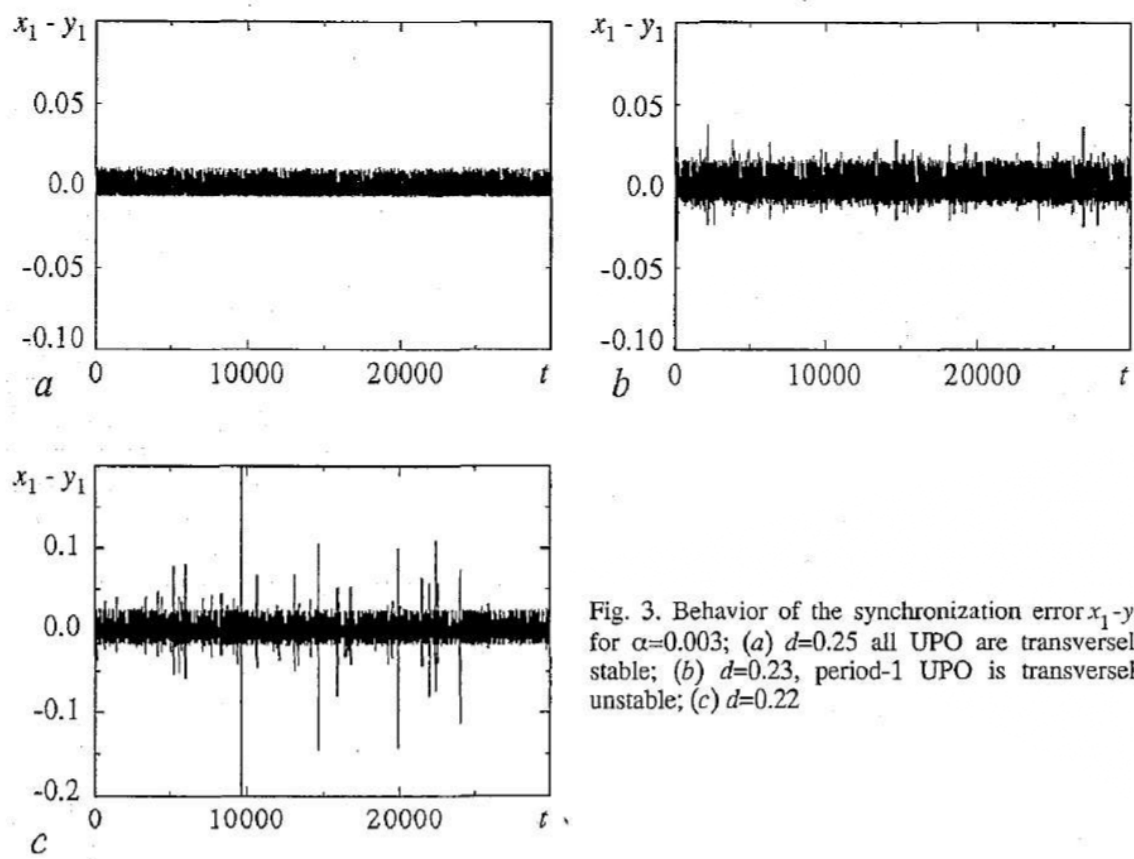

Fig. 3. Behavior of the synchronization error x_1-y_1 for α=0.003; (a) d=0.25 all UPO are transversely stable; (b) d=0.23, period-1 UPO is transversely unstable; (c) d=0.22

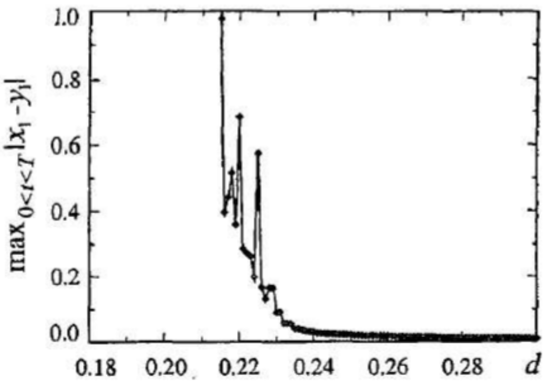

Fig. 4. Transverse growth of the attractor with decreasing of d; α=0.003

can be seen that the attractor grows rapidly in transverse direction already after the riddling bifurcation.

In the case of the ideal coupled systems the chaotic attractor A located at the invariant manifold $x=y$ can have locally or globally riddled basins of attraction. A is an attractor[2] with *locally riddled* basin if there is neighborhood U of A such that in any neighborhood V of any point in A, there is a set of points in $V \cap U$ of positive measure which leaves U in a finite time. The trajectories which leave neighborhood U can either go to the other attractor (attractors) or after a finite number of iterations be diverted back to A. If there is neighborhood U of A such that in any neighborhood V of any point in U, there is a set of points of positive measure which leaves U and goes to the other attractor (attractors), then the basin of A is *globally riddled.*

Conclusions

In conclusion, we investigated the effect of riddling bifurcation on the chaotic attractor of the coupled systems with the parameter mismatch. After the onset of bifurcation, the system trajectory shows intermittency-like behavior with bursts away from the manifold $x=y$. These bursts grow rapidly resulting in the growth in size of the chaotic attractor. Contrary to the case of the coupled ideal systems we have not observed globally riddled basins of the chaotic attractor located in the neighborhood of the manifold $x=y$.

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[2] Here we assume the attractor in the sense of Milnor [9].

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РИДЛИНГ В ПРИСУТСТВИИ МАЛОЙ РАССТРОЙКИ ПО ПАРАМЕТРУ

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Ридлинг-бифуркация приводит к потере синхронизации хаоса в связанных идентичных системах. В статье обсуждается проявление ридлинг-бифуркации для случая малой расстройки по параметру между связанными системами. Показано, что для немного неидентичных связанных систем уширение синхронного аттрактора обуславливается трансверсальной бифуркацией неустойчивых периодических орбит, встроенных в аттрактор.

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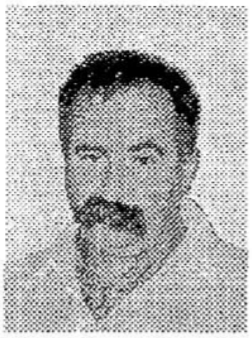

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