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INVESTIGATION OF TRANSIENT CHAOS IN GYRO-BACKWARD-WAVE-OSCILLATOR SYNCHRONIZED BY THE EXTERNAL SIGNAL

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In this work we explore the transient chaos in non-autonomous, distributed active medium (gyro-backward-wave oscillator synchronized by the external signal). The transient chaos characteristics near the synchronization tongue boundaries are investigated. Special attention is payed to the building of long time series which is used to appreciate the characteristics of system dynamics. The time series is constructed by gluing of short time realizations which characterize the transient chaos observed in the distributed system.

Introduction

In our days great interest causes the questions of microwave signals generation and amplification in gyroresonance devices with the travelling wave and the backward wave, based on the interaction of the unmoderated electromagnetic waves with the spiral electron beam (gyro-BWO and gyro-BWT). Such devices are actively examined theoretically and experimentally [1-5].

The non-autonomous active medium «spiral electron beam - backward electromagnetic wave» demonstrates several non-linear effects like periodical and chaotical modulation of the output signal, synchronization of the gyro-BWO by the external signal etc. [5-9]. In this work we show the possibility of appearance of a phenomenon of transient chaos in such system. As distinct from the «classic» dynamical chaos (its image in phase space is strange attractor and the phase trajectories tend to it with $t\to\infty$), under the term «transient chaos» [10-12] the follow phenomena is ment: in the phase space of the system there exists the so-called chaotic saddle - a chaotic set which is unstable in one of directions. The phase path starting from the points situated near the chaotic saddle for a long time demonstrates chaotic behaviour and henceforth quits from its vicinity and reaches the attractor which may be regular of chaotic.

The unstable chaotic set might be characterized by the same parameters as a strange attractor (dimension, Lyapunov exponent etc.) In this case the characteristics can be carried out from ensemble of short time series describing the transient chaotic process in the investigated system. In this case usually the procedure of gluing of the time series is used. Mostly, the transient chaos is investigated in simple finite-dimensional systems with discrete and continuous time.

In this work we investigate transient chaos in a distributed (and therefore, infinite-dimensional) active medium containing oscillations-electrons. The characteristics of transient chaos in non-autonomous gyro-BWO are examined. Special attention is payed to the problem of correct building of long time series using the short series derived from the distributed system.

The structure of the work is the follows. In sec. 1 the mathematical model of the investigated system is set, the admissions used in its constructing and the boundaries of its adaptability are discussed. In sec. 2 the oscillation regimes in non-autonomous gyro-generator with backward wave are discussed. The parameters' interval, in which the regime of transient chaos realizes, is explored. Sec.3 is devoted to the question of building of long time series produced by distributed system in transient chaos regime. In sec. 4 some characteristics of transient chaos (dimension, maximal Lyapunov exponent) are derived from the constructed time series.

1. General formalism

When the spiral electron beam interacts with the TE-modes of the waveguide and the waves synchronizm condition is fulfiled

$$\omega \approx \hat{\omega}, \quad \hat{\omega} + \beta_0(\hat{\omega})v_{\parallel} - \omega_c = 0, \tag{1}$$

we can observe high-frequency generation [13, 14]. Here $\hat{\omega}$ is the synchronism frequency, ω_c is the cyclotron frequency, $v_{\parallel}$ is the electrons longitudinal velocity, i.e. the velocity which is parallel with the applied magnetic field, $\beta_0(\hat{\omega})$ is the distribution constant of the waveguide without electron beam.

In such system accelerative grouping of the electrons takes place. It is caused by relativistic non-isochronism of the electrons-oscillators of the spiral (or polyspiral) beam. One of the peculiarities of this system is the possibility of retuning the generation frequency by the changing of the longitudinal electrons velocity $v_{\parallel}$ or the static magnetic field B_0. In real systems for such purpose it is necessary to change the geometry of the waveguide and the value of the magnetic field along the interaction space [15, 16]. From this sight the model described in our paper is idealized.

The interaction between the weakly relativistic spiral beam and the backward wave is described by the self-consistent system of movement equation [13] and stimulation equation [17]

$$d\beta/d\xi - j\,\mu(1 - |\beta|^2\beta = F, \tag{2}$$

$$\partial F/\partial\tau - \partial F/\partial\xi = -I, \quad I = {}^1\!/_{2\pi}\int_0^{2\pi} \beta d\theta_0, \tag{3}$$

where $\beta = r\exp(j\theta)$ is the complex radius of the trajectories of the ensemble electrons, which are initially distributed by phases relatively the HF field, $F=F(\xi,\tau)$ is the slowly changing complex non-dimensional amplitude of the field in the beam section, $I=I(\xi,\tau)$ is the first harmony of the grouped current, $\xi=\beta_0(\hat{\omega})\varepsilon z$ is the nondimensional longitudinal coordinate, $\tau=\hat{\omega}\varepsilon(t-z/v_{\parallel})(1+v_{\parallel}/|v_g|)^{-1}$ is the non-dimensional time in the coordinate system moving with the longitudinal beam velocity $v_{\parallel}$, $\hat{\omega}$ is the frequency satisfying the synchronism condition (1), $\beta_0(\hat{\omega})$ is the coefficient of propagation of the backward wave with the frequency $\hat{\omega}$ in the system without electron beam, v_g is the wave group velocity on the frequency $\hat{\omega}$.

Besides we bring in the following parameters: $\mu=(v_{\parallel}/c)/2\varepsilon$ is the non-isochronism parameter, characterizing the system phase non-linearity,

$$\varepsilon = [(I_0K/4V_0)(1 + v^2_{\perp 0}/v^2_{\parallel})]^{0.5} << 1$$

is the interaction parameter,

$$\omega_c = (eB_0/m_0c)(1 - {}^1\!/_2(v_{\parallel}^2 + v^2_{\perp 0})/c^2)$$

is the cyclotron frequency with ξ=0, K is the coupling impedance, $v_{\perp 0}$ is the initial transversal electron velocity, I_0 and V_0 are the constant constituents of the beam current and voltage.

Equations (2) and (3) are solved within the follow initial and boundary conditions:

$$F(\xi,\tau=0) = f^0(\xi), \qquad I(\xi,\tau=0) = 0, \tag{4}$$

$$\beta(\xi=0) = \exp(j\,\theta_0), \qquad \theta_0 \in [0, 2\pi], \tag{5}$$

where the initial distribution f^0 is taken as

$$f^0(\xi) = \delta_0 \sin(\pi(A - \xi)/2). \tag{6}$$

The external controlling signal

$$F(\xi=A,\tau) = F_0 \exp[j\,\Omega\tau] \tag{7}$$

is added on the collector boundary of the system $\xi=A$, where A is the length of the system, F_0 is the external signal amplitude, Ω is the mismatch between the external signal frequency and the «cold» synchronism frequency $\hat{\omega}$.

The model described by the equations (2)-(5), is correct only within the following conditions: the EM field in the beam cross section is uniform, the longitudinal velocity $v_{\parallel}\approx$const (i.e. the interaction between the electrons-oscillators and the HF components of the magnetic field is neglected), the non-stationary process is assumed to be narrow-band, hence in the active frequency band it is necessary to take into account only the interaction of the spiral beam with the backward wave.

In our work we investigate the gyro-BWO within the following parameter values: μ=4 and A=3.0. In the autonomous system these values correspond to the regime of a periodical self-modulation of the exit signal. The numerical scheme parameters for the equations (2) and (3) were taken as $\Delta\xi=8\cdot 10^{-3}$ (coordinate step), $\Delta\tau=4\cdot 10^{-3}$ (time step).

2. Oscillation regimes and the transient chaos in gyro-BWO

In the works [9, 18, 19] the influence of different types of external control signal on the dynamics of the simple gyro-BWO model is investigated. In reference [20] the problem of chaotic auto-oscillation synchronization in the system «spiral electron beam - backward elecrtomagnetic wave» is analyzed. The importance of this problem is conditioned by the practical aspects of elaborating gyro-devices with controlled parameters and also by theoretical interest attracted to the investigation of auto-oscillation synhronization in distributed active medium.

In Fig. 1 we introduce the regimes map of the gyro-BWO synchronized by the external signal on the parameters plane «frequency - amplitude of the external influence» (the external signal parameters are foregoing) [18]. Different symbols on the map marks the areas of different oscillation regimes. With Ω>-2.0 we derive the synchronization regime, i.e. stationary generation on the frequency of external signal. The dashed line marks the boundary of frequency capture area. In this case the device may demonstrate periodical or chaotic self-modulation of the output signal.

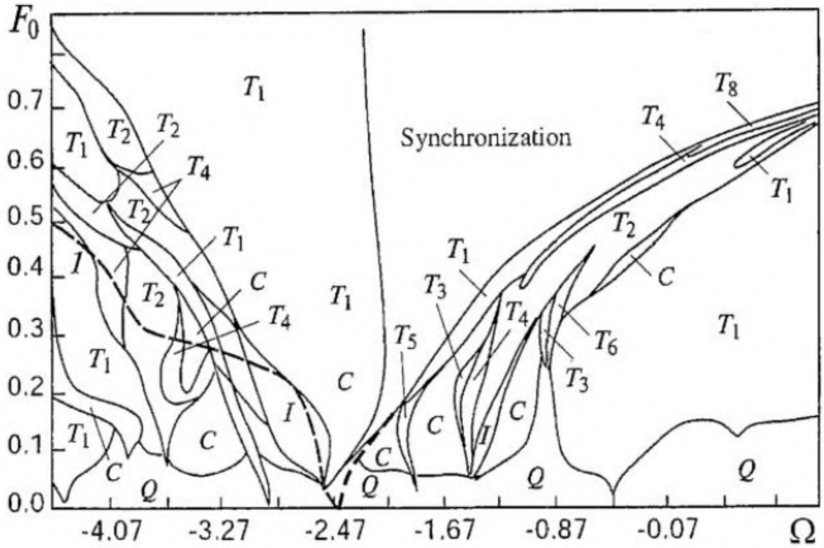

Fig. 1. Typical regimes of the non-autonomous oscillations in gyro-BWO on the controlling parameters plane frequency Ω - amplitude F_0. By the dashed line the quasi-synchronization area is marked

The region of frequency capture from the side of higher frequencies coincides with the boundary of the self-modulation region (and, accordingly, the synchronization region), and, on the side of smaller frequencies, the frequency of the external signal, at which the frequency capture takes place lays, essentially more to the left of from the boundary of the stationary generation regime. The regions of the regimes map marked by the symbols T_n corresponds to the periodic automodulation of the output signal with the period n. And, at last, the regions marked by the symbols C and Q corresponds to the chaotic generation and generation with several incommensurable automodulation base spectral components (quasiperiodic automodulation).

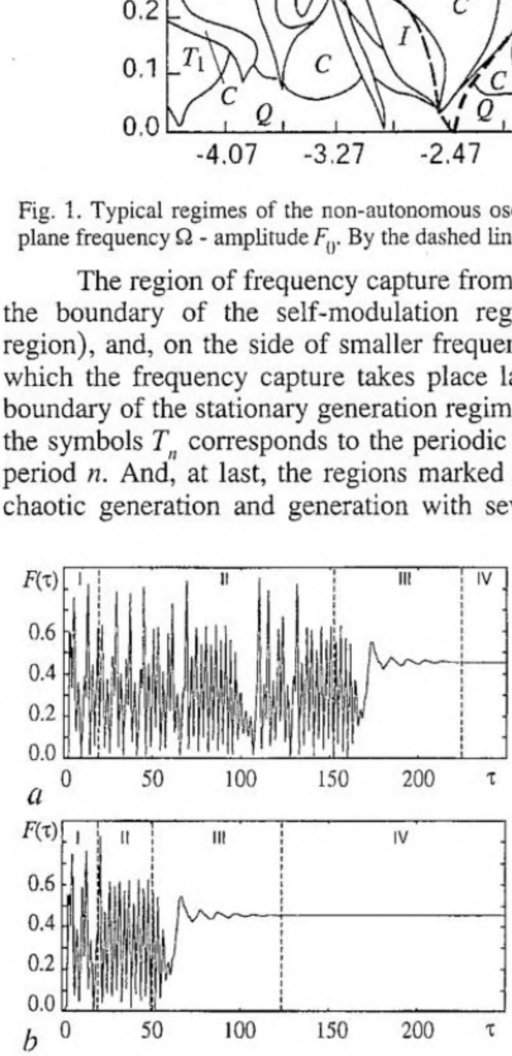

Fig. 2. Typical time series of the output signal amplitude of non-autonomous gyro-BWO. The signals are constructed at the following values of the initial perturbation amplitude: (*a*) δ_0=0.0019 and (*b*) δ_0=0.0072. By vertical shaped lines the typical parts of time series are divided: the part II is the region of the transient chaos which we analyse

Near the right boundary of the synchronization tongue (in the region of larger frequencies) the appearance of transient chaos considered the Introduction takes place. We explore the transient chaos in the regimes map point with the following values of the external signal control parameters: Ω=2.0 and F_0=0.62. In Fig. 2 we represent the typical time series of the output signal $F(\xi=0,\tau)$ in the transient chaos regime obtained at the different starting conditions (6), namely at different amplitudes of initial perturbation δ_0.

In Fig. 2 one can see that depending on the initial perturbation amplitude δ_0 the transient time duration is various, but finally the regime of stationary generation on the external signal frequency is stated. The transient process is rather irregular, what testifies the presence of the phenomenon of transient chaos in a system.

We analyse the transient chaos characteristics, for which it is necessary to create an artificially long time series consisting of the «sewed» short segments of time series corresponding to the transient chaos. For analysing the transient chaos regime and the procedure of gluing of short time series characterizing the unstable transient chaos regime we use the time series generated by the field amplitude oscillations $|F(\tau)|$, taken from the exit of the system $\xi=0$.

3. Constructing of an artificially long time series

Time series generated by the explored system in the transient chaos regime, can be «divided» into four parts (see Fig. 2): I - evolution of the system from the initial state to the unstable chaotic state, II - naturally, the transient chaos regime, III - the exiting of the system on an asymptotic regime and IV - the final asymptotically stable state. The information concerning the unstable chaotic saddle we can obtain from the part II of the time series, while the parts I, III and IV correspond to other states of the system. Therefore, for the analysis of characteristics of the chaotic saddle existing in the system phase space, we need to «cut out» the parts I, III and IV of the time series [12], and then to «sew» the parts corresponding to the chaotic unstable regime. This approach allows then to apply the standard methods of the analysis to the obtained artificially long time series (see, for example, [21, 22, 23]).

One of possible methods of deriving an artificially long time series is immediate combination of the truncated time series with each other. In this case the variable F of the derived new time series can have break points i.e.

$$\lim_{t\to T_n - 0} F(t) \neq \lim_{t\to T_n+0} F(t), \tag{8}$$

Where T_n are the points of the uniting of the series.

Yet when we restore an attractor using the delay method (Takens method) [24, 25] by an artificially derived long time series, J false points appear on an attractor [12]:

$$J = d(T/\Delta\tau)(n - 1), \tag{9}$$

where T is the delay time of the Takens method, n is the amount of the truncated time series, d is the dimension of the space of embedding, $\Delta\tau$ is the time step.

Another, more correct method of deriving of similar long time realization is the method of «gluing» of two different time series. Let $x(t)$ and $y(t)$ be the «glued» time series. If the attractor is restored in d-measure phase space, the following condition must be fulfilled for the «gluing» of the phase trajectories:

$$[\Sigma_{i=0}^{d-1} (x(t_1 + i\times T) - y(t_2 + i\times T))^2]^{1/2} < \varepsilon, \tag{10}$$

where ε is the «gluing» precision (we have chosen $\varepsilon=2.5\times10^{-3}$), t_1 and t_2 are the times of «gluing» for $x(t)$ and $y(t)$. The condition (10) can be replaced by the similar

$$|x(t_1 + i\times T) - y(t_2 + i\times T)| < \varepsilon, \quad i = \overline{0,\ldots,d\text{-}1}. \tag{11}$$

For procedure (11) it is required the value of the phase space dimension d, in which the attractor corresponding to the transient chaos is embedded. Let's estimate the value of d, calculating the correlation dimension D [21, 22], of an attractor restored by an artificially long time series derived without special method of «gluing» for different, increasing values of embedding space dimension d=2,3,4,.... The phase space dimension is equal to the value of embedding space dimension d with which the correlation dimension D is saturated [26].

Of course, such estimation is not absolutely precise, taking into account the presence of «false» points in the phase space. However, as was shown in [27], the estimation of correlation dimension by short segments of chaotic time series is rather reliable. Therefore the offered procedure can be used for the estimation of phase space dimension d. Further, using the procedure of correct gluing of short time series, we shall test the obtained results on calculation correlation dimension using simulated long chaotic time series.

The correlation dimension of an attractor D is a function of a scale of observation ε:

$$D(\varepsilon) = \lim_{\varepsilon\to 0} (\ln C(\varepsilon,d) / \ln\varepsilon), \tag{12}$$

where $C(\varepsilon,d)$, the number of pairs of points, distance between which in d-measured phase space is less than ε (the reduced correlation integral), is derived from the following relation

$$C(\varepsilon,d) = (1/MN) \sum_{j=1}^{M} \sum_{i=1, i\neq j}^{N} H(\varepsilon - \|\mathbf{x}_i - \mathbf{x}_j\|). \tag{13}$$

Here M is the number of reduction points, N is the number of points in the time series, H is the Heviside function, $\mathbf{x}$ is the position vector in the phase space restored by Takens method.

In Fig. 3, *a* one can see the results of calculation of the correlation dimension $D(\varepsilon)$ by the transient chaos time series combined without «gluing» for different values of embedding space dimension d. Time series length was chosen N=6·10^4, and number of points of a reduction M=10^4. The time series is combined from three short ones and the number of false points in pseudo-phase space J=3200 according to (9).

From fig. 3 one can see that the chaotic attractor corresponding to the transient chaos, is strongly inhomogeneous, because there is no scaling region on the function of correlation integral inclination depending on the scale of observation. However, beginning from the embedding space dimension d=3÷4 the shape of curves $D(\varepsilon)$ does not vary. Therefore as an estimate of embedding space dimension d=4 can be taken. This value would further be used for constructing artificially long time series by gluing the short ones.

When the parts of short time series are glued it's necessary to obtain the value of the T. We realized the glue of short time series with different values of T and analysed the effectiveness of the method in each case. One of the time series $x(t)$ (we shall call it «x-realization») was cut into two parts $x_1(t)$ and $x_2(t)$ at the time $\hat{t}$ so, that

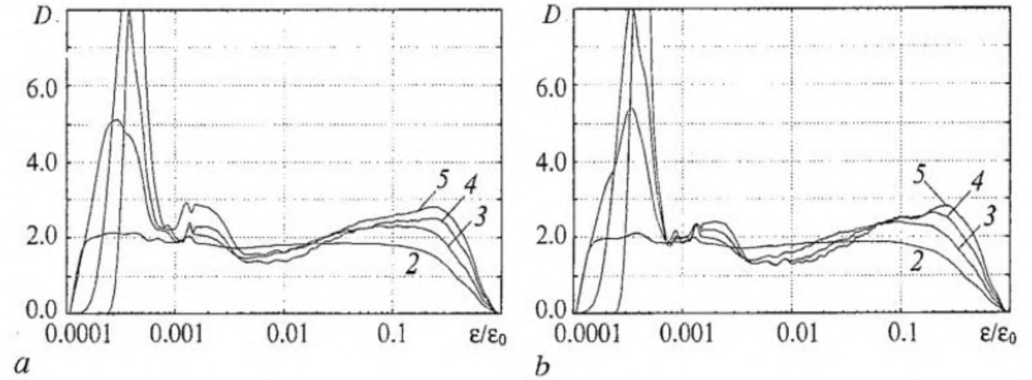

Fig. 3. Correlation dimension D as a function of observation scale: (*a*) for attractor restored on the time series derived by simple combining of short ones without special «gluing» method (11) and (*b*) for attractor restored on correctly glued long time series. The numbers correspond to different embedding space dimensions d

$$x(t) = \begin{cases} x_1(t), & \text{if } t \le \hat{t}, \\ x_2(t), & \text{if } t > \hat{t}. \end{cases} \tag{14}$$

After that the time series $x_1(t)$ was glued together with another time series $y(\theta)$, which we shall call «y-realization» (see (11)), so that the following condition should be satisfied:

$$\hat{t}_1 = t - (d-1) \times T. \tag{16}$$

In the case of x- and y-realizations are equivalent, i.e. $x(t) \equiv y(\theta)$, the following relation takes place

$$x_2(t) \equiv y(t_2 + (d-1) \times T + (t - \hat{t})), \qquad t \ge \hat{t}. \tag{16}$$

In a case of phase trajectories corresponding to x- and y-realizations are close to each other in d - measured phase space (but are not equivalent), by virtue of instability they will disperse with time and after a slice of time Δt will separate on distance exceeding some value ε_2 (in our case $\varepsilon_2 = 2.5 \times 10^{-2}$):

$$|x_2(\hat{t} + \Delta t) - y(t_2 + (d-1) \times T + \Delta t)| > \varepsilon_2. \tag{17}$$

The value of time interval during which the difference between «glued» time series $x(t)$ and $y(\theta)$ is less than ε_2, characterizes quality of the procedure of time series «gluing».

In Fig. 4 the dependance of the average value of Δt on the delay time T is represented. One can see that the average value of interval Δt during which the glued time series becomes practically identical, does not depend on the chosen magnitude of a delay time T. Only at small delay times the average value Δt diminishes.

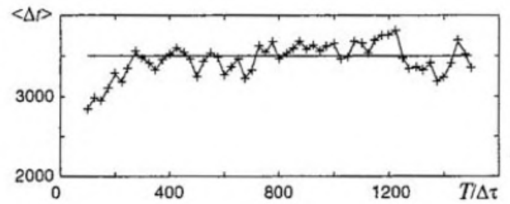

Fig. 4. Dependence of the average magnitude $\langle \Delta t \rangle$ on the value of a delay time T. Average was carried out on 300 points of «gluing»

The analysis of normalized distribution (see Fig. 5) of the magnitude of Δt shows that at a delay time T=1.6 the part of «unsuccessfully» glued time series is minimal. In this case the number of glued time series with little interval Δt is small. Simultaneously, the exploration carries out that with the increase of the delay time T the number of glue points diminishes. Therefore the delay time for the procedure of gluing is chosen T=1.6. Besides, with such a choice of delay time T=1.6 the duration of time interval $(d\text{-}1)\cdot T$ becomes comparable with the typical oscillations time scale.

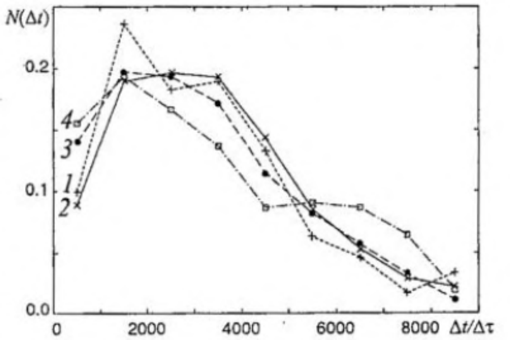

Fig. 5. Normalized distribution of Δt at the different values of a delay time T: curve 1 (+) corresponds to the delay time T=0.8; curve 2 (×) - to the T=1.6; curve 3 (•) - to the T=2.4; curve 4 (□) - to the T=3.2. The allocations are constructed on 300 glued time series for each delay time T

Let's consider now the glued time series. On Fig. 6 the examples of the most successful (a) and unsuccessful (b) gluing with delay time T=1.6 are represented. One can see from the figure that even in case of «unsuccessful» glue the y-realization well agrees with the x-realization, with which it is glued.

The important circumstance is that all the time series used to construct the artificially long one are generated by the distributed system. Hence, that fact, that

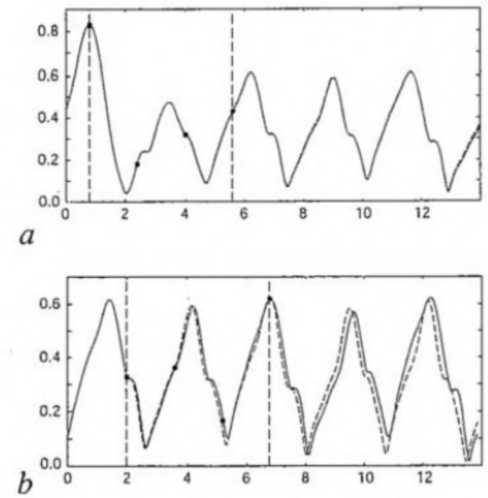

Fig. 6. Successful (a) and unsuccessful (b) gluing of the time series. The delay time in both cases T=1.6. Time interval Δt, during which the distinction between x- and y-realizations does not exceed the value ε_2, is equal to Δt_a=30.09 and Δt_b=0.28 time unities, accordingly. The vertical dashed lines restrict the time interval (d-1)T, on which the glue of the x- and y-time series is carried out. The circles (•) correspond to the points of gluing (see relation (11)). x-realization is shown by solid line, y-realization - by dashed one

two time series are well glued, yet does not guarantee that the states of an initial distributed system are close in times t_1 and t_2 for x- and y-realizations, accordingly. Therefore it is necessary to consider the spatial distributions of amplitude of the field $|F(\xi)|$ and current $|I(\xi)|$ in gyro-BWO at the time corresponding to the points of glue of time series $x(t)$ and $y(\theta)$.

Fig. 7 illustrates the spatial distributions of the values $|F(\xi)|$ and $|I(\xi)|$ in the case of «successful» glue of time series at the chosen value of delay time T=1.6 (see also Fig. 6, a) and the corresponding distributions after the time Δt_a=30.09 passes and the x- and y-realizations dispersed on the distance ε_2. Similar dependences for the most «unsuccessful» glue are represented on Fig. 8 (see Fig. 6, b); the time interval Δt_a is chosen the same Δt_a=30.09. In spite of the fact that in a case of unsuccessful glue of x- and y-realizations, the spatial distributions $|F(\xi)|$ and $|I(\xi)|$ explicitly differ from each other, these differences are quantitative but not

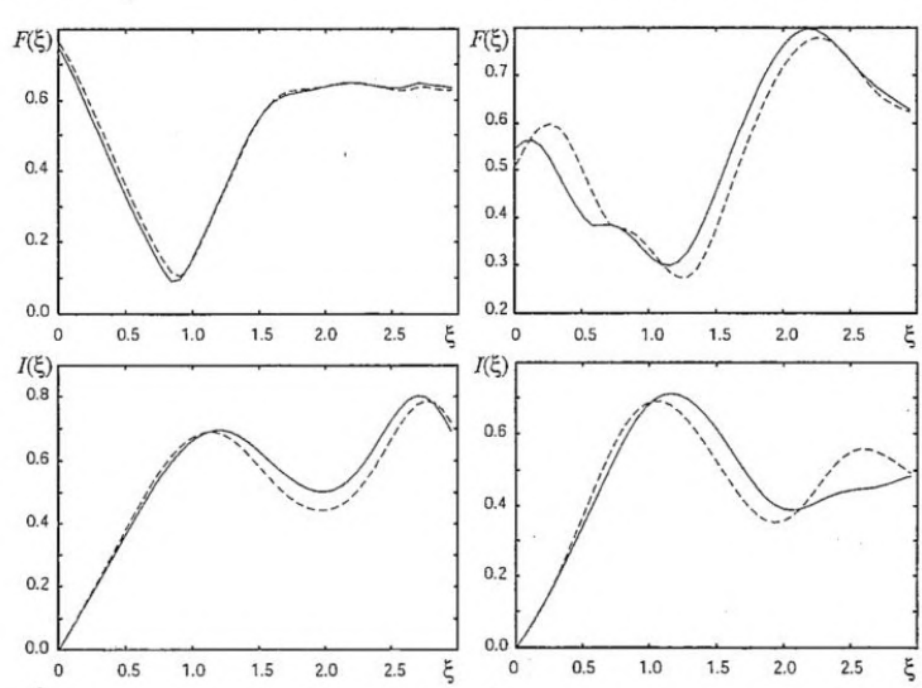

Fig. 7. Spatial distributions of the field amplitude F and current I for the case of «successful» glue of time series with T=1.6. The time series with initial amplitude Δ_0=0.0041 (solid line) at time τ=29.36 and Δ_0=0.0012 (dashed line) at the time τ=106.16 (Fig. a) are glued. The Fig. b illustrates the divergence of time series and corresponds to the time interval passed from the moment corresponding to Fig. a, Δt_a=30.09

qualitative. At the same time the structures existing in a distributed system which generates x- and y- realizations, at the moment of gluing agree with each other. In the case of «good» glue after the disperse of the two time series on distance ε_2 in both cases we derive the double-peak distribution of the first harmonic of the grouped current, what is equal to two electronic structures (two phase bundles of electrons - oscillators of the spiral beam) on the length of interaction space, though the value of the second maximum of the grouped current strongly differs in both cases.

For the case of «bad» glue of time series after the pass of the same time Δt_a, as in the first case, essentially stronger discrepancy of system dynamics takes place. Comparing the distributions introduced in Fig. 8, *a* and the Fig. 8, *b*, one can see that not only quantitative, but also qualitative distinction of the interior beam and field structures takes place. However, if we take the time interval Δt_b=0.28, through which the time series disperse on the distance ε_2, it is possible to make the same deduction, as earlier: the difference between states is only quantitative. Qualitatively the behaviour of the system is identical in both cases. Thus, the quality of gluing first of all renders influence on the length of the time interval Δt, during which it is possible to consider the two time series identical (compare the times Δt_a and Δt_b for the case of a good and bag glue accordingly). It is obvious, that the duration Δt is defined, first of all, by quality of gluing, and, then, by the magnitude of the maximum Lyapunov index λ_1, which is the measure of the velocity of the disperse of neighboring phase trajectories of a chaotic set corresponding to transient chaos.

In a view of above-stated it is possible to make a deduction that the offered procedure of glue of truncated time series generated by a distributed system, allows to create a correct long time series, on which it is possible to determine the characteristics of transient chaos.

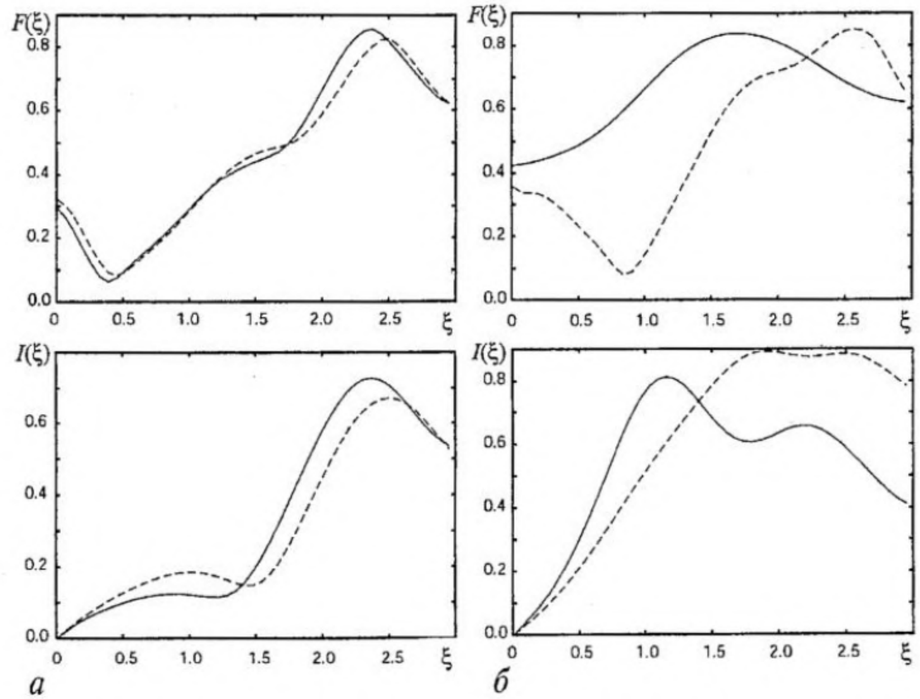

Fig. 8. Spatial distributions of the field amplitude F and current I for the case of «bad» glue of time series with delay time T=1.6. The time series with initial amplitude Δ_0=0.0033 (solid line) at time τ=54.97 and δ_0=0.0012 (dashed line) at the time τ=111.80 (Fig. *a*) are glued. The Fig. *b* illustrates the divergence of time series and corresponds to the time interval passed from the moment corresponding to Fig. *a*, Δt_a=30.09

4. Characteristics of transient chaos

Let's compare the results obtained at calculation of the correlation dimension of the attractor restored by «not glued» long time series at presence of false points, and the result calculated from correctly glued time series. For correct comparison the time series length $N=6\cdot10^4$ and the number of points of reduction $M=10\cdot4$ were chosen the same, as in the previous case. The results of calculation are represented in fig. 3, *b*, on which one can see the curves $D(\varepsilon)$ for the embedding space dimension d=2,3,4 and 5. It is obvious that the earlier obtained estimated results agree with the more precise calculation of correlation dimension. The embedding space dimension d in this case also don't exceed 4. The last means that the pseudo-phase spaces dimension, evaluated in previous section, was chosen correctly.

Also it is possible to make a conclusion that for the estimation of correlation dimension of the attractor restored from short time series the procedure of glue is not required and the small number of false points in the phase space does not render essential influence to the calculation of correlation dimension.

Let's consider now such important characteristics of transient chaos as the maximum Lyapunov exponent λ_1. Its estimation was produced with the help of procedure offered in [28, 29]. According to it the magnitude of λ_1 is defined as

$$\lambda_1 = \lim_{t\to\infty}(1/t)\ \ln(\chi(t)/\chi(t_0)), \tag{18}$$

where $\chi(t)$ is the distance between two points $\mathbf{x}'$ and $\mathbf{x}''$ in phase space at the time t. We suppose that at the initial time these points are close, i.e. $\|\mathbf{x}'-\mathbf{x}''\|=\chi(t_0)<<R$, where R is the typical geometrical size of an attractor in phase space. The positive value of the maximum Lyapunov index λ is evidence of the chaotic dynamics of the system. Through the time interval $\tau\approx\ln(R/\chi(t_0))/\lambda$ the behaviour of the system becomes unpredictable, i.e. the magnitude of the Lyapunov index characterizes the measure of instability and complexity of the chaotic process.

Now, keeping up for the system dynamics after starting from the points $\mathbf{x}'$ and $\mathbf{x}''$ and analyzing the distance $\chi(t_0+m\Delta\tau)=\|\mathbf{x}'(t_0+m\Delta\tau)-\mathbf{x}''(t_0+m\Delta\tau)\|$ between the current states of the system, we find the time interval $m\Delta\tau$, during which the trajectories disperse on the distance larger than $\chi_{\max}$. Then a new point $\mathbf{x}_m''$ on the attractor, which is close to the point $\mathbf{x}'(t_0+m\Delta\tau)(\|\mathbf{x}'(t_0+m\Delta\tau)-\mathbf{x}_m''\|=\chi(t_0+m\Delta t)<<R)$, also is moved from it to the direction of the vector $\mathbf{x}''(t_0+m\Delta\tau)-\mathbf{x}'(t_0+m\Delta\tau)$ is found. Then the procedure is repeated.

To define the value of the maximum Lyapunov index average on the attractor the above described procedure it is necessary to iterate M times before reaching by magnitude

$$\langle\lambda_1\rangle = (1/M\Delta\tau)\ \Sigma_{m=1}^{M}\ln(\chi(t_0+m\Delta\tau)/\chi(t_0+(m-1)\Delta\tau)) \tag{19}$$

the asymptotic value.

Using the above described procedure for the «correctly» glued time series we obtained the value of the maximum Lyapunov index $\lambda_1=0.098\pm0.011$.

Let's remark, that if we are moving a little bit to the area of the chaotic generation by changing the generation parameters $\Omega=0.1$, $F_0=0.62$ (let's remind that we have studied the transient chaos with $\Omega=2.0$, $F_0=0.62$), the typical characteristics of the chaotic attractor as the restored attractor and Fourier power spectrum are similar to the characteristics of the transient chaos.

However, the maximum Lyapunov exponent of the chaotic attractor $\lambda_1=0.002$. I.e. the chaotic set (transient chaos) is a more unstable (and, hence, more «chaotic») regime than the chaotic attractor existing «at neighbourhood» in the parameters space.

Conclusion

In this work the transitional chaos found in non-autonomous distributed active medium «spiral electron beam - electromagnetic wave» (gyro-BWO, synchronized by external signal) near the boundary of synchronization tongue was investigated. The analysis of the characteristics of the transient chaos in explored distributed system was carried out. For this purpose we modified the procedure of combining of short time series generated by the distributed auto-oscillation system and obtained at the different starting conditions of integration of the model equations (2)-(6). Great attention was payed to the examination of the correctness of procedure of short time series «gluing» for the purpose of constructing an artificially long time series. It is possible to make a conclusion that the procedure of gluing offered in our paper is effective at constructing the long time series by a set of short time series generated by distributed systems.

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ИССЛЕДОВАНИЕ ПЕРЕХОДНОГО ХАОСА В ГИРОЛАМПЕ СО ВСТРЕЧНОЙ ВОЛНОЙ, СИНХРОНИЗИРУЕМОЙ ВНЕШНИМ СИГНАЛОМ

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В работе изучается переходный хаос в неавтономной распределенной активной среде (гиролампа со встречной волной (гиро-ЛВВ), синхронизируемая внешним сигналом). Исследуются характеристики переходного хаоса в гиро-ЛВВ вблизи границы области синхронизации. Особое внимание уделяется проблеме построения искусственной длинной временной реализации, по которой оцениваются характеристики хаотической динамики, и которая строится путем сшивания коротких временных реализаций, характеризующих переходный хаос, порождаемый распределенный системой.

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