Applied problems of nonlinear oscillation and wave theory

Official English translation

Chaos and order in athmospheric dynamics
Part 2. Interannual rhythms of the El Niño – Southern oscillation

I. V. Serykh[1], *D. M. Sonechkin*[1,2]

[1] Shirshov Institute of Oceanology, Russian Academy of Sciences
36, Nahimovskiy prospekt, 117997 Moscow, Russia
[2]Hydrometeorological Research Centre of the Russian Federation
11–13, Bol'shoi Predtechenskii per., 123242 Moscow, Russia
E-mail: iserykh@ocean.ru, dsonech@ocean.ru

Received 19.05.2017

Processes of the El Niño – Southern Oscillation (ENSO) are investigated based on the mathematical theory of the so-called the strange nonchaotic attractor (SNA) in the quasi-periodically forced dynamic systems, and using the sea surface temperature and the atmospheric sea-level pressure data for the 1870–2014 year period. It is found that ENSO is influenced not only by the annual Sun-induced periodic heating of the climate system, but also by the three more other external forces which periods are incommensurable to the annual period. These forces are induced by the 18.6-year Luni-Solar nutation of the Earth's rotation axis, the 11-year cycle of the solar activity and the Chandler wobble in the Earth's pole motion (the period 1.2 years). Because of the reciprocal incommensurability of the periods of these forces, all of them affect the climatic system in «improper» time moments. As a result, the dynamics of the indices representing the ENSO processes look to be very complex (strange in mathematical terms), but not chaotic. It is shown that power spectra of the ENSO indices have some bands of the increased spectral density located on sub- and superharmonics of above-mentioned periods. On the basis of some special considerations of structure of the power spectra of the ENSO indices the evidence of the discreteness of these spectra, i.e. the spectra being nonchaoticity, is received. Nobody assumed this circumstance earlier. Despite complexity of the ENSO processes, the dynamics of the temporal variations of these process indices reveals an internal ordering similar to that internal order which is known to be inherent to the SNA dynamics. This ordering reveals itself in the existence of spectral density peaks in the ENSO power spectra, and some rhythms corresponding to these peaks in the temporal variations of the ENSO indices. Acceptance of the SNA model for ENSO means that there are no predictability limits for ENSO in principle. In practice, it opens an opportunity to predict ENSO for several years ahead.

Keywords: Power spectra of the El Niño – Southern Oscillation, quasi-periodicity of the external climate system drivers, the Chandler wobble, the Luni-Solar nutation, the Solar activity, strange nonchaotic attractor.

References: Serykh I.V., Sonechkin D.M. Chaos and order in athmospheric dynamics. Part 2. Interannual rhythms of the El Niño – Southern oscillation. *Izvestiya VUZ. Applied Nonlinear Dynamics*. 2017. Vol. 25. Issue 5. P. 5–25. DOI: 10.18500/0869-6632-2017-25-5-5-25

The research was carried out partially under the financial support by the grant of Russian Science Foundation (project No. 14-50-00095).

© *I.V. Serykh, D.M. Sonechkin*

Introduction

Energy spectra estimations of climate fluctuations on a wide range of time scales– from a year to thousands and even hundreds of thousands years – have been published repeatedly. Probably, the very first of them is the estimation by J.E. Kurzbach and R.A. Bryson [1, 2], which could be found in the book by A.S. Monin and Yu.A. Shishkov [3]. Indeed, at present this estimation looks very naive. Nevertheless, it is quite relevant for the purpose of this work because the climate spectrum in it is shown to be continuous, the idea accepted in almost all subsequent estimations published until recently. A relatively low-frequency part (periods of more than a thousand years) in the Kurzbach–Bryson spectrum was defined as «red noise», and a relatively high-frequency part (periods from a year to one hundred years) was defined as «white noise», that is, all local increases and decreases in spectral density (with the exception of only the delta peak at the annual period) were interpreted as insignificant.

In the years following the publication of Kurzbach and Bryson a lot of new instrumental meteorological observations have been acquired and numerous paleoclimatic reconstructions have appeared. This made it possible to substantially refine the form and quantitative characteristics of the climatic energy spectrum (e.g. see [4]). One of the most recent and accurate estimation of the climate energy spectrum is given in [5]. In its interpretation, the continuity of the spectrum is emphasized and all climate fluctuations are identified as chaotic. In the relatively high-frequency part (periods from a year to about a hundred years), the average inclination of the spectrum (in double logarithmic coordinates) is estimated by the authors of [5] as equal to 0.37 and 0.58, and in the relatively low-frequency part (periods of more than a hundred years) it is estimated as equal to 1.64 and 1.29, based on data from continents at high latitudes and oceans at low latitudes, respectively. This confirms the conclusion of Kurzbach and Bryson on the «white» and «red» nature of high- and low-frequency climatic «noises». At the same time, authors in [5] recognize the presence of statistically significant peaks in spectral density associated with responses of the global climate system to changes and redistribution of incoming solar radiation over the Earth's surface, while possible responses to other periodically changing external forces are not mentioned.

In this paper the subject matter is only a part of the climate spectrum that corresponds to interannual changes in the El Niño – Southern Oscillation (ENSO) processes. Note that a methodologically similar consideration for a part of the climatic spectrum corresponding to the geological time scale of tens and hundreds of thousands of years is made in [6–11], where the main conclusion is the absence of chaotic behavior.

Important role of seasonality in the ENSO dynamics and, accordingly, in the formation of ENSO spectra has long been recognized. However, a quantitative consideration of this role was made only more recently by two groups of Western experts in Atmospheric and Oceanic Physics. One group was informally led by M. Gil, and the other by E. Tziperman. In 1994, these groups published their work in the same issue of the Science Magazine [12]. Simultaneity of publications indicates that Science Magazine editors understood the extreme importance of stating the problem of the ENSO nature given in these articles. In fact, for the first time, there were used mathematical conclusions about

© *I.V. Serykh, D.M. Sonechkin*

bifurcations of invariant torus as an attractor of a periodically forced nonlinear dynamical system. In the following years there were published several more articles developing this idea [13–17]. In them the ENSO rhythm, known from observations, was explained as a seasonal «phase capture» – a phenomenon proved by mathematicians as a typical property of periodically forced systems with torus attractors.

It is curious that there are very few references to all these publications in the following works on the ENSO problem. This suggests that the international community of meteorologists has not yet properly appreciated the idea expressed by Gil and Tziperman and their colleagues. In the Russian meteorological literature bifurcations of invariant tori were used to explain mechanisms of excitation in two-year and even longer-period oscillations in atmospheric models much earlier (see book [18] and articles [19, 20]), but there was almost no reaction to these works by the wide range of Russian researchers.

It is important to underline that the ENSO rhythm modeled by aforementioned groups of American scientists reproduced the real ENSO rhythm only qualitatively. Peaks in the ENSO energy spectrum were obtained in multiple year periods of the type 2:1 (two-year cycle), 3:1 (three-year cycle), 5:2 (two cycles in five years), etc. The main peak in real ENSO spectra is usually estimated as equal to 3.6 years (see, for example, [21]). The reason for this is that ENSO is clearly influenced not only by the annual course of solar heat flux, but also by other periodic external forces, their periods being different from the annual period and probably incommensurable with it. Apparently, this was first suggested in [22]. Another external force essential for ENSO is Chandler oscillation of the Earth's poles with a period of about 1.2 years. N.S. Sidorenkov has also paid a lot of attention to the Chandler oscillation as an ENSO mover (see [23] and following publications).

Taking into account a lot of periodic external forces is probably important not only for describing and predicting ENSO, but also for many other long-period processes in the atmosphere, since in a nonlinear system such as climate all processes generally interact with each other. Since the 1990s, American researchers R.J. Curry and S. Hamid [24, 25] have been champions for this idea.

1. Initial data and processing methods

In order to calculate the ENSO energy spectra and determine periodicities that affect ENSO, in this study we used English monthly global databases of Sea Surface Temperature (SST) with resolution of $1^\circ \times 1^\circ$ (HadISST) and atmospheric pressure at sea level (APS) with resolution $5^\circ \times 5^\circ$ (HadSLP2). These bases are publicly available for the entire period of instrumental observations: 1870–2014 for temperature and 1850–2014 for pressure [26, 27]. To calculate energy spectra of various ENSO indices we chose 1870–2014 as the main period. This period seems to be optimal, since, on the one hand, it has a duration already sufficient for detecting ENSO rhythms in the range from a year to ten years; on the other hand, observational data related to this period are fuller and more reliable than during earlier instrumental meteorological observations. To verify the results, we used the data from SST COBE2 (1850–2014) and ERSSTV4 (1854–2015), APS NOAA-CIRES 20th Century Reanalysis v2c (1851–2011) [28] and ERA-20C (1900–2010).

© *I.V. Serykh, D.M. Sonechkin*

Spectra calculated from these data have peaks at the same periods as those based on the English data, as confirmed by the results obtained.

ENSO indices were calculated as average SST anomalies in the Nino1+ 2, Nino3, Nino3.4 and Nino4 regions, thereon the spectra were calculated (Table 1). Based on the completeness of the data available in the HadISST database, Nino2, Nino3, and Nino3.4 regions were combined in the form of a sum, i.e., SST anomalies were considered on average for the region (5°N–5°S, 170°–80°W). Based on these anomalies, the time series of the index, called the Extended Oceanic Nino Index (EONI), was calculated. In contrast to the frequently used ONI index based on a rather scarce base of vessel observations in the center of the equatorial zone of the Pacific Ocean (Nino3.4 region), while calculating the EONI we took into account numerous observations from the vessels leaving the Panama Canal towards Asia and Australia.

To study the atmospheric component of the ENSO processes, the Equatorial Southern Oscillation Index (ESOI) was chosen. It is calculated as a difference between the average APS anomalies between the region of Indonesia (5°N–5°S, 90°–140°E) and the eastern part of the equatorial Pacific Ocean (5°N–5°S,130°–80°W). In our opinion, this index is more representative than the standard SOI (the difference of the APS between city of Darwin, Australia and island of Tahiti), since it covers large areas located at the equator.

Energy spectra of these indices were calculated by two methods: The Fast Fourier Transform (FFT) of the series and the Fourier Transform of the pre-calculated time corre-

Table 1

Periods (in years) of the spectral peaks of the indices Nino1+2, Nino3, Nino3.4 and Nino4 for the HadISST, ERSSTV4 and COBE2 databases arranged in descending order of the spectral density

	HadISST 1870–2014 Nino				ERSSTV4 1870–2015 Nino				COBE2 1870–2014 Nino			
№	1+2	3	3.4	4	1+2	3	3.4	4	1+2	3	3.4	4
1	3.62	5.70	5.69	5.66	3.56	5.70	5.68	5.65	3.60	5.70	5.69	9.03
2	3.60	3.54	3.53	12.95	13.55	3.54	2.88	12.92	3.59	3.54	3.54	5.67
3	3.61	3.63	2.88	3.54	3.60	3.63	13.16	20.88	3.56	3.60	8.98	12.98
4	3.56	3.60	3.79	20.63	6.60	3.60	3.54	6.42	5.72	3.62	13.10	3.63
5	5.71	5.17	3.64	5.08	5.72	5.19	5.16	2.88	5.20	3.78	3.62	3.55
6	3.79	3.79	5.13	3.64	52.67	2.88	3.79	3.64	3.79	5.17	3.60	3.56
7	6.56	2.88	3.60	2.88	5.19	3.79	3.64	3.55	6.56	2.87	2.87	3.57
8	5.19	4.28	13.09	4.77	2.88	13.36	6.39	3.56	6.53	4.28	3.79	3.58
9	6.52	4.27	4.79	3.79	4.33	6.41	3.60	3.57	2.88	4.27	5.13	28.94
10	2.88	4.80	4.26	3.60	3.40	2.32	4.24	3.58	4.33	13.37	4.78	2.88

© *I.V. Serykh, D.M. Sonechkin*

lation function (PFC). The FFT method has a higher frequency resolution, while the PCF method allows a more reliable estimation of peak amplitudes. To increase the reliability of spectral estimations the spectra were calculated many times by successively shortening the series in question until they were reduced to 1/3 of the original length. Moreover, the windows by which the shortened rows were determined ran through the original rows from their beginning to the end. Then all the obtained spectra were averaged and, in some cases, smoothed. After calculating all the spectra of a certain type, their average was calculated.

Spectra were calculated both for a complete series of monthly average EONI and ESOI, and separately for each month of the year: only Januaries, only Februaries, etc. Calculation of spectra for complete series (month after month and year after year) is generally accepted in climatology. Calculation for series of certain months is consistent with dynamic system study during the period of one of the external forces (discrete «mapping»), accepted in the mathematical theory of periodically forced dynamic systems. This mapping excludes from explicit consideration a higher frequency dynamics compared to the period of the selected external force. Whereas, seasonality dynamics was studied by comparing mappings specific to different months of the year. This method enabled to study the ENSO energy spectra separately for the phase of the strongest development of El Niño and La Niña events (months from October to February).

The results' sensitivity to changes in the data series length was investigated. For this, the spectra of indices were considered separately for the periods: 1850–2014, 1870–2014, 1900–2014 and 1920–2014 (Tables 2 and 3).

Table 2

Periods (in years) of the spectral peaks of the EONI for the HadISST, ERSSTV4 and COBE2 databases arranged in descending order of the spectral density

	HadISST EONI			ERSSTV4 EONI				COBE2 EONI			
№	1870–2014	1900–2014	1920–2014	1854–2015	1870–2015	1900–2015	1920–2015	1850–2014	1870–2014	1900–2014	1920–2014
1	5.70	3.58	3.57	5.71	5.70	3.59	3.59	5.72	5.70	3.58	3.58
2	3.54	5.64	5.09	2.87	3.54	5.66	5.13	3.79	3.54	5.64	5.09
3	3.63	5.15	5.65	3.79	3.63	5.18	5.70	3.53	3.60	5.15	5.66
4	5.16	6.51	4.84	3.54	3.60	6.48	4.74	2.87	3.62	6.51	4.78
5	3.79	4.84	4.79	5.20	2.88	4.21	6.41	3.63	3.62	4.21	4.84
6	2.88	4.21	2.87	3.64	5.18	13.04	2.87	3.64	3.78	4.84	4.76
7	4.28	4.23	4.19	13.02	3.79	2.88	4.16	8.92	2.87	4.86	2.87
8	4.27	4.88	2.45	6.35	13.29	4.78	2.45	3.60	5.16	4.88	4.15
9	4.80	2.87	6.44	3.59	6.40	4.80	12.93	5.20	8.93	4.91	2.12
10	13.22	3.78	2.12	3.39	2.32	3.78	2.12	3.39	13.29	3.77	6.44

Table 3

Periods (in years) of spectral peaks of the ESOI for the HadSLP2, NOAA-CIRES 20CRv2c and ERA-20C databases in descending order of the spectral density

	HadSLP2 ESOI				NOAA-CIRES 20CRv2c ESOI				ERA-20C ESOI	
№	1850–2014	1870–2014	1900–2014	1920–2014	1851–2011	1870–2011	1900–2011	1920–2011	1900–2010	1920–2010
1	5.70	5.68	5.65	5.09	5.73	5.72	5.66	3.59	3.58	3.58
2	61.00	59.38	53.50	66.00	5.71	3.54	3.58	5.07	5.66	5.09
3	2.88	3.54	3.59	3.60	3.53	2.88	5.10	5.67	5.16	5.67
4	3.53	2.88	5.18	5.68	2.88	3.63	5.03	2.44	6.53	4.83
5	5.19	5.18	6.49	4.84	3.64	3.79	2.57	53.42	13.16	13.04
6	3.80	3.79	4.86	4.80	3.79	51.50	6.52	2.57	4.20	6.40
7	3.39	3.63	4.85	2.45	3.40	13.02	2.45	6.40	4.20	2.57
8	3.64	3.60	2.87	6.38	9.01	8.95	4.20	2.86	4.89	2.45
9	4.81	6.47	3.77	2.87	8.99	41.04	4.20	2.17	3.79	2.86
10	3.60	6.46	2.45	31.50	2.78	2.32	12.85	2.15	9.12	2.12

2. Results

Figure 1 shows EONI energy spectra (above) and ESOI energy spectra (below), obtained by the Fast Fourier Transform method for a full series of monthly average values of these indices during 1870–2014. Previously, the annual move was excluded from the data. Also it shows 95 percent and 5 percent confidence intervals for the corresponding red noise spectra. In the interannual range the spectra themselves exhibit numerous peaks, more precisely, more or less narrow bands of increased spectral density. One of the reasons for the broadening the bases of these peaks is quite obvious: it is the finite and relatively short length of the series in question. Another reason, far from obvious, is that in fact each wide peak visible in Figure 1 can be composed of many closely spaced delta peaks.

Position on the time period axis and ratio of the amplitudes of all bands with increased spectral density are consistent with statistically significant peaks in many previously published spectra of ENSO indices for the range from two to ten years [29]. The most powerful in both spectra are the statistically significant bands of increased spectral density, occurring in periods of about 5.1, 3.8, 3.6 and 2.9 years. The EONI spectrum also contains bands with significance of 95% for periods of about 2.4, 2.1, 1.8, 1.5, and 1.2 years, and the ESOI spectrum for periods of about 5.6 years.

To be sure of the peak's periods that are implied by these bands, one may use the self-similarity indication of SNA energy spectra for the quasiperiodically forced dynamic

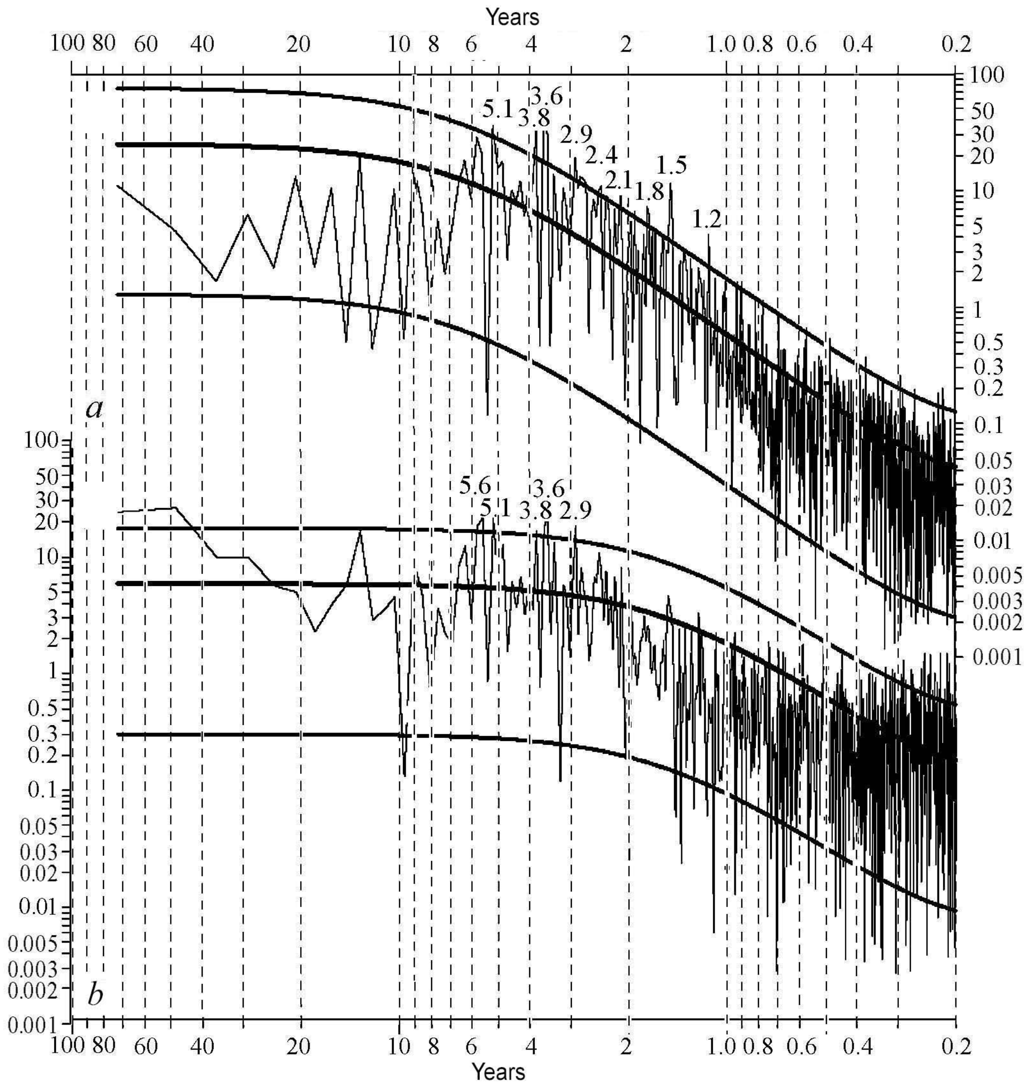

Fig. 1. Power spectra of the time series of the monthly mean values (detrended) of the EONI (*a*) and ESOI (*b*) estimated over the 1870–2014 year period. The significance limits of 5% (a smooth line below) and of 95% (a smooth line upper) as well as the respective red-noise spectrum (a smooth line in between) also are shown

system theory with so-called strange non-chaotic attractors (SNA). Self-similarity means that after a corresponding rescaling ratios of peak periods and peak amplitudes in the SNA spectra are the same for different parts of SNA spectra.

Thus, usually the power ratio of the annual band to the half-year band and the power ratio of the half-year to the quarter-year band are approximately equal. The same is true for the power ratio of the 3.6-year band to the 2.9-year band and the 2.9-year band to the 2.4-year band. Accordingly, the band period of 3.6 years refers to the band period of 2.9 years as approximately 1.2. The ratio of the 2.9-year period to the 2.4-year period is the same. We can go further and compare the power of the bands and their periods for superharmonics with those of subharmonics.

All these ratios are close in magnitude. Therefore, it is conceivable that self-similarity of EONI and ESOI spectra does exist. This confirms the reality of the considered bands of increased spectral density, without regard to the formal 95 percent statistical evaluation of their significance.

Let us turn now to Figure 2, that shows energy spectra for series of monthly average values of EONI and ESOI in the range of periods from one year to ten years, calculated by multiple recalculation, as described in Section 2. These spectra are much more detailed than in Figure 1, making it possible to discuss the nature of the bands with increased spectral density visible in them. Once again, we note that the formal statistical significance of the peaks in the spectra of El Niño indices has already been discussed in many previously published works. For example, in the spectrum of the Niño3 index shown in [21], there are nine peaks, their significance level being over 95%. Of these, the significance is greatest at peaks during periods of 3.5, 5.6, 2.8 and 1.4 years. The same peaks are the

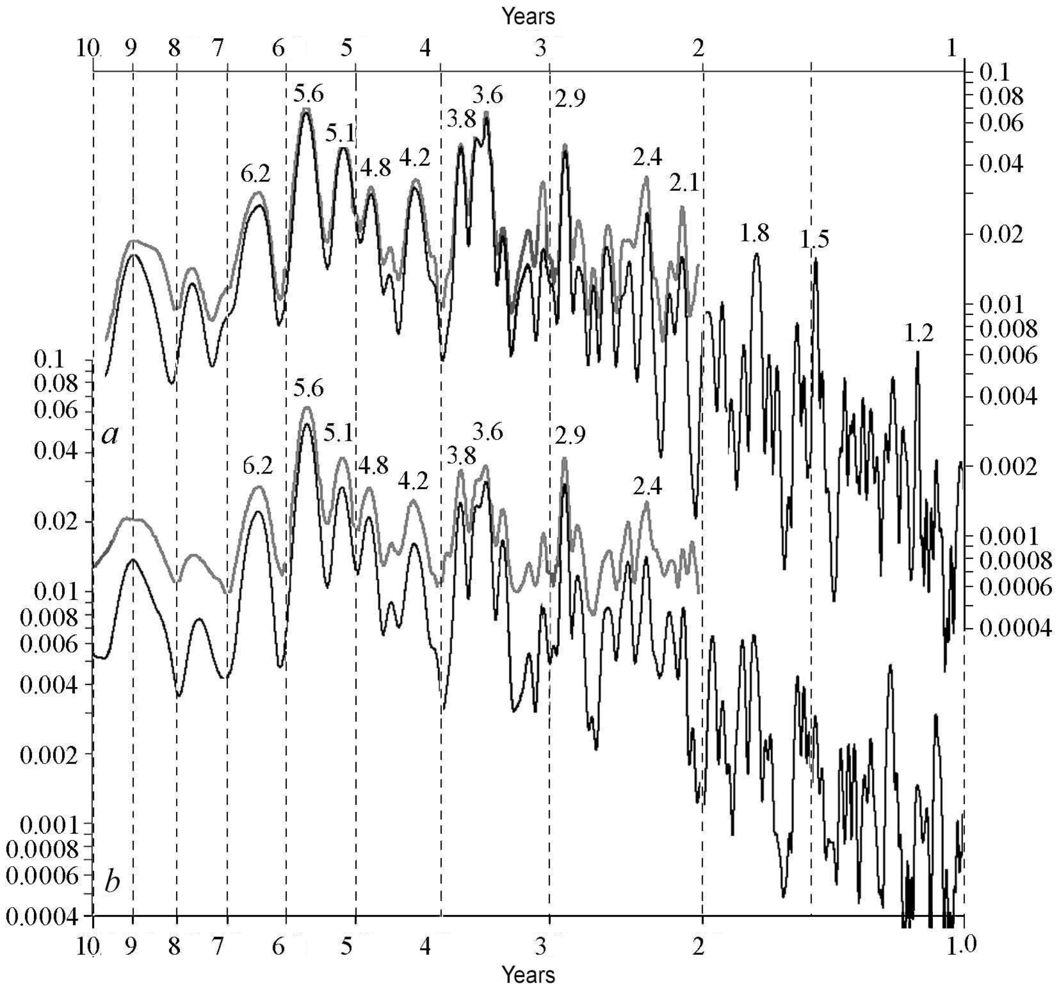

Fig. 2. Power spectra of the time series of the monthly mean values of EONI (*a*) and ESOI (*b*) represented as mappings on the annual period (grey line) as well as represented with maximal spectral resolution (dark line), and calculated over the 1870–2014 year period

main ones in the detailed EONI and ESOI spectra (see Figure 2). However, at a period of 5.1 years there is a peak added, that in the spectrum [21] has almost reached the level of 95%.

To determine the origin of all these bands and peaks of increased spectral density, it is useful to take into account the indication of the Dynamical systems theory that dynamics of autonomous (with constant external forces) systems alone is not able to produce spectral density peaks. This matter is crucial for the purpose of this study. Therefore, we give it a little more notice.

The fact is that the energy spectrum of the variable z in the famous three-mode model with a strange attractor used by E.N. Lorenz [30] seems to contain delta peak. This is also known about the spectrum of another famous model with a strange attractor (model used by O.E. Roessler [31]). However, these two exceptions are caused by the fact that both models have a symmetry group: their solutions are determined up to a change in the signs of the model variables. Moreover, as Lorenz himself showed [32], the peak in the spectrum of the variable z is actually a very narrow band of increased spectral density. Lorenz concluded that it is improbable for an autonomous dynamical system to have peaks in the spectrum after the onset of chaos. In other words, if there is no symmetry group, each peak visible in the spectrum should be associated with some external periodic force or be a combination harmonic of several such forces.

In the case of EONI, the main band visible in the spectra in Figure 1 is significant at a 95 percent level and has an increased spectral density for a period of about 3.6 years; this band could be easily associated with tripling the period of the known Chandler oscillation of the Earth's poles, its period approximately 1.2 years (435 days). In [22] it is noted that the semi-annual course of solar heat flux dominates in the equatorial zone of the Earth, and the total period of three Chandler oscillations exposure to El Niño approximately coincides with the general period of seven solar half-year influences. This can lead to a resonance of these effects. Statistically significant bands of increased spectral density at periods of about 4.8 and 2.4 years correspond to four periods and two periods of the Chandler oscillation, respectively. In order to better appreciate the physical mechanism of the Chandler oscillation's influence on the El Niño rhythm see [33].

Note that ratio of the annual period to the Chandler period is extremely close to the «very» irrational number 0.839... [34], therefore, in the EONI spectrum there should be bands of increased spectral density at the combination harmonics of these periodicities with each other and with a predominant natural oscillation of ESOI processes. The dynamical systems theory indicates that the bands visible in the spectrum will be all the more numerous, the disproportionate will be the period of the most unstable intrinsic «mode» of the considered dynamical system with both external periodicities (ratios of their periods are 1/1.839... and 0.839...).

Regarding the possible period of ESOI natural «mode», there is a research [35] indicating a period of 20 months (about 1.7 years) as a probable one. Turning to Figure 2, it may be seen that near this period there is indeed some increase in spectral density (labeled as 1.8 years).

However, close to the ESOI spectra there is another increase over a period of about 1.9 years. Similar statistically significant (95% level) increases are also visible in the spectrum of [21]. Another powerful band is visible in the spectra shown in Figure 2 (and in the spectrum [21]) for a period of about 1.5 years. It is difficult to say which of these three peaks should be considered the main «mode» of ENSO processes. Maybe all of them have the right to be considered as such, for the issue of several varieties of El Niño is being actively discussed. It is possible that each of them has its own mode of oscillation. For example, it has long been known that in the late 19th and early 20th centuries, when more El Niño events of the eastern type were observed, longer-period fluctuations prevailed, and in the second half of the 20th century, when more El Niño of the central type began to appear (named Modocs), periods of oscillations reduced slightly. Considering all these «modes» to be real, the fourth most powerful band over a period of 5.1 years, from numbers visible in Figure 2, can be explained as the combination harmonic of the oscillations during the natural period of 1.5 years and the Chandler period of 1.2 years: $1/1.2 - 1/1.5 \approx 1/5.1$ in reverse years.

According to N.S. Sidorenkov [23], the longest-running rhythms of ENSO are determined by the superharmonics of the luni-solar nutation of the Earth's poles, their period being 18.6 years. The spectra in Figure 2 confirm this, since they have peaks at periods close to 9.1 and 6.2 years, corresponding to 1:2 and 1:3 superharmonics of lunar-solar nutation. However, these peaks do not reach the 95% level of statistical significance. A possible explanation for this is the insufficient length of available meteorological data series.

The first, third, and fifth peaks in amplitude in Figure 2, with their periods approximately equal to 5.6, 2.9, and 3.8 years, may be explained by superharmonics of the 1:2, 1:4, and 1:3 11-year cycle of solar activity. Note that a number of researchers have already pointed out the possible synchronization of the ENSO rhythm with solar activity (see, in particular, [24, 25, 36]).

Here it should be also noted that 1.5-year period mentioned above as one of natural «modes» can also be superharmonic of the solar cycle 1:8. The same may be said about other weak bands of high spectral density close in periods. For example, the peak at a period of 1.8 years is superharmonic 1:2 of the most important of those shown in Figure 2 bands for a period of 3.6 years. The band often mentioned in the literature for a period of 2.1 years might be superharmonic 1:2 of the already mentioned band for a period of 4.2 years. The band of the 4.8-year period shown in [21] to be very short of a significance level of 95%, might be a subharmonic of the 2:1 band in the 2.4-year period, that in [21] exceeds the significance level of 95% and corresponds to a double Chandler oscillation.

It was suggested in [24, 25] that frequencies of second-order peaks in atmospheric energy spectra in the range of interannual time scales could be calculated according to a simple rule: $\omega_{i\pm j} = 1/P_i \pm (j+1)/12$. The second term on the right-hand side of this formula describes contributions of the superharmonics of the annual solar flux ($j = 0, 1, 2, 3, 4, 5$) existing at all latitudes, and the first term describes contributions of the Chandler pole oscillation, its doubling and tripling (all periods are set in months here). All of the above allows to add to this list of frequencies those that arise as superharmonics of the 11-year cycle of solar activity and lunar-solar nutation of the Earth's rotation axis with a period of about 18.6 years. After all, although it is impossible to discern in real estimations of the spectra, there must be peaks of the third and fourth order, when frequencies of all four external forces (the annual solar flux, the 14-month Chandler oscillation of the poles of the Earth, the 11-year solar cycle activity and 18.6-year-long lunar-solar nutation) and their sub- and superharmonics are algebraically added. As a result, we might expect that there is an infinite, but countable (it can be renumbered) set of peaks. Due to the limited lengths of the observational series and the noisiness of their data, these peaks merge, forming more or less wide bands of increased spectral density in real estimations of the energy spectra.

Does the EONI spectrum under consideration have a continuous base or are there gaps separating adjacent bands within which the spectral density is zero? The answer to this question is important from the point of view of ENSO predictability: the spectrum continuity indicates chaotic nature and, therefore, unpredictability of ENSO for a more or less long time in advance. Modern practice of ENSO prediction, as it seems, confirms such a pessimistic conclusion. But maybe it is just the wrong methodology of today's prediction models?

To answer this question, we expand the usage of the well-known test of statistical significance of energy spectrum estimations against the null hypothesis of «noise», its' spectrum having the form $\omega = (1 - \alpha^2)/(1 + \alpha^2 - 2\alpha\cos(2\pi\omega/N))$, $\omega = 0, 1, ..., N/2$. The shape of the spectrum is substantially determined by α equal to the value of the temporal correlation of this noise at a unit shift. If α is zero, then the noise is «white». The bigger is α, the «redder» is the noise.

Temporal correlation function of a number of EONI's monthly mean values is shown in Figure 3. At a time shift of one month its value is 0.92, so that the «noise» of ENSO could be considered «red», as, in particular, it was accepted when testing the hypothesis about the noise nature of ENSO in the above-mentioned paper [21]. However, in Figure 3

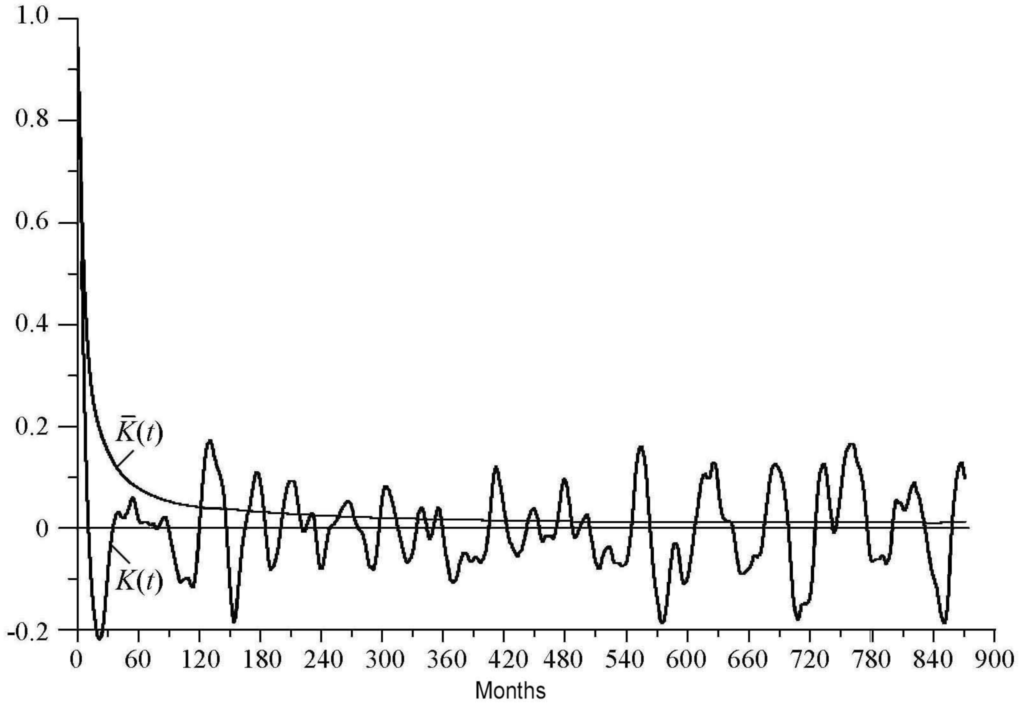

Fig. 3. Temporal correlation function of the monthly mean values of EONI calculated over the 1870–2014 year period ($K(t)$), and the squared accumulated sum of this function ($\bar{K}(t)$)

it could be seen that with time shifts of more than a year correlations sharply damp out to zero and remain as such with a further increase in the shift. Thus, in general, for the range of interannual time scales, the «noise» of ENSO should be regarded as «white». Accepting this, EONI and ESOI spectra for the annual period were calculated, mean for all months (see Figure 2, light lines). All these spectra exhibit peaks in the periods that were listed earlier while considering the spectra of EONI and ESOI monthly mean values shown in Figure 2 (dark lines). Figure 4 shows the spectra separately for the period from October to February, during which El Niño events usually reach their maximum, together with the average spectrum for these months. Spectra of white noise calculated by the above formula for $\alpha = 0$ are shown in Figure 5.

To test the null hypothesis of the noise nature of energy spectra, the value of the 95 percent quantile is found from the distribution table χ^2 for the corresponding numbers of degrees of freedom. However, on an equal basis such a test may be applied to spectral

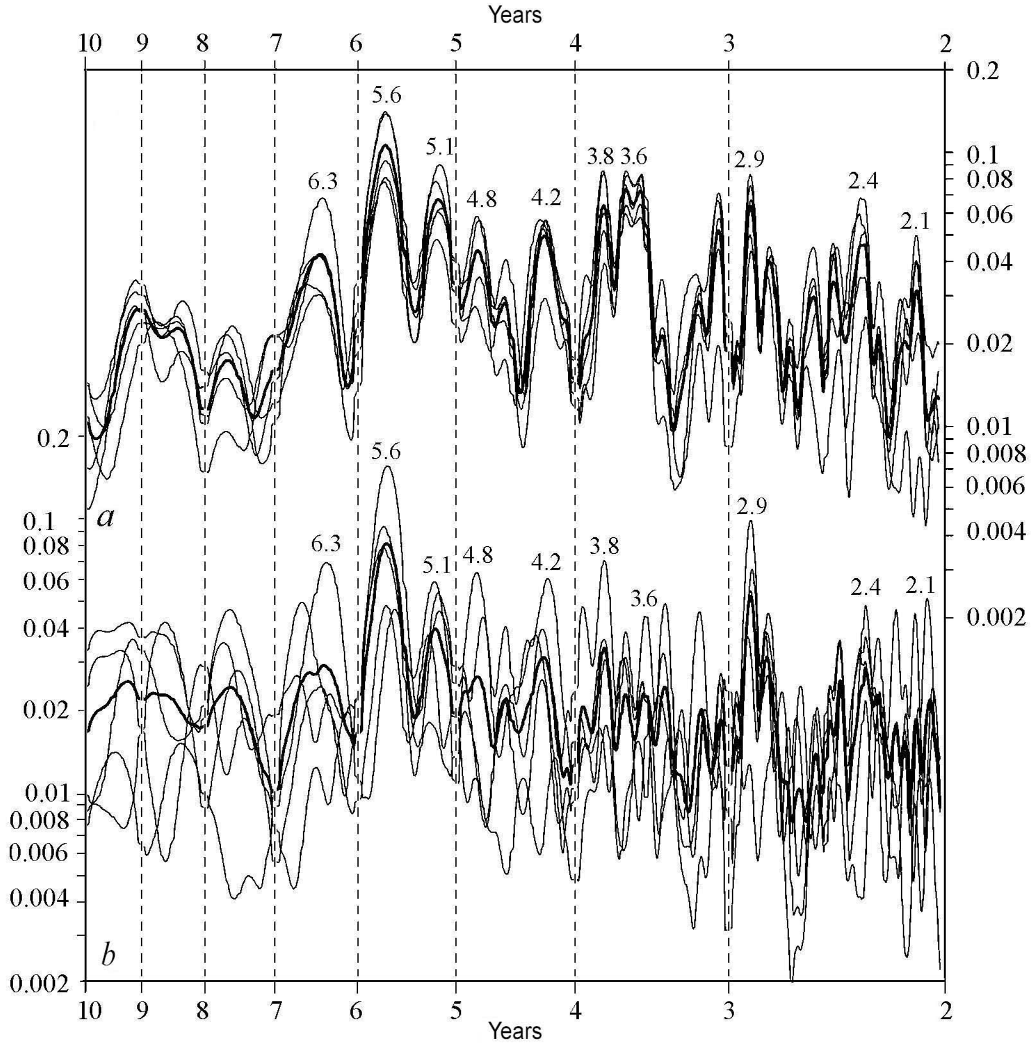

Fig. 4. Power spectra of the mappings on the annual period of EONI (*a*) and ESOI (*b*) for separate months from October to February (thin lines), and their mean values (fat lines)

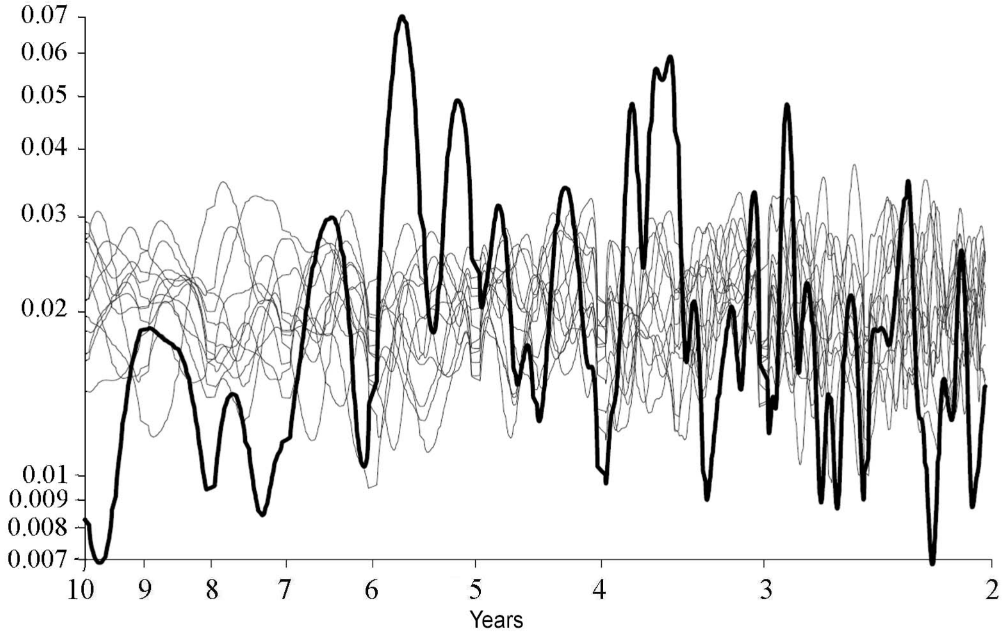

Fig. 5. The mean power spectrum of the mappings on the annual period for EONI for all months calculated by means of the FFT-method (fat line) and power spectra of the twenty realizations of the respective mapping of the white noise (thin lines)

density gaps. To do this, the 5 percent quantile is to be found. This was done, and Figure 1 shows both quantiles. They demonstrate that not only the most powerful peaks in the spectra of the indices considered exceed the level of the 95 percent quantile, but also some lacunae in these spectra fall below the 5 percent quantile. Both of these facts allow us to reject the hypothesis of the noise nature of the spectra under consideration.

To definitely reject this hypothesis and recognize the spectrum under consideration as consisting of a discrete set of spectral density peaks, the average spectrum of the EONI mapping over the annual period for all months of the year was calculated. It is shown in Figure 5 along with spectra calculated from twelve random realizations of «white» noise corresponding to all months of the year. Even more clearly than in the case of using χ^2 approximations for distributions of sample spectral density estimations, it can be seen that many peaks and many gaps in the considered EONI spectrum go beyond the limits restricting the spectral density estimations for «white» noise.

Returning to the time correlation function shown in Figure 3, let us indicate its property to oscillate at large time shifts around zero with a non-decreasing range (within approximately $+0.2$). Continuous oscillations of the temporal correlations are inherent in periodic and quasiperiodic time series. However, in [37] it was pointed out that this may also be characteristic of the so-called strange non-chaotic dynamics. To distinguish between these cases, in the same work it was recommended to plot accumulated sums of time correlation squares. If the dynamics is periodic or quasiperiodic, then this accumulated sum does not decrease with an increase in the time shift, but if the dynamics is more complex, then it decreases approaching zero. Figure 3 shows that for ENSO temporal correlations it is the latter case.

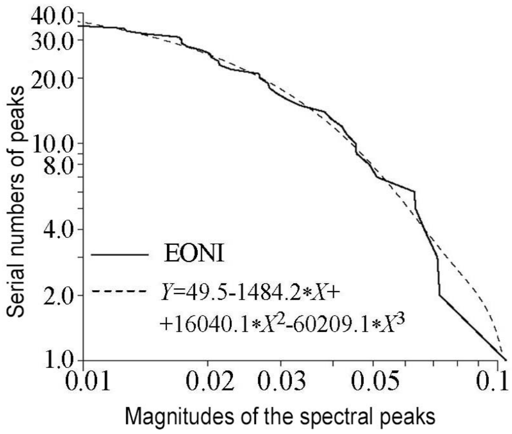

Fig. 6. The mean relationship between logarithms of the spectral peak magnitudes (abscissa) and serial numbers of these peaks (ordinate) in the power spectra of the EONI-mappings for October – February (fat line). An approximation of this relationship also is shown as it represented by a cubic polynomial (dotted line)

Another way to answer the question about the continuity or discreteness of the energy spectra of quasiperiodically forced nonlinear dynamical systems was indicated in [38, 39]. It focuses on considering the relationship between powers of the spectral density peaks and their serial numbers (the peaks should be sorted in decreasing order of their power). Figure 6 shows how this relation looks like for the average of the five spectra of EONI mappings corresponding to the cold period of the year for the northern hemisphere, when the EONI variability is the greatest (October–February). The average ratio was calculated by the method of least squares and is described by a third-order polynomial. Generally speaking, this should indicate the quasiperiodic dynamics of ENSO, because in cases of strange non-chaos and deterministic chaos this average should be a straight line on a double logarithmic scale. However, it is difficult to say exactly what the dynamics of ENSO are, because the greatest shift of the mean graph from a straight line occurs in its upper left part, and not in the lower right part, being determined by the most powerful peaks of spectral density, and that is required according to the existing theory of strange non-chaos [38, 39].

In any case, whether that be a «simple» quasiperiodicity or strange non-chaos, the result obtained indicates in principle a possibility of long-term (for years to come) ENSO forecasts.

Conclusion

Results of this study are as follows:

- Energy spectra of ENSO processes were studied in detail basing on the methodology of the mathematical theory of nonlinear dynamic systems with quasiperiodic external forcing. We have found that all the main peaks, more precisely, bands of increased spectral density, may be interpreted as the results of responses of the global climate system to the combined effect of the annual solar flux and three other external forces, periods being significantly incommensurable with the annual period.

- A part of the bands with increased spectral density is assigned to the subharmonics of the Chandler oscillation of the Earth's poles. Another part is assigned to the superharmonics of the lunar-solar nutation, and the third part – to the superharmonics of the 11-year cycle of solar activity. Statistical significance of these bands, already assumed by many researchers, is confirmed.
- Based on special considerations of the ENSO energy spectra's structure, we have obtained an evidence of discreteness of ENSO energy spectra, that is, their non-chaotic nature, previously not anticipated.
- A principle possibility of long-term (for years to come) ENSO predictions follows from discreteness of the ENSO energy spectra.

References

1. Kurzbach J.E., Bryson R.A. Variance spectrum of Holocene climatic fluctuations in the North Atlantic sector. *J. Atmoc. Sci.* 1974. Vol. 183. No.4126. P. 709–714.
2. Mitchell J.M. Note on solar variability and volcanic activity as potential sources of climatic variability. *The Physical Basis of Climate and Climate modeling. GARP Bull. Series.* 1975. No. 16. P. 127–131.
3. Monin A.S., Shishkov Yu.A. Istoriya klimata. L.: Gidrometeoizdat, 1979. 407 s. (in Russian).
4. Pelletier J. The power-spectral density of atmospheric temperature from time scales of 10^{-2} to 10^{6} yr. *Earth Planetary Sci. Lett.* 1998. Vol. 158. P. 157–164.
5. Huybers P., Curry W. Links between annual, Milankovitch and continuum temperature variability. *Nature.* 2006. Vol. 441. P. 329–332.
6. Vakulenko N.V., Sonechkin D.M., Ivashchenko N.N., Kotlyakov V.M. On periods of multiplying bifurcation of early Pleistocene glacial cycles. *Doklady Earth Sciences.* 2011. Vol. 436. No.2. P. 245–248.
7. Vakulenko N.V., Sonechkin D.M. Evidence of the upcoming end of the contemporary interglacial. *Doklady Earth Sciences.* 2013. Vol. 452. No. 1. P. 926–929.
8. Vakulenko N.V., Sonechkin D.M., Kotlyakov V.M. Increase in the global climate variability from about 400 ka bp until present. *Doklady Earth Sciences.* 2014. Vol. 456. No.2. P. 745–748.
9. Vakulenko N.V., Ivashchenko N.N., Kotlyakov V.M., Sonechkin D.M. On the period-multiplying bifurcation of glacial cycles in the Pliocene – Pleistocene. *Izvestiya VUZ. Applied nonlinear dynamics.* 2013. Vol. 21. No. 2. S. 88–112 (in Russian).
10. Vakulenko N.V., Sonechkin D.M., Kotlyakov V.M. Is climate predictable on a geological time scale? *Doklady Earth Sciences.* 2015. Vol. 460. No.1. P. 68–72.
11. Ivashchenko N.N., Kotlyakov V.M., Sonechkin D.M. et al. On bifurcations inducing glacial cycle lengthening during Pliocene/Pleistocene epoch. *Intern. J. Bifurcation and Chaos.* 2014. Vol. 24. No.8. 1440018 (8 pages).
12. Jin F.F., Neelin J.D., GhilM. El Nino on the devil's: annual subharmonic steps to chaos. *Science.* 1994. Vol. 264. P. 70–72.
13. Jin F.F., Neelin J.D., Ghil M. El Nino/Southern Oscillation and the annual cycle: Subharmonic frequency locking and aperiodicity. *Physica D.* 1996. Vol. 98. P. 442–465.

© *I.V. Serykh, D.M. Sonechkin*

14. Tziperman E., Stone L., Cane M.A. et al. El Nino chaos: overlapping of resonances between the seasonal cycle and the Pacific ocean – atmosphere oscillator. *Science.* 1994. Vol. 264. P. 72–74.
15. Tziperman E., Zebiak S.E., Cane M.A. Mechanisms of seasonal – ENSO interaction. *J. Atmos. Sci.* 1997. Vol. 54. P. 61–71.
16. Tziperman E., Cane M.A., Zebiak S.E. et al. Locking of El Nino's peak time to the end of the calendar year in the delayed oscillator picture of ENSO. *J. Climate.* 1998. Vol. 9. P. 2191–2199.
17. Eccles F., Tziperman E. Nonlinear effects on ENSO's period. *J. Atmos. Sci.* 2004. Vol. 61. P. 474–482.
18. Sonechkin D.M. Stokhastichnost' v modelyakh obshchey tsirkulyatsii atmosfery. L.: Gidrometeoizdat, 1984. 280 s. (in Russian).
19. Datsenko N.M., Zimin N.E. *Proceedings of the USSR Hydrometeorological Center*. 1987. Issue. 290. S. 111–127 (in Russian).
20. Vinogradskaya A.A., Vlasova, Datsenko N.M., Sonechkin D.M. *Proceedings of the USSR Hydrometeorological Center*. 1988. Issue. 297. S. 166–174 (in Russian).
21. Torrence C., Compo G.P. A practical guide to wavelet analysis. *Bull. Amer. Meteorol. Soc.* 1997. Vol. 79, No.1. P. 61–78.
22. Sonechkin D.M., Ivashchenko N.N. On the role of a quasiperiodic forcing in the interannual and interdecadal climate variations. *CLIVAR Exchanges.* 2001. Vol. 6, No.1. P. 5–6.
23. Sidorenkov N.S. The interaction between Earth's rotation and geophysical processes. 2009. Wiley-VCH & Co. KCaA, Weinheim, 305 p.
24. Currie R.G., Hameed S. Evidence of quasi-biennial oscillations in a general circulation model. *Geophys. Res. Lett.* 1988. Vol. 15. P. 649–652.
25. Hameed S., Currie R.G., LaGrone H. Signals in atmospheric pressure variations from 2 to $\sim$ 70 months: Part 1. Simulations by two coupled ocean-atmosphere GCMs. *Intern. J. Climatol.* 2007. Vol. 15. No.8. P. 853–871.
26. Allan R.J., Ansell T.J. A new globally-complete monthly historical gridded mean sea level pressure data set (HadSLP2): 1850–2004. *J. Climate.* 2006. Vol. 19. P. 5816–05846.
27. Rayner N.A., Parker D.E., Horton E.B., Folland C.K., Alexander L.V., Rowell D.P., Kent E.C., Kaplan A. Global analyses of sea surface temperature, sea ice, and night marine air temperature since the late nineteenth century. *J. Geophys. Res.* 2003. Vol. 108, No. D14, 4407 10.1029/2002JD002670.
28. Compo G.P., Whitaker J.S., Sardeshmukh P.D. et al. The Twentieth Century Reanalysis Project. *Quarterly J. Roy. Meteorol. Soc.* 2011. 137. P. 1–28.
29. Braganza, K., Gergis J.L., Power S.B., Risbey J.S., Fowler A.M. A multiproxy index of the El Nino–Southern Oscillation, A.D. 1525–1982. *J. Geophys. Res.* 2009. 114. D05106.
30. Lorenz E.N. Deterministic nonperiodic flow. *J. Atmos. Sci.* 1963. Vol. 20. P. 130–141.

© *I.V. Serykh, D.M. Sonechkin*

31. Roessler O.E. Continuous Chaos. In: Synergetics, Proceedings Intern. Workshop, Schloss Elmar, Bavaria, 1977: Berlin, 1977. P. 184–197.
32. Lorenz E.N. A very narrow spectral band. *J. Statist. Phys.* 1984. Vol. 36. No. 1–2. P. 1–14.
33. Serykh I.V., Sonechkin D.M. Confirmation of the oceanic pole tide influence on El Niño. *Sovremennye problemy distantsionnogo zondirovaniya Zemli iz Kosmosa.* 2016. Vol. 13, No.2. S. 44–52 (in Russian).
34. Kim S.-H., Ostlund S. Renormalization of mappings of the two-torus. *Physical Review Letters.* 1985. Vol. 55, No.1. P. 1165–1168.
35. Tsonis A.A., Roebber P.J., Elsner J.B. A characteristic time scale in the global temperature record // *Geophys. Res. Lett.* 1998. Vol. 25, No.15. P. 2821–2823.
36. Vakulenko N.V., Sonechkin D.M. Evidence of the solar activity's effect on El Nino – Southern oscillation // *Oceanology.* 2011. Vol. 51, No. 6. P. 935–939.
37. Feudel U., Kuznetsov S., Pikovsky A. Strange nonchaotic attractors. Singapore: *World Scientific Series on Nonlinear Science*, Series A. 2006. Vol. 56.
38. Bondeson A., Ott E., Antonsen T.M. Quasiperiodic forced damped pendula and Schroedinger equations with quasiperiodic potentials: implications of their equivalence // *Phys. Rev. Lett.* 1985. Vol. 55. No. 20. P. 2103–2106.
39. Ditto W.L., Spano M.L., Savege H.T. et al. Experimental observation of a strange nonchaotic attractor // *Phys. Rev. Lett.* 1990. Vol. 65. No 5. P. 533–536.

© *I.V. Serykh, D.M. Sonechkin*