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On the conditions for safe connection to hub-cluster power grids

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Abstract. *Purpose* of this work is studying of the dynamics of a power grid model that results from the expansion of a highly centralized grid, i.e. a hub-cluster, by adding a small subgrid. The main attention is paid to the study of possible power grid operation regimes and their characteristics. *Methods.* Numerical simulation of power grid operation, the dynamics of which is described by the Kuramoto equations with inertia, is used. *Results.* Various power grid operation regimes and the boundaries of their existence in the parameter space are given. The main characteristics of these regimes, such as the probability of realization and the magnitude of oscillations of regime variables, are considered. The conditions for safe connection to hub-cluster power grids are obtained. *Conclusion.* The dynamics of power grid consisting of two subgrids and its operation regimes are considered. Based on the characteristics of these regimes, their safety for subgrids is determined. The results obtained made it possible to formulate conditions for a safe connection to hub-cluster power grids.

Keywords: power grids, synchronous machines, Kuramoto model, synchronization.

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Introduction

The main task of the study of power grids is to ensure their stable and uninterrupted operation [1–3] in accordance with the synchronous mode of interaction between consumers and generators of electricity . One of the reasons for the violation of the synchronous mode is a change in the topology of the power grid, that is, the composition of its elements and transmission lines between them [4–8]. Currently, there is an active expansion of existing power grids due to the addition of new generators and consumers of electricity, which leads to the problem of their safe connection, in which, on the one hand, the synchronous mode of operation of the original grid should be maintained, and on the other hand, synchronous operation mode should be established throughout the extended grid.

The dynamic approach has become the most widely used in the study of the stability of power grids [4–20]. In this approach, power grids are considered as dynamic grids. Their active node-element is a synchronous machine — the basic element of the power grid [9]. Transmission lines play the role of links that interact between grid nodes. The dynamics of each synchronous machine can be described in a simplified way by the equation of motion of its rotor, which has a small mathematical dimension, which facilitates the study of models of real power grids consisting of a large number of elements. The dynamic approach was used in the study of both fairly large power grids [4, 6, 8, 11–15] and small subgrids [5, 10, 16–18] that are part of them. The most complete overview of the results obtained through this approach can be found in [9, 19, 20]. In the works [18, 21–23], the dynamics of power grids consisting of one generator (consumer) and several consumers (generators) connected with it. Such grids are called hub clusters. A hub cluster is a model of a highly centralized power grid consisting of a powerful power plant (nuclear power plant , hydroelectric power station) supplying a large number of consumers. As a rule, large power grids consist of several hub clusters. The work [24] investigated the dynamics of a small part of the power grid consisting of two generators and one consumer connected to each other. This subgrid forms the so-called motif. It represents a typical option for expanding the existing power grid by introducing additional power plants that can operate, for example, at the expense of alternative energy sources. It was assumed that all elements of the power grid are in synchronous mode, except for the elements of the motif, which significantly limits the results of the work.

In this paper, we consider a grid formed as a result of the expansion of the power grid in the form of a hub cluster by connecting a motif to it. At the same time, no restrictions are imposed on the operating mode of the hub cluster. The main operating modes of the grid are established, as well as the conditions for the safe connection of the motif to the hub cluster are found. The characteristic of each mode from the point of view of its safety for the operating of the power grid is given. The values of transmission lines capacity of the grid have been determined, at which, depending on the initial conditions, various modes of its operation can be established, which makes it possible to identify the conditions for the safe connection of new elements to the power grid with a hub topology.

1. Model

Consider a power grid in the form of a graph, the nodes of which correspond to synchronous machines that act as consumers or generators of electricity, and the edges are transmission lines connecting these machines. We believe that the behavior of each synchronous machine is determined by the equation of motion of its rotor, and assume that it interacts with other machines through transmission lines having purely inductive impedance [9, 19, 20]. Then the state of the ith synchronous machine is determined by the phase of its rotor Θ_i, counted in the coordinate system rotating with the reference frequency of the grid. It obeys the dimensionless Kuramoto equation with inertia [9, 19, 20]

$$\frac{d^2\Theta_i}{dt^2} = P_i - \alpha\frac{d\Theta_i}{dt} + \sum_{j=1}^{N} K_{i,j}\sin(\Theta_j - \Theta_i), \tag{1}$$

where $i = 1, 2, ..., N$, N — the number of synchronous machines. The parameter P_i characterizes the power supplied to the rotor shaft of the ith synchronous machine or removed from it. If mechanical power is supplied, that is, $P_i > 0$, then the machine works as a generator, and if it is removed, that is, $P_i < 0$, the machine works as a consumer. The term $\alpha d\Theta_i/dt$ characterizes the loss power. The parameter α — is a damping coefficient that generically reflects the influence of all damping factors. The term $K_{i,j}\sin(\Theta_j - \Theta_i) = P_{i,j}$ represents the power exchanged between i-i and j-i synchronous machines connected by a transmission line with a capacity of $K_{i,j}$ $(K_{i,j} = K_{j,i})$, equal to the maximum power transmitted over this line. If the machines are not connected by a transmission line, then $K_{i,j} = 0$.

The main modes of operation of the power grid are: synchronous, asynchronous and quasi-synchronous. These modes are characterized by different behavior of phase differences $\Theta_i - \Theta_j = \Theta_{ij}$ and the powers depending on them $P_{i,j}$.

- If the phase difference is $\Theta_{ij} = \text{const}$, then the power is $P_{i,j} = \text{const}$. In this case, a synchronous mode of interaction of the i and j elements of the power grid is implemented. If similar conditions are met for all interconnected elements of the power grid, then a synchronous mode is implemented in it, which is normal.
- If the phase difference Θ_{ij} decreases or increases in time, then the power of $P_{i,j}$ is constantly changing. In this case, the asynchronous mode of interaction of the i and j elements of the power grid is implemented. This mode is emergency.
- If the phase difference Θ_{ij} oscillates around some average value, so that $|\Theta_{ij}(t)| < \pi$, then the power of $P_{i,j}$ also oscillates. In this case, a quasi-synchronous mode of interaction of the i and j elements of the power grid is implemented. With small amplitudes of power fluctuations $P_{i,j}$, the quasi-synchronous mode is relatively safe.

2. Power grid model

Consider a grid formed by two subgrids: the motif [24], consisting of a single consumer ($c_1,\ P_1 < 0$) and two generators ($g_{2,3},\ P_{2,3} > 0$), and a hub cluster [18, 22, 23] consisting of one generator ($g_4,\ P_4 > 0$) and $N-4$ consumers ($c_j,\ P_j < 0,\ j = 5, 6, ..., N$) (fig. 1). Let's consider the equations describing the dynamics of the grid, and for convenience we will introduce new variables and parameters

$$\begin{aligned} &\phi_1 = \Theta_2 - \Theta_1, \quad \phi_2 = \Theta_3 - \Theta_1, \quad \phi_3 = \Theta_4 - \Theta_1, \quad \phi_{j-1} = \Theta_j - \Theta_4, \\ &K_{1,2} = K_{1,3} = K, \quad K_{2,3} = B, \quad K_{1,4} = C, \quad K_{4,j} = H, \quad j = 5, 6, ..., N, \end{aligned} \tag{2}$$

Taking into account (2), the equations (1) will take the form

$$\begin{cases} \dot{\phi}_i = y_i, \quad i = 1, 2, ..., N, \\ \dot{y}_1 = P_2 - P_1 - \alpha y_1 - 2K\sin(\phi_1) - K\sin(\phi_2) - C\sin(\phi_3) - B\sin(\phi_1 - \phi_2), \\ \dot{y}_2 = P_3 - P_1 - \alpha y_2 - K\sin(\phi_1) - 2K\sin(\phi_2) - C\sin(\phi_3) - B\sin(\phi_2 - \phi_1), \\ \dot{y}_3 = P_4 - P_1 - \alpha y_3 - K\sin(\phi_1) - K\sin(\phi_2) - 2C\sin(\phi_3) + \sum\limits_{k=4}^{N-1} H\sin(\phi_k), \\ \dot{y}_j = P_{j+1} - P_4 - \alpha y_j + C\sin(\phi_3) - H\sin(\phi_j) - \sum\limits_{k=4}^{N-1} H\sin(\phi_k), \quad j = 4, 5, ..., N-1, \end{cases} \tag{3}$$

where the dot denotes the time derivative t. The system (3) is defined in a cylindrical phase space $G = S^{N-1} \times R^{N-1}$. It is not difficult to show that there is an attraction region in it

$$\begin{aligned} &G^+ = \{\phi_i \in S^1,\ y_i \in [y_i^-, y_i^+],\ i = 1, 2, ..., N-1\}, \\ &y_{1,2}^{\pm} = (P_{2,3} - P_1 \pm [3K + C + B]), \\ &y_3^{\pm} = (P_4 - P_1 \pm [2K + 2C + (N-4)H]), \\ &y_j^{\pm} = (P_{j+1} - P_4 \pm [C + (N-3)H]),\ j = 4, 5, ..., N-1. \end{aligned} \tag{4}$$

The variables $\phi_{1,2},\ y_{1,2}$ describe the motif, and the variables $\phi_3,\ y_j,\ j = 4, 5, ..., N-1$ — hub cluster. The variables $\pi_3,\ y_3$ describe the interaction of the motif with the hub cluster. The parameters K, B determine the capacities of the transmission lines connecting the generators $g_{2,3}$ with the consumer c_1 and among themselves, respectively, and characterize the motif. The parameter H determines the capacity of transmission lines connecting the generator g_4 with consumers c_j, $j = 5, 6, ..., N-1$, and characterizes the hub cluster. The parameter C — is the bandwidth of the transmission line connecting the consumer c_1 with the generator g_1, that is, the motif with the hub cluster.

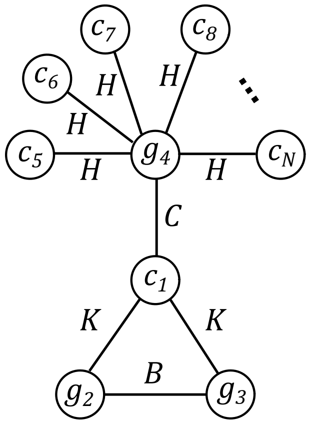

Fig. 1. Schematic representation of the studied power grid in the form of a graph

We assume that the power balance condition necessary for the existence of a synchronous mode is fulfilled for the motif and the hub cluster separately and, therefore, is fulfilled for the entire grid, that is

$$\sum_{k=1}^{3} P_k = 0, \quad \sum_{j=4}^{N} P_j = 0, \quad \sum_{i=1}^{N} P_i = 0. \tag{5}$$

The condition (5) allows the existence of synchronous modes in the motif and hub cluster in the absence of a connection between them ($C = 0$), that is, before connecting one subgridsto another.

Let's set the parameters of the motif and hub cluster at $C = 0$ based on the following considerations.

The dynamics of the motif at $C = 0$ was studied in [24]. In particular, it was found that asynchronous modes exist in it, which can lead to the loss of synchronous mode in the grid to which it is connected ($C > 0$). The probability of implementing asynchronous modes in a motif depends on its parameters. It can be assumed that when connected to a hub cluster ($C > 0$), the motif can significantly affect its dynamics, if there is an asynchronous mode in the motif at $C = 0$ and the probability of its implementation is quite high. Therefore, based on the results of [24], we fix the parameters $\alpha = 0.8$, $P_1 = -2/3$, $P_2 = P_3 = 1/3$ (satisfy the condition (5)). With these parameters, the probability distribution of synchronous mode implementation is known $P_{\text{sync}} = P_{\text{sync}}(B, K)$ [24]; accordingly, the probability of asynchronous mode implementation in the motif $P_{\text{async}} = 1 - P_{\text{sync}}$. Let's choose the parameters $B = 0.2$ and $K = 0.4$, for which $P_{\text{async}} > 0.8$.

Consider the hub cluster parameters. It is convenient to choose the value of the parameter P_4 of the same order as the values $|P_i|$, $i = 1, 2, 3$, so that the grid is more homogeneous, that is, it consists of consumers and generators that differ slightly from each other in terms of consumed and generated capacities. Fix $P_4 = 0.6$ and in order to fulfill the condition (5), we put

$$P_j = P_{\text{con}}(N), \quad P_{\text{con}}(N) = -\frac{P_4}{N-4}, \quad j = 5, 6, ..., N. \tag{6}$$

The capacity H, on the one hand, should meet the needs of consumers, and on the other — should not be excessive, so we will limit ourselves to the values of the parameter

$H \in [H_{\text{start}}, H_{\text{fin}}]$, $H_{\text{start}} = 1.1|P_{\text{con}}|$, $H_{\text{fin}} = 2|P_{\text{con}}|$, which provide the necessary capacity of the transmission lines of the hub cluster.

Thus, we consider the dynamics of the grid at the following parameter values:

$$\begin{gathered} P_1 = -2/3, \quad P_2 = P_3 = 1/3, \quad P_4 = 0.6, \; P_j = P_{\text{con}}, \quad j = 5, 6, ..., N, \\ \alpha = 0.8, \quad B = 0.2, \quad K = 0.4, \quad H \in [H_{\text{start}}, H_{\text{fin}}]. \end{gathered} \tag{7}$$

3. Operating modes of the power grid and conditions for safe connection

Here, we consider the dynamics of system (3). The attraction region G^+ contains a single locally stable state

$$\begin{gathered} O(\phi_i = \phi_i^{(0)}, \; y_i = 0, \; i = 1, 2, ..., N-1), \\ \phi_{1,2}^{(0)} = \arcsin\left(\frac{P_{2,3}}{K}\right), \quad \phi_3^{(0)} = 0, \quad \phi_j^{(0)} = \arcsin\left(\frac{P_{\text{con}}}{H}\right), \quad j = 4, 5, ..., N-1, \end{gathered} \tag{8}$$

which exists for $K > P_{2,3}$ and $H > |P_{\text{con}}|$. It corresponds to the synchronous mode of operation of the entire grid. The rest steady states are saddles. Note that with the parameters (7), there is always an stable state of O.

Along with synchronous, modes arise in the considered power grid, which are various combinations of asynchronous and quasi-synchronous modes of interaction of individual elements of the grid. Such modes correspond to rotational-type attractors in the phase space of the system (3). The setting of a particular mode depends on the initial conditions and the values of the system parameters (3). Therefore, consider the partition of the (B,K)-parameter plane into regions corresponding to different modes of operation of a grid consisting of $N = 10$ elements (Fig. 2, *a*).

- If $(H, C) \in a$, then the stable state of O is globally asymptotically stable, that is, under any initial conditions, a synchronous mode of interaction between all connected elements of the grid is realized. Therefore, this parameter region is safe.
- If $(H, C) \in b_1$, then mode *1* can be established in the grid, in which there is an asynchronous mode of interaction between the elements of the hub cluster: consumers c_j, $j = 5, ..., N-1$ and the generator g_4, as well as a quasi-synchronous mode of interaction between the elements of the motif: generators $g_{2,3}$ and the consumer c_1 (Fig. 2, *b*). That is, asynchronous mode is set in the hub cluster, and quasi-synchronous mode is set in the motif.
- If $(H, C) \in b_2$, then in the grid, along with the mode *1*, mode *2* can be established, in which an asynchronous mode of interaction is observed both between the elements of the hub cluster and between the elements of the motif (Fig. 2, *c*). Asynchronous mode is set throughout the grid.
- If $(H, C) \in b_3$, then in the grid, along with the modes *1* and *2*, mode *3* can be established, in which there is a quasi-synchronous mode of interaction between the elements of the hub cluster, as well as asynchronous the mode of interaction between the elements of the motif (Fig. 2, *d*). That is, a quasi-synchronous mode is set in the hub cluster, and an asynchronous mode is set in the motif.
- If $(H, C) \in b_4$, then the *3* mode can be set in the grid (see Fig. 2, *d*).

In each of the modes *1*, *2*, *3*, an asynchronous mode of interaction is observed between the hub cluster generator g_4 and the consumer of the motif c_1, which are connected by a transmission line with a capacity of C (see Fig. 2, *b*–*d*).

There is a threshold value of the parameter $C = C_1$, when exceeded, synchronous mode is always set in the grid, if the value of the parameter H allows its existence (see Fig. 2, *a*).

For the values of the parameter $H > H_1$ (see Fig. 2, a), it is possible to bring the grid parameters to theregion of a, thereby guaranteeing the establishment of synchronous mode, by setting a relatively small (compared to K, B and H) parameter value $C > C_0$.

Thus, a safe connection of the motif to the hub cluster is implemented in the power grid if the parameters meet the conditions

$$\begin{cases} C > C_{\mathrm{th}}(H), & \text{если } H_{\mathrm{start}} \leqslant H < H_1, \\ C > C_0, & \text{если } H_1 \leqslant H \leqslant H_{\mathrm{fin}}. \end{cases} \tag{9}$$

Note that $C_0 \leqslant C_{\mathrm{th}}(H)$, moreover $C_0 \approx 0.0430$ is several times less than the characteristic value of $C_1 \approx 0.3365$. Therefore, with a sufficient margin of capacity $H \geqslant H_1$ of transmission lines in the hub cluster, the safe connection of the motif requires the creation of a transmission line with a lower capacity value C than in the case of $H < H_1$.

However, in practice, it is not always possible to realize the necessary reserve of transmission line capacity, which is especially important when expanding existing power grids. Therefore, along with the safe parameterregion a, it is necessary to take into account the less safe, from the point of view of establishing synchronous mode, parameter regions b_i, $i = 1, 2, 3, 4$, where the modes *1*, *2* or *3* can be set. Let's now turn to their characteristics.

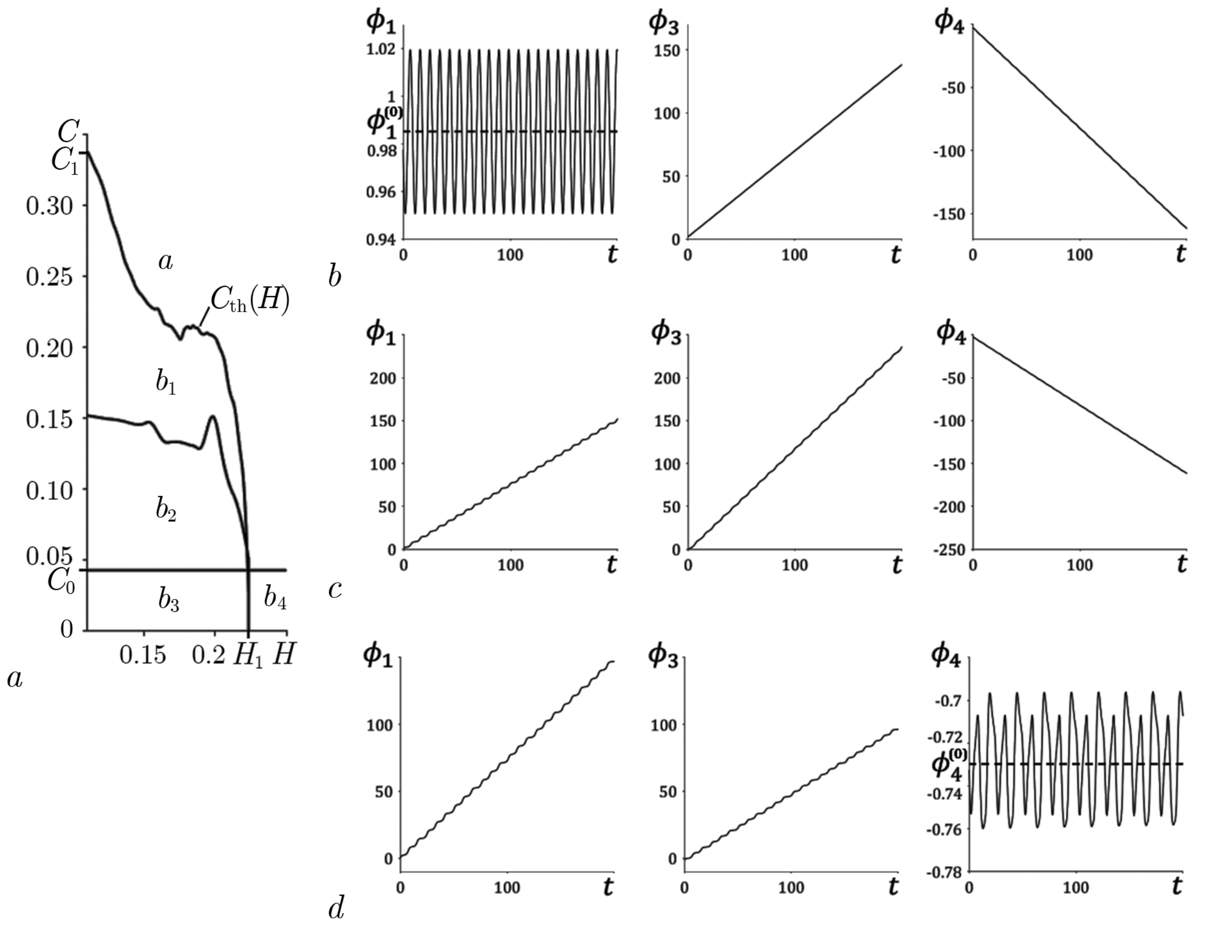

Fig. 2. a — Partition of (H, C) — parameter plane into the regions with different working regimes of the grid. Characteristic oscillograms of variables $\phi_1(t)$, $\phi_3(t)$, $\phi_4(t)$ in case of establishment in the grid b — regime *1*, c — regime *2*, d — regime *3*. The oscillograms of the regimes are given for parameter values $H = 0.15$, $C = 0.02$. Parameter values $C_0 \approx 0.0430$, $H_1 \approx 0.2234$, $C_1 \approx 0.3365$

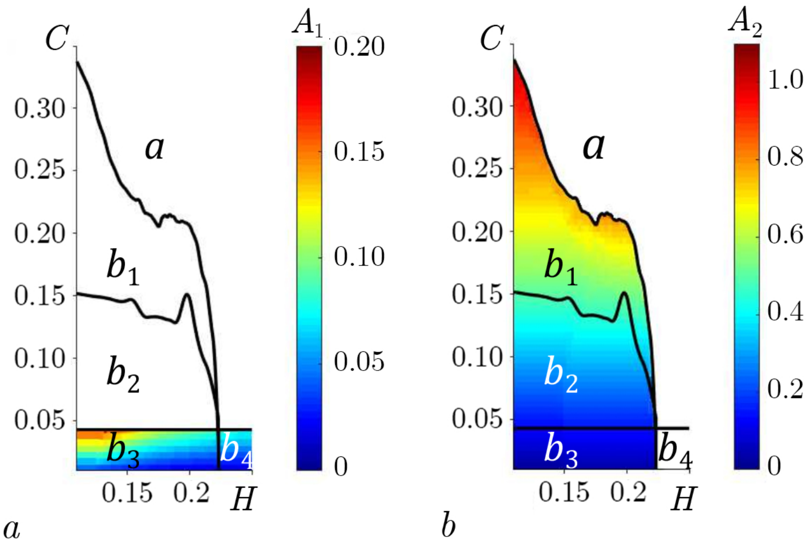

Fig. 3. The magnitude of oscillations a — A_1, b — A_2 depending on parameters H and C (color online)

Obviously, the *m*ode 2 is an emergency for the entire grid. The *1* and *3* modes provide for quasi-synchronous modes of interaction of individual grid elements and can be relatively safe for one of the subgrids. It is convenient to characterize the quasi-synchronous mode of interaction using the oscillation range of the corresponding variables $\phi_i(t)$. Denote by $A_1 = |\max(\phi_1(t)) - \min(\phi_1(t))|$ — the swing of the variable ϕ_1 in the case of setting the mode *1*, and after $A_2 = |\max(\phi_4(t)) - \min(\phi_4(t))|$ — the swing of the variable ϕ_4 in the case of setting the mode *3* (fig. 3). The calculation shows that $A_1 < 0.2$, and $A_2 < 1.1$ in the considered parameter range H, C (see Fig. 3), which is significantly less than the value of 2π, which is the limit for the oscillation range in quasi-synchronous mode. Thus, the *1* mode is relatively safe for the motif, and the *3* mode is— for the hub cluster. At the same time, the *1* mode is emergency (dangerous) for the hub cluster, and the *3* mode is— for motif.

Since the hub cluster is a large centralized grid, and the motif is a variant of its expansion, maintaining the safe mode of operation of the hub cluster is a higher priority than maintaining a similar mode in the motif. Therefore, the *3* mode is preferable to the modes *1* and *2*.

The implementation of the synchronous mode or one of the three modes mentioned above depends on the values of the parameters H, C and the initial conditions. In general, the initial conditions are arbitrary. Therefore, it is convenient to determine the probability of the realization of each detected mode. To do this, we fix the parameters H and C, randomly select n initial conditions from the attraction region G^+ and determine the number of initial conditions leading to the mode number i, denote it by n_i, $i = 1, 2, 3$. Then the probability of setting the mode i will be defined as $p_i = n_i/n$. In Fig. 4 the probability distributions $p_i(H, C)$ are given for $n = 1000$. Based on them, we will divide the b_i parameter regions into dangerous and relatively safe for the hub cluster operation.

- With parameters from the b_1 region the mode *1* has the highest probability of being established, which is an emergency for the hub cluster, so this parameterregion is— dangerous.
- With parameters from the region b_2 , the *2* mode has the highest probability of being established, which is an emergency for the entire grid, so this parameterregion is also dangerous.
- With parameters from the domain b_3, the modes *h*ave the highest probability of being

established 2 and *3*. With the growth of the parameter H, the probability of p_2 decreases, and p_3 increases. Therefore, the region b_3, with the appropriate selection of the H parameter, can be used for relatively safe operation of the hub cluster.

- With parameters from the region b_4 the *3* mode has the highest probability of being established, which is relatively safe for the hub cluster, as is the b_4 region itself.

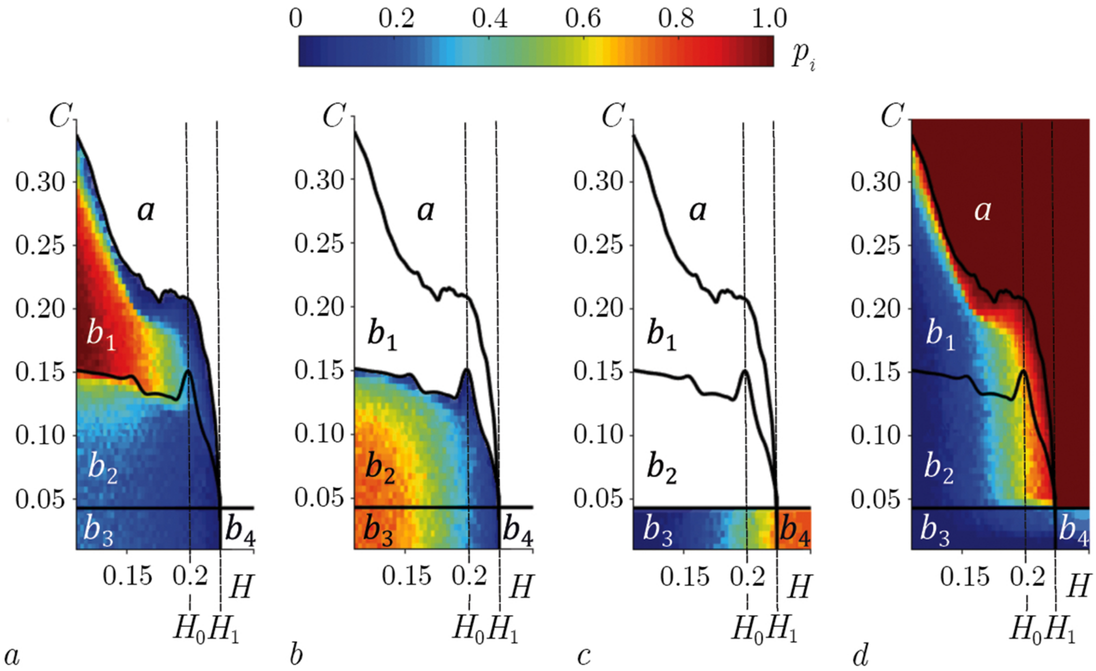

Fig. 4. Probability of realization of a — regime *1*, b — regime *2*, c — regime *3*, d — the synchronous regime depending on parameters H and C. Parameter value $H_0 \approx 0.2$ (color online)

When expanding existing power grids, by combining severalsubgrids, it is possible to choose the parameters of transmission lines that should connect the subgrids. In this case, the parameters of the lines of individual subgrids, as a rule, remain unchanged. That is, when attaching a motif to a hub, you can vary the parameter C, for example, multiplying or decreasing it by changing the number of transmission lines between the consumer c_1 and the generator g_4, as described in [23]. Then, depending on the value of the parameter H, we can distinguish three main scenarios of the behavior of the interconnected grid when changing the parameter C (see Fig. 4).

1. If $H_{\text{start}} < H < H_0$, then when the parameter C is increased from zero, the point (H, C) first falls into the region b_3, namely in the part where it is most likely to establish dangerous modes for the hub cluster *1* and *2*. Next, the point (H, C) falls into the regions of b_2 and b_1, where only dangerous modes for the hub cluster are most likely to be implemented. However, with a further increase in the parameter C, the point (H, C) falls into theregion of a, and synchronous mode is established throughout the grid.
2. If $H_0 < H < H_1$, then when the parameter C is increased from zero, the point (H, C) first falls into the region b_3, namely in that part of it where it is most likely to establish a relatively safe mode for the hub cluster *3*. Next, the point (H, C) falls into the regions b_2 and b_1, but unlike the previous case, with H selected in these regions, the most likely mode is synchronous. With a further increase in the parameter C, the point (H, C) falls into the safe region a.

3. If $H > H_1$, then when the parameter C increases from zero, the point (H, C) first falls into theregion b_4, where the relatively safe mode for the hub cluster *3* is most likely set, and then into the safe region a.

These scenarios show that for different values of the parameter H, the connection of a motif to a hub cluster via a transmission line with a capacity of C can lead to the establishment of a synchronous mode in the entire grid or modes that can be relatively safe (mode *3*) or emergency (modes *1*, *2*) for the hub cluster and even for the entire grid as a whole (*2* mode).

Conclusion

The paper considers a model of a power grid formed as a result of joining the grid in the form of a hub cluster of a motif of two generators and one consumer. Such a model is a typical example of the expansion of a highly centralized power grid by introducing additional sources of electricity into it, for example, alternative ones.

For a grid of $N = 10$ elements, a partition of the parameter space into regions corresponding to different modes of grid operation is constructed. In particular, the region of global stability of the synchronous mode of the grid is obtained. The conditions for the safe connection of the motif to the hub cluster are obtained. The classification of grid operation modes, from the point of view of their safety for subgrids, that is, the hub cluster and the motif, is carried out. The main characteristics of the modes are established, such as the probability of their realization and the magnitude of oscillations of variables in the quasi-synchronous mode of interaction of individual elements of the grid.

Based on the results obtained, three main scenarios of the behavior of the power grid were determined when the transmission line capacity connecting the subgrids changes.

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