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Estimation of impulse action parameters using a network of neuronlike oscillators

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Abstract. *Aim* of the study is to develop a method for estimating the parameters of an external periodic impulse action using a spiking network of neuronlike oscillators. *Methods.* The spiking activity of a network consisting of coupled nonidentical neuronlike FitzHugh–Nagumo oscillators was studied, depending on the parameters of the periodic impulse action. To estimate the amplitude of the external impulse signal, we detuned the FitzHugh–Nagumo oscillators, which were in a stable state of equilibrium in the absence of an external action, by the threshold parameter responsible for the excitation of the oscillator. To estimate the frequency of excitatory pulses, we detuned the FitzHugh–Nagumo oscillators by the parameter characterizing the ratio of time scales, the value of which determines the natural frequency of oscillators. We also changed the duration of external pulses. *Results.* It is shown that the number of spikes generated by a network of nonidentical FitzHugh–Nagumo oscillators has a monotonic dependence on the amplitude of the external pulse signal and a nonmonotonic dependence on the frequency of the pulse signal. The number of spikes generated by the network remains constant over a wide range of external pulse durations. A method for estimating the amplitude and frequency of impulse action is proposed. The method efficiency is demonstrated in numerical simulations and in a radio physical experiment. *Conclusion.* The proposed method allows one to estimate the amplitude of an external pulse signal, knowing its frequency, and estimate the frequency of this signal, knowing its amplitude. The method can be used in robotics when solving the problems of information processing related to the motion control of mobile robots.

Keywords: neuronlike oscillators, FitzHugh–Nagumo model, spiking neural network, periodic impulse action, radio physical experiment.

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Introduction

Artificial neural networks are built on the principle of the organization and functioning of networks of brain neurons and consist of interacting artificial neurons. Artificial neural networks are widely used in a variety of scientific disciplines to solve problems of identification, classification, forecasting and adaptive management. Initially, the formal neurons [1–6] were used as node elements of artificial neural networks, which were threshold elements that transform the input–output [7]. However, such formal neurons are too simple and do not have their own dynamics, and the networks consisting of them are very far from the neural networks of the brain.

To simulate the processes of brain activity, more adequate models of neurons in the form of nonlinear dynamic systems are required [8]. The most well-known dynamic models of neural activity are discussed in detail in the review [9]. These are Hodgkin models–Huxley, Morris–Lekar, Hindmarsh–Rose and Fitzhugh–Nagumo, described by ordinary differential equations, and the models of Izhikevich, Rulkov and Kurbazh–Nekorkin, described by point maps. The parameters of such model neurons have a physiological meaning. These neurons themselves are capable of demonstrating the complex dynamics inherent in real neurons, including spikes (impulses) that occur when the membrane potential of a neuron exceeds a certain threshold level.

Neural networks constructed from such neural-like oscillators are called spike or pulse neural networks. They allow you to effectively simulate the processes of processing and memorizing information by the brain [10]. Spike neural networks are successfully used in practice for automatic recognition of sound and visual information [11, 12], classification of characteristic patterns in biomedical signals [13–16] and for solving information processing problems related to robot motion control [17–20]. Spike neural networks usually require fewer neurons to solve tasks compared to other artificial neural networks.

External stimulation of neurons can lead to changes in the amplitude and frequency of spikes generated by them [21, 22]. The external influence applied to the neural network can affect the processing of information in the network, since spikes play an important role in the transmission of information between neurons [23–25]. Thus, the task of assessing the parameters of external influence is of great interest, for example, in robotics and neurophysiology. Note that spike neural networks were previously used for threshold classification of external influences applied to the neurons of the [26] network, and for estimating the amplitude of the external harmonic signal [27]. There are also known methods for restoring the parameters of external influence based on the reconstruction of the model equations of the oscillators [28–31].

In this paper, we consider for the first time the problem of estimating the amplitude and frequency of the pulse action applied to the spike neural network in both numerical and radiophysical experiments. The influence of the shape and duration of pulses on the generation of spike activity is also investigated.

1. Dynamics of the Fitzhugh–Nagumo neuropodic oscillator under external periodic impulse action

As the nodal element of the spike neural network, we choose a neuron-like oscillator described by the simplified differential equations Fitzhugh–Nagumo [32, 33]. The dynamics of a neuron-like oscillator under external impulse action is described by the following model equations:

$$\begin{aligned} \varepsilon \dot{u}(t) &= u(t) - \frac{u^3(t)}{3} - v(t) + y(t), \\ \dot{v}(t) &= u(t) + a \end{aligned} \tag{1}$$

In this equation, $u(t)$ describes the dynamics of the membrane potential of the neuron; $v(t)$ is responsible for restoring the resting potential of the membrane; ε — the parameter of the ratio of time scales, which is usually a small value; a — threshold parameter; $y(t)$ — external impact, which is rectangular pulses. The Fitzhugh–Nagumo equations are a reference model of excitable neuron dynamics. Unlike the Fitzhugh–Nagumo equation notation given in the [9] review, the second equation in (1) lacks the $-bv$ term, which greatly simplifies the analysis of the [34] system and its implementation in a radiophysical experiment.

In the absence of external influence, the oscillator (1) at $a > 1$ is in a stable state of equilibrium. In this case, the action of an external signal can cause the system to generate spikes (excitation pulses), so this state is called excitable. At $a < 1$ and $y(t) = 0$, the oscillator (1) demonstrates periodic self-oscillations that occur as a result of the Andronov–Hopf bifurcation at $a = 1$ [32]. We will consider only the cases $a > 1$ corresponding to the excitable state of the Fitzhugh–Nagumo oscillator, in which the generation of spikes is absent without external influence.

The dynamics of the Fitzhugh–Nagumo oscillator under external harmonic influence was studied, including experimentally, in the works [22, 27]. In this paper, we will consider in detail the case of an external periodic pulse action. Such an effect is used, for example, with electrical stimulation of brain regions to control the level of synchronization of neurons in the treatment of certain brain pathologies [35, 36].

The type of periodic pulse signal $y(t)$ supplied to the Fitzhugh–Nagumo oscillators is shown in Fig. 1. The signal $y(t)$ is a rectangular pulse with an amplitude of B, a period of T and a duration of M. For convenience, we use in the article to describe the pulses their frequency $f = 1/T$ and the relative duration $D = M/T$. The case when the pulse amplitude varies from 0 to 1 is mainly considered. Such a unipolar effect simulates a situation in which, after passing a pulse with a duration of M, the neurooscillator is in autonomous mode for a time of $T - M$ (Fig. 1, a). However, this kind of external influence on real neurons is not always possible. For example, in the treatment of patients with Parkinson's disease using pulsed stimulation of brain regions, bipolar electrical stimuli [36] are mainly used, consisting of two rectangular pulses with different polarity (Fig. 1, b). This avoids the accumulation of charge in living brain tissues due to the equality of the total current of external stimuli to zero. The optimal type of external pulses for desynchronization of network oscillators is investigated in [37].

In Fig. 2, a different colors show how many spikes N during $t = 100$ demonstrates the neuropodic Fitzhugh–Nagumo oscillator (1) depending on the amplitude and frequency of the external pulse action $yappliedtoit(t)$. The figure is constructed when the frequency f and the

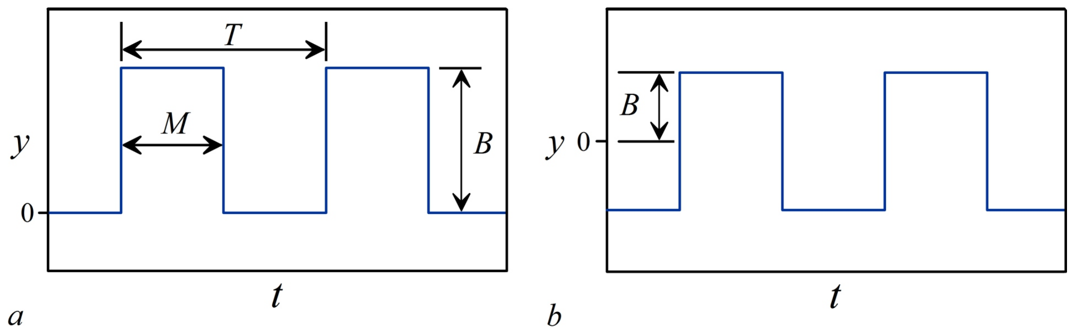

Fig. 1. Вид импульсного воздействия $y(t)$ при $M = T/2$ ($D = 0.5$). a — Однополярный импульсный сигнал. b — Двухполярный импульсный сигнал

Fig. 1. Shape of impulse action $y(t)$ at $M = T/2$ ($D = 0.5$). a — Unipolar pulse signal. b — Bipolar pulse signal

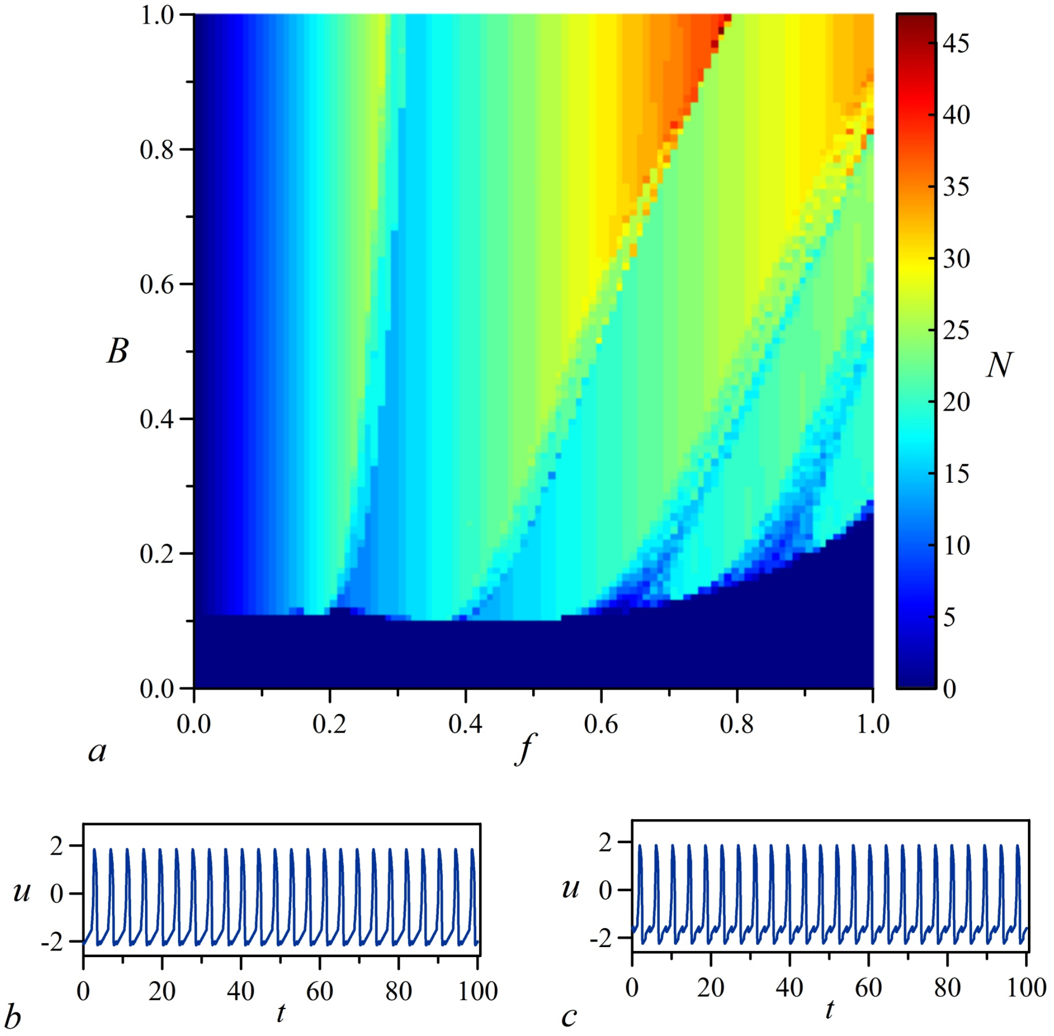

Fig. 2. *a* — Количество спайков N, генерируемое осциллятором (1) за время $t = 100$ при $a = 1.1$ и $\varepsilon = 0.1$, в зависимости от частоты f и амплитуды B внешнего импульсного воздействия $y(t)$ при $D = 0.5$ (цвет online). *b* — Временная реализация $u(t)$ при $B = 0.5$ и $f = 0.24$. *c* — Временная реализация $u(t)$ при $B = 0.5$ и $f = 0.48$

Fig. 2. *a* — The number of spikes N generated by the oscillator (1) over time $t = 100$ at $a = 1.1$ and $\varepsilon = 0.1$, depending on the frequency f and amplitude B of external impulse action $y(t)$ at $D = 0.5$ (color online). *b* — Time series of $u(t)$ at $B = 0.5$ and $f = 0.24$. *c* — Time series of $u(t)$ at $B = 0.5$ and $f = 0.48$

amplitude B of the pulse signal change from 0 to 1 with a step of 0.01 for the case $D = 0.5$ and oscillator parameters: $a = 1.1$ and $\varepsilon = 0.1$. The initial conditions are the same for each point on the parameter plane (f, B).

As can be seen from Fig. 2, *a*, for small values of B, the oscillator (1) does not generate spikes and, accordingly, the number of spikes $N = 0$. That is, at small B, the external stimulus is too weak to excite a neuron-like oscillator. When the amplitude of B exceeds a certain threshold value of B_p, which depends on the frequency of f exposure, the Fitzhugh–Nagumo oscillator begins to generate spikes. The number of spikes increases with an increase in B and at a fixed t reaches the maximum value of $N_{\max}$, which also depends on the value of f. For example, at the pulse frequency $f = 0.24$, close to the natural frequency f_s of periodic self-oscillations demonstrated by the oscillator (1) at $\varepsilon = 0.1$, $a < 1$ and $y(t) = 0$, the value of $N_{\max} = 24$ at $t = 100$. This means that each impact pulse causes the generation of a spike. Temporary implementation of the dynamic variable $u(t)$ at $B = 0.5$ and $f = 0.24$ are shown in Fig. 2, *b*. In this figure, the frequency of the spikes coincides with the frequency of the external pulse signal, and $N = 24$.

At $B > B_p$ and small values of f, each pulse acting on the oscillator causes the generation of a spike. For example, at $f = 0.01$ during the observation of $t = 100$, one pulse comes to the oscillator and causes one spike, at $f = 0.02$ during the time of $t = 100$, two pulses act on the oscillator, causing two spikes, and so on. As a result, N depends linearly on f in the domain of $f \leqslant f_s$.

Real neurons, as well as their models in the form of nonlinear dynamic systems, have the property of refractoriness. It consists in the fact that after the spike generation, the membrane potential of the neuron remains a small value for some time, and during this period of refractoriness, the neuron does not respond to external influences [9]. When the frequency of the external influence becomes sufficiently large, some impulses affect the neuropodic oscillator at the moment when it is non-excitable due to refractoriness, and therefore do not cause the generation of a spike. For this reason, when $f > f_s$, the dependence $N(f)$ ceases to be monotonous. For example, in Fig. 2, *c* shows a temporary implementation of the dynamic variable $u(t)$ at $B = 0.5$ and $f = 0.48$. In this figure, the frequency of spikes is two times less than the frequency of the external pulse signal and $N = 24$, as in Fig. 2, *b*. With sufficiently large f, the frequency of spikes can be three or more times less than the frequency of the external pulse signal.

Thus, the spike activity of the Fitzhugh–Nagumo neuron-like oscillator depends both on the parameters of the oscillator itself and on the parameters of external impulse action. In the following sections, we will consider networks consisting of interconnected non-identical Fitzhugh–Nagumo(1) oscillators and show the possibility of using them to estimate the parameters of an external periodic pulse action.

2. Using a network of Fitzhugh–Nagumo oscillators to estimate the parameters of pulse action in numerical simulation

Consider a spike neural network consisting of interconnected neural-like Fitzhugh–Nagumo oscillators, the dynamics of which is described by model equations of the following form:

$$\begin{aligned}\varepsilon_i \dot{u}_i(t) &= u_i(t) - \frac{u_i^3(t)}{3} - v_i(t) + \sum_{j=1 (j \neq i)}^{L} k_{i,j}(u_j(t) - u_i(t)) + y(t), \\ \dot{v}_i(t) &= u_i(t) + a_i,\end{aligned} \tag{2}$$

where $i = 1, ..., L$ is the number of the oscillator; L is the number of oscillators; $k_{i,j}$ is the coupling coefficient from the jth element to the ith. In general, all oscillators of the network are non-identical, but they are under the influence of the same external impulse action $y(t)$.

In numerical simulation, we investigate a network consisting of $L = 50$ oscillators (2). We choose the architecture of the connections in such a way that each of the oscillators has ten other oscillators, the numbers of which are randomly selected. This means that in the equation (2) out of 49 coupling coefficients $k_{i,j}$ for each oscillator, 39 coefficients are zero. The values of all nonzero coefficients of connections in the network are chosen the same: $k_{i,j} = 0.01$. At such small values of $k_{i,j}$, spikes in the network oscillators do not occur without external influence. For all the examples discussed in this section, the architecture of the connections in the network and the strength of the connections are the same.

The spike activity of the neural network under study depends in a complex way on the parameters of the external periodic pulse action. Such a network can generate the same number of spikes with different parameters of the pulse signal. Therefore, simultaneous evaluation of all parameters of the external stimulus $y(t)$ by the response of the neural network is generally impossible. We will consider a simpler situation in which it is required to estimate the amplitude of the pulse signal $y(t)$, knowing its frequency, and a situation where it is required to estimate the frequency of the signal $y(t)$, knowing its amplitude. Separately, we will consider the effect of pulse duration on the generation of spike activity.

2.1. Estimation of the amplitude of the pulse action. As shown above, in section 1, even by the response to the external influence of one neuron-like oscillator in an excitable state, it is possible to estimate the amplitude of the stimulus very roughly. The use of a spike network of non-identical oscillators makes it possible to increase the accuracy of estimating the amplitude of an external periodic pulse effect, and the greater the number of L oscillators in the network, the more accurate the estimate of B. To estimate the value of B, the network oscillators should be detuned by the parameter a_i, on the value of which the threshold value B_p depends, as well as the number of spikes generated by each i oscillator.

In the studied network, all Fitzhugh–Nagumo oscillators were non-identical and differed in the value of the parameter a_i, which took values from $a_1 = 1.1$ to $a_{50} = 1.2715$ in increments of 0.0035. At such values, there is no generation of spikes in the absence of external influence. We changed the amplitude B of the pulse signal from 0 to 1 in increments of 0.01 and for each value B we calculated the total number of spikes generated by all 50 oscillators during the time $t = 100$. In Fig. 3 dependencies are constructed for three different values of f at $D = 0.5$ and $\varepsilon_i = \varepsilon = 0.1$. Such graphs allow us to estimate the unknown amplitude B of an external pulse action at a known frequency of action f.

By counting the number of spikes N observed in the network for the selected time, you can determine B. For example, if $N = 800$ with $f = 0.24$, then $B = 0.42$. The more smoothly N grows, the more accurate the estimate of B. Of the three curves shown in Fig. 3, the slowest growth demonstrates the dependence of $N(B)$ at the pulse frequency $f = 0.24$. This curve reaches a plateau only at large B, and the number of spikes on this plateau is the maximum possible ($N = 1200$) for a network of 50 oscillators at $f = 0.24$. As noted in section 1, the frequency f_s of self-oscillators (1) at $\varepsilon = 0.1$ and $a < 1$ is approximately equal to 0.24. Thus, if it is possible to

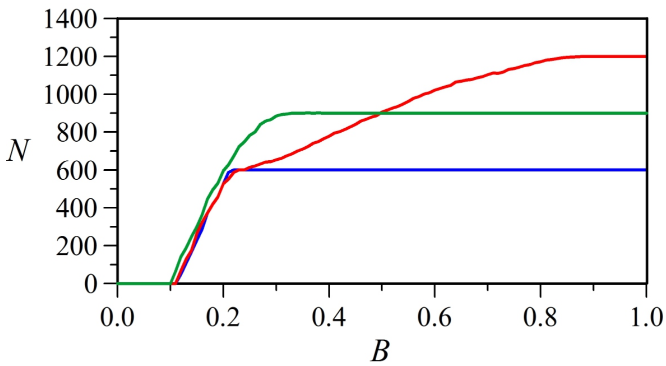

Fig. 3. Зависимости количества спайков N, генерируемых сетью (2) за время $t = 100$, от амплитуды B внешнего импульсного воздействия $y(t)$ при $D = 0.5$, $L = 50$, $\varepsilon_i = \varepsilon = 0.1$, $a_i \in [1.1; 1.2715]$ для $f = 0.12$ (синяя кривая), $f = 0.24$ (красная кривая) и $f = 0.36$ (зеленая кривая) (цвет online)

Fig. 3. Dependences of the number of spikes N generated by the network (2) over time $t = 100$ on the amplitude B of external impulse action $y(t)$ at $D = 0.5$, $L = 50$, $\varepsilon_i = \varepsilon = 0.1$, and $a_i \in [1.1; 1.2715]$ for $f = 0.12$ (blue line), $f = 0.24$ (red line), and $f = 0.36$ (green line) (color online)

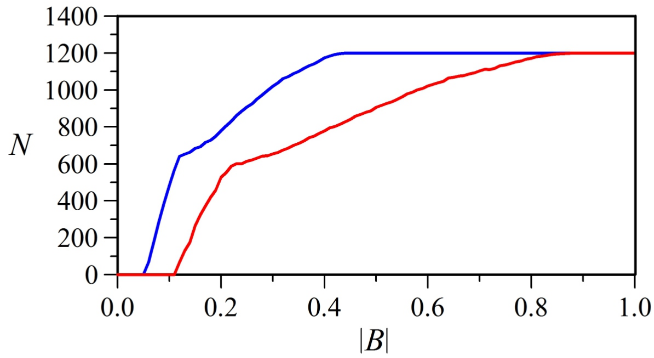

Fig. 4. Зависимости количества спайков N, генерируемых сетью (2) за время $t = 100$, от $|B|$ для случаев однополярного (красная кривая) и двухполярного (синяя кривая) импульсного воздействия с частотой $f = 0.24$. Параметры сети такие же, как для рис. 3 (цвет online)

Fig. 4. Dependences of the number of spikes N generated by the network (2) over time $t = 100$ on $|B|$ for the cases of unipolar (red line) and bipolar (blue line) impulse action with the frequency $f = 0.24$. The network parameters are the same as for Fig. 3 (color online)

choose the frequency of exposure to the spike network, then $f \approx f_s$ should be used for a more accurate estimate of B. If the frequency of the signal $y(t)$ cannot be changed, but is known in advance, then for a more accurate estimate of B, it is necessary to select the parameters of the oscillators in such a way that their natural frequencies are also close to the frequency of exposure $f_s \approx f$.

We investigated the effectiveness of the method with different communication architectures in a spike neural network and with a different number of connections between oscillators. It is established that these characteristics have little effect on the accuracy of the estimation of the amplitude of the external pulse action. The values of the coupling coefficients $k_{i,j}$ between the oscillators have a more significant effect on the accuracy of the method. For large $k_{i,j}$, the accuracy of the method decreases, as the dependence of $N(B)$ becomes steeper, quickly moves from the minimum to the maximum value of N.

We compared the spike activity of a network of neural-like oscillators (2) for cases of pulse amplitude changes from 0 to 1 and from -1 to 1.

The obtained results are illustrated by Fig. **??**, on which $|B|$ is deposited along the abscissa axis. As expected, a pulse action with an amplitude of B, varying from -1 to 1, has a more noticeable effect on the spike activity of the network. With such a bipolar pulse action, the threshold value B_p, at which the network begins to generate spikes, turns out to be less than with a unipolar pulse action. In addition, with bipolar pulses, the number of spikes reaches a maximum value at lower values of $|B|$ than with unipolar pulses.

2.2. Evaluation of the pulse frequency. Let us now consider the problem of estimating the frequency f of an external pulse signal $y(t)$, assuming that we know the amplitude of this signal. To solve this problem, we will detuned the oscillators of the network (2) not by the parameter a_i, as in section 2.1, but by the parameter ε_i, the value of which determines the frequency f_s of periodic self-oscillations of the Fitzhugh–Nagumo oscillators (1) at $a < 1$ in the absence of external influence.

All Fitzhugh–Nagumo oscillators in the studied network of 50 elements were non-identical and differed in the value of the parameter ε_i, which took values from $\varepsilon_1 = 0.02$ to $\varepsilon_{50} = 0.51$ in increments of 0.01. In Fig. 5, *a* $N(f)$ dependencies are constructed for two different values of B at $D = 0.5$ and $a_i = a = 1.1$.

In contrast to the dependencies $N(B)$, shown in Fig. 3, dependencies $N(f)$ are not monotonic. For different values of f, the network can generate the same number of N spikes. We

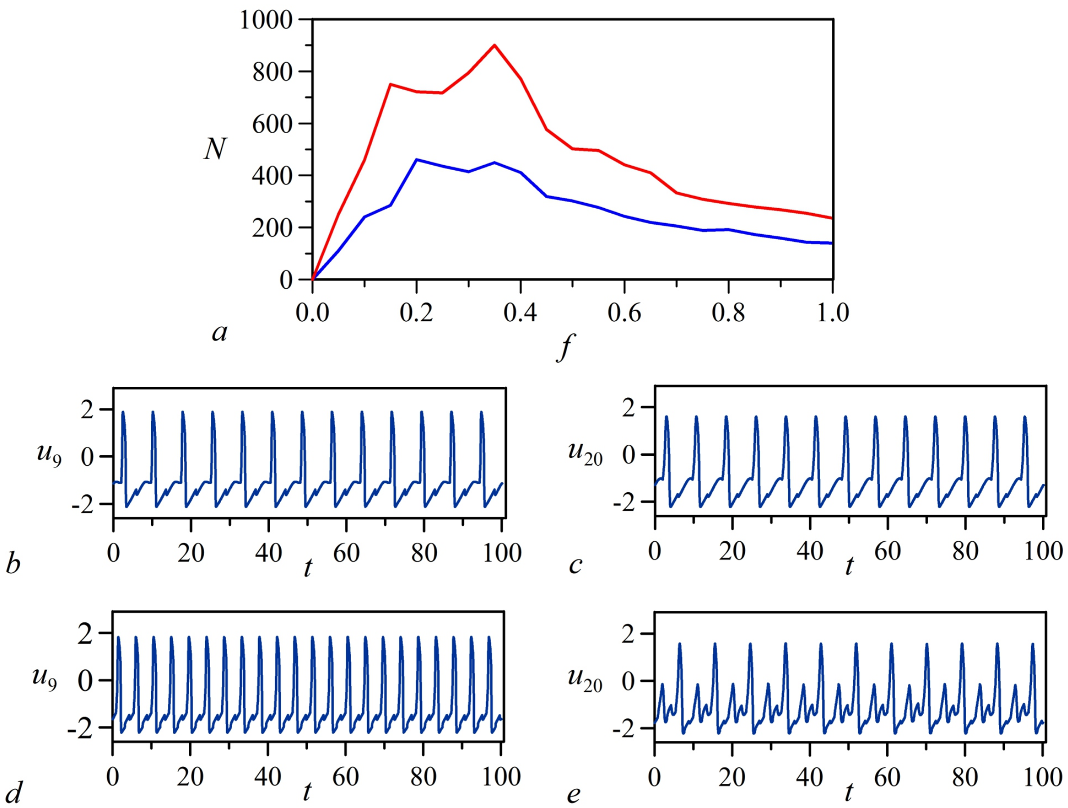

Fig. 5. a — Зависимости количества спайков N, генерируемых сетью (2) за время $t = 100$, от частоты f внешнего импульсного воздействия $y(t)$ при $D = 0.5$, $L = 50$, $a_i = a = 1.1$, $\varepsilon_i \in [0.02; 0.51]$ для $B = 0.2$ (синяя кривая) и $B = 0.4$ (красная кривая) (цвет online). b — Временная реализация $u_9(t)$ при $\varepsilon_9 = 0.1$, $B = 0.4$, $f = 0.13$. c — Временная реализация $u_{20}(t)$ при $\varepsilon_{20} = 0.21$, $B = 0.4$, $f = 0.13$. d — Временная реализация $u_9(t)$ при $\varepsilon_9 = 0.1$, $B = 0.4$, $f = 0.44$. e — Временная реализация $u_{20}(t)$ при $\varepsilon_{20} = 0.21$, $B = 0.4$, $f = 0.44$

Fig. 5. a — Dependences of the number of spikes N, generated by the network (2) over time $t = 100$, on the frequency f of external impulse action $y(t)$ at $D = 0.5$, $L = 50$, $a_i = a = 1.1$, and $\varepsilon_i \in [0.02; 0.51]$ for $B = 0.2$ (blue line) and $B = 0.4$ (red line) (color online). b — Time series of $u_9(t)$ at $\varepsilon_9 = 0.1$, $B = 0.4$, and $f = 0.13$. c — Time series of $u_{20}(t)$ at $\varepsilon_{20} = 0.21$, $B = 0.4$, and $f = 0.13$. d — Time series of $u_9(t)$ at $\varepsilon_9 = 0.1$, $B = 0.4$, and $f = 0.44$. e — Time series of $u_{20}(t)$ at $\varepsilon_{20} = 0.21$, $B = 0.4$, and $f = 0.44$

observed a similar pattern for a single Fitzhugh–Nagumo oscillator (see Fig. 2). Therefore, to estimate f, it may not be sufficient to simply count the number of spikes in the network. For example, when $B = 0.4$ the number of spikes $N = 635$ can be observed both at $f = 0.13$ and at $f = 0.44$ (see Fig. 5, a). In order to determine the frequency of the pulse action, additional analysis is necessary.

We found that in the areas of dependence growth $N(f)$ all or almost all oscillators of the network (with the exception of oscillators with the smallest or largest values of ε_i) generate the same number of spikes at a fixed value of f. In areas where the dependency is $N(f)$ decreases, oscillators having a large detuning relative to each other by the parameter ε_i demonstrate a different number of spikes at a fixed value of f. So, to estimate the value of f by the number of spikes N in the case of ambiguity of the dependence $N(f)$ it is necessary to additionally compare the number of spikes N_i generated by two oscillators whose parameters ε_i differ significantly. With the selected range of the parameter ε_i, you can compare N_i, for example, for oscillators with $\varepsilon_9 = 0.1$ and $\varepsilon_{20} = 0.21$. If for $N = 635$ we have $N_9 = N_{20}$, then $f = 0.13$, and if $N_9 \neq N_{20}$, then $f = 0.44$.

In Fig. 5, b, c are the temporary implementations of the variables $u_9(t)$ and $u_{20}(t)$,

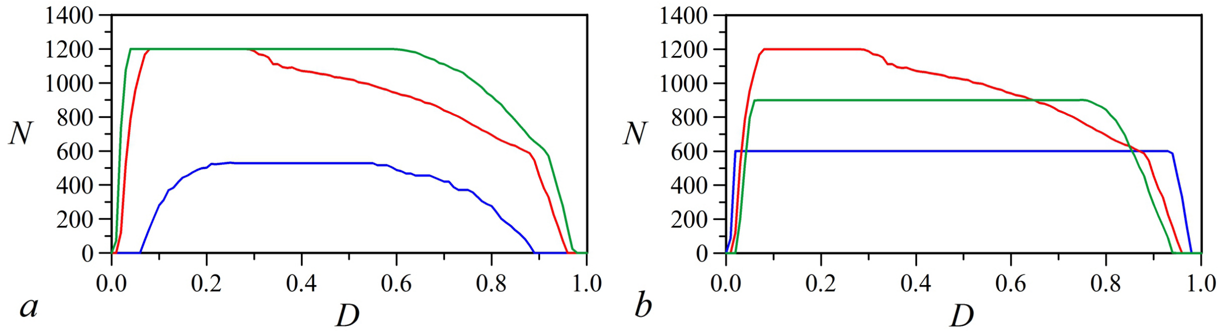

Fig. 6. Зависимости количества спайков N, генерируемых сетью (2) за время $t = 100$, от относительной длительности D импульсов сигнала $y(t)$ при $L = 50$, $\varepsilon_i = \varepsilon = 0.1$, $a_i \in [1.1; 1.2715]$. a — $f = 0.24$, $B = 0.2$ (синяя кривая), $B = 0.6$ (красная кривая), $B = 1$ (зеленая кривая). b — $B = 0.6$, $f = 0.12$ (синяя кривая), $f = 0.24$ (красная кривая), $f = 0.36$ (зеленая кривая) (цвет online)

Fig. 6. Dependences of the number of spikes N generated by the network (2) over time $t = 100$ on the relative duration D of impulse signal $y(t)$ at $L = 50$, $\varepsilon_i = \varepsilon = 0.1$, and $a_i \in [1.1; 1.2715]$. a — $f = 0.24$ and $B = 0.2$ (blue line), $B = 0.6$ (red line), and B=1 (green line). b — B=0.6 and f=0.12 (blue line), f=0.24 (red line), and f=0.36 (green line) (color online)

respectively, for $B = 0.4$ and $f = 0.13$. On both charts, the number of spikes is the same: $N_9 = N_{20}$ =13. In this case, the frequency f of the signal $y(t)$ corresponds to the ascending branch of the dependence $N(f)$ at $N = 635$. In Fig. 5, *d, e* are the temporary implementations of the variables $u_9(t)$ and $u_{20}(t)$, respectively, for $B = 0.4$ and $f = 0.44$. The number of spikes in Fig. 5, *d* and fig. 5, *e* differs: $N_9 = 22$ and $N_{20} = 11$. In this case, the frequency f of the signal $y(t)$ corresponds to the descending branch of the dependence $N(f)$ at $N = 635$.

2.3. The effect of pulse duration on the generation of spike activity. The spike activity of the Fitzhugh–Nagumo neural oscillator network (2) depends not only on the amplitude and frequency of the external pulse signal $y(t)$, but also on the pulse duration. We investigated the effect of the relative duration of D pulses on the number of spikes generated. The Fitzhugh–Nagumo oscillators, as in section 2.1, were detuned by the parameter a_i, which took values from $a_1 = 1.1$ to $a_{50} = 1.2715$ in increments of 0.0035. We changed the duration of the D pulses from 0 to 1 in 0.01 increments and for each D we calculated the total number of spikes generated by all 50 oscillators during the time $t = 100$. In Fig. 6, *a* $N(D)$ dependencies are constructed for three different values of B at $f = 0.24$ and $\varepsilon_i = \varepsilon = 0.1$.

Under the action of very short pulses, the oscillators, due to their inertia, do not have time to generate spikes, so $N = 0$ for small values of D. With an increase in D, a rapid growth of N is observed, and the dependencies of $N(D)$ reach a plateau, the value of N on which depends on the amplitude of the pulses. In a fairly wide range of values of D, the value of N remains the maximum, and then decreases with the growth of D. The case $D = 1$ corresponds to a constant external force acting on the oscillators. With this effect, there is no spike activity.

In Fig. 6, *b* $N(D)$ dependencies are constructed for three different values of f at $B = 0.6$. The form of these dependencies qualitatively coincides with the graphs of $N(D)$ in fig. 6, *a*. Graphs $N(D)$ in fig. 6, *b* agree well with the dependencies of $N(B)$ in fig. 3. From Fig. 6 it follows that, in general, it is not possible to estimate the duration of a periodic pulse effect based on the network response. At the same time, the number of spikes generated by the network remains constant over a wide range of D values.

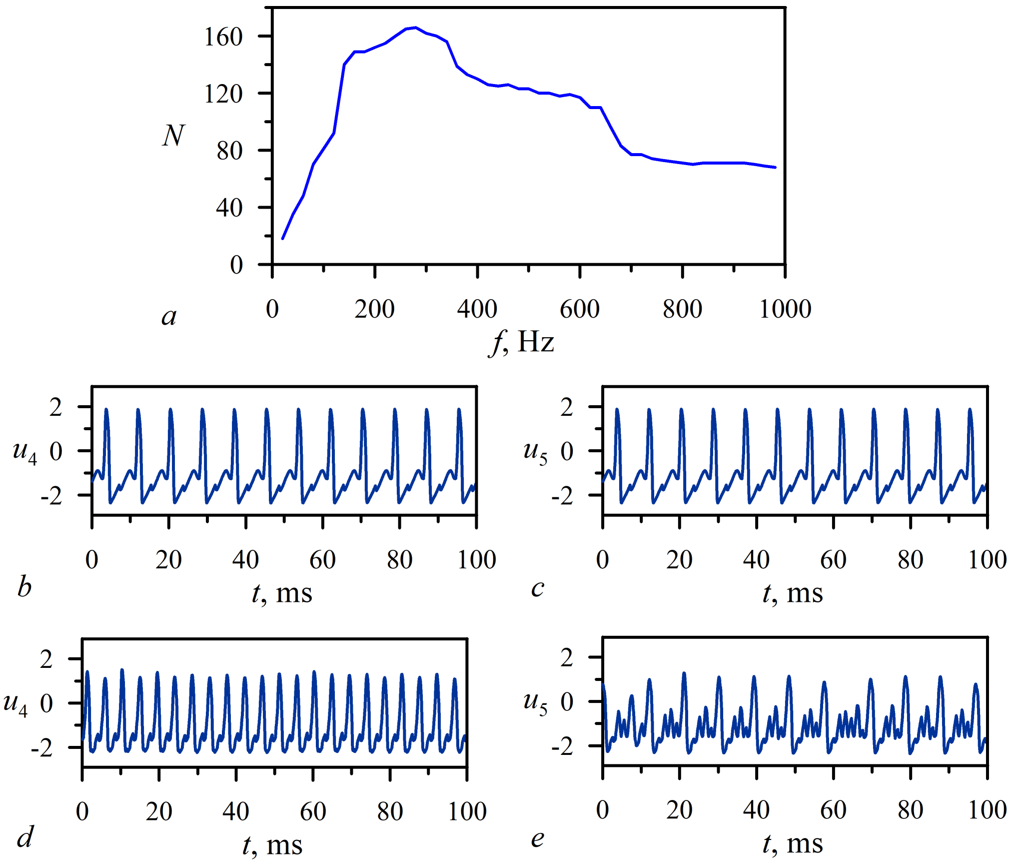

Fig. 7. a — Зависимость количества спайков N, генерируемых сетью из 10 радиотехнических генераторов за время $t = 100$ мс, от частоты f внешнего импульсного воздействия при $B = 0.6$ В, $D = 0.5$, $a_i = a = 1.1$, $\varepsilon_i \in [0.05; 0.56]$. b — Временная реализация $u_4(t)$ при $\varepsilon_4 = 0.18$ мс, $f = 120$ Гц. c — Временная реализация $u_5(t)$ при $\varepsilon_5 = 0.25$ мс, $f = 120$ Гц. d — Временная реализация $u_4(t)$ при $\varepsilon_4 = 0.18$ мс, $f = 660$ Гц. e — Временная реализация $u_5(t)$ при $\varepsilon_5 = 0.25$ мс, $f = 660$ Гц

Fig. 7. a — Dependence of the number of spikes N generated by a network of 10 radio technical generators over time $t = 100$ ms on the frequency f of external impulse action at $B = 0.6$ V, $D = 0.5$, $a_i = a = 1.1$, and $\varepsilon_i \in [0.05; 0.56]$. b — Time series of $u_4(t)$ at $\varepsilon_4 = 0.18$ ms and $f = 120$ Hz. c — Time series of $u_5(t)$ at $\varepsilon_5 = 0.25$ ms and $f = 120$ Hz. d — Time series of $u_4(t)$ at $\varepsilon_4 = 0.18$ ms and $f = 660$ Hz. e — Time series of $u_5(t)$ at $\varepsilon_5 = 0.25$ ms and $f = 660$ Hz

3. Estimation of parameters of external pulse action using a small network of Fitzhugh–Nagumo oscillators in a radiophysical experiment

We have built a radio engineering installation for experimental investigation of spike activity in a network consisting of connected non-identical neuron-like oscillators described by the Fitzhugh–Nagumo(2) equations. The schematic diagram of the Fitzhugh–Nagumo radio generators implemented by

by us is described in detail in [38]. In contrast to the numerical studies described above, a small network consisting of 10 coupled generators was investigated in the radiophysical experiment. To implement the connections between the generators, we used the approach proposed by us, which is based on a software method for generating signals responsible for the connection between the generators [39]. The essence of this approach is that the voltage signals from the output of each generator are fed to the analog inputs of a multichannel analog-to-digital converter and digitized for further processing. Then, with the help of a program on LabVIEW, the conversion of these signals is carried out, and the signals responsible for the communication of the generators are formed. A digitized pulse signal is added to each of the communication signals in accordance with the equation (2). The signals generated in this way are converted to analog form using a multi-channel digital-to-analog converter and fed to the input of each generator.

This approach allows you to set an arbitrary architecture and the type of connections between generators. The communication architecture was chosen in such a way that two other generators operated on each of the generators, the numbers of which were chosen randomly. In accordance with the model equation (2), a simple linear connection between generators is implemented in the installation, simulating an electrical synaptic connection between neurons. In analog modeling, such a connection corresponds to the connection of two generators via a resistor [40, 41].

First, we upset the Fitzhugh–Nagumo generators by the parameter a_i, which took values from $a_1 = 1.1$ In up to $a_{10} = 1.28$ In increments of 0.02 In. We changed the amplitude B of the pulse signal from 0 to 1 V in increments of 0.02 V and calculated the total number of spikes generated by all ten generators in 100 ms at a fixed value of B. In Fig. 8 the dependence $N(B)$ is constructed at $f = 240$ Hz, $D = 0.5$, $\varepsilon_i = \varepsilon = 0.1$ ms. The values of all non-zero communication coefficients in the network were chosen the same: $k_{i,j} = 0.01$. The constructed graph allows us to estimate the unknown amplitude B of the external pulse action by counting the number of spikes N generated in the network. For example, if $N = 156$, then $B = 0.16$ B. The results obtained in the radiophysical experiment are in good agreement with the above results of numerical studies of a network of neural-like oscillators (2).

Then we detuned the Fitzhugh–Nagumo generators by the parameter ε_i, which took values from $\varepsilon_1 = 0.05$ ms to $\varepsilon_{10} = 0.56$ ms. The change step of ε_i was uneven and was determined by the capacitance of the available capacitors. In the experiment, we changed the frequency of the pulse signal f from 20 Hz to 1000 Hz in increments of 20 Hz and for each value of f, we calculated the total number of spikes generated by all ten generators in 100 ms. In Fig. 7, *a* the dependency $N(f)$ is constructed for $B = 0.6$ B, $D = 0.5$, $a_i = a = 1.1$.

Just as in the numerical experiment, the dependence of $N(f)$ is non-monotonic, and for different values of f, the network can generate the same number of spikes. For example, the number of spikes $N = 94$ can be observed in the network at $f = 120$ Hz and at $f = 660$ Hz. To estimate the value of f, you first need to calculate the total number of spikes N in the network, and then compare the number of spikes N_i generated by two generators having different values ε_i. We compared the 4th and 5th generators, for which $\varepsilon_4 = 0.18$ ms and $\varepsilon_5 = 0.25$ ms, respectively.

In Fig. **??**, *b, c* are the temporary implementations of the variables $u_4(t)$ and $u_5(t)$,

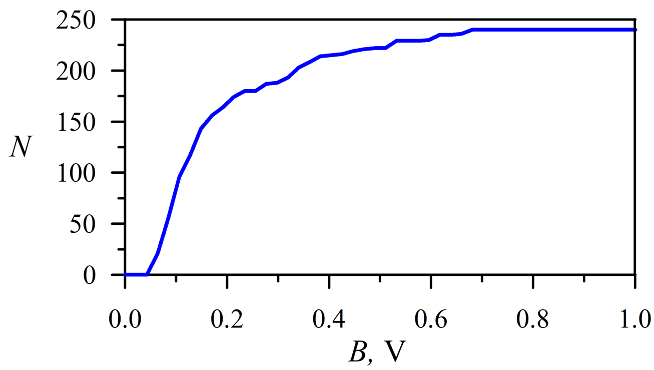

Fig. 8. Зависимость количества спайков N, генерируемых сетью из 10 радиотехнических генераторов за время t=100 мс, от амплитуды B внешнего импульсного воздействия при f=240 Гц, D=0.5, ε_i=ε=0.1 мс, $a_i \in [1.1; 1.28]$

Fig. 8. Dependence of the number of spikes N generated by a network of 10 radio technical generators over time $t = 100$ ms on the amplitude B of external impulse action at $f = 240$ Hz, $D = 0.5$, $\varepsilon_i = \varepsilon = 0.1$ ms, and $a_i \in [1.1; 1.28]$

respectively, at $f = 120$ Hz. On both charts, the number of spikes is the same: $N_4 = N_5 = 12$. In this case, the frequency f of the pulse signal corresponds to the ascending branch of the dependence $N(f)$ at $N = 94$. In Fig. **??**, *d, e* are the temporary implementations of the variables $u_4(t)$ and $u_5(t)$, respectively, at f=660 Hz. The number of spikes in Fig. **??**, *d* and fig. **??**, *e* is different: N_4=22 and N_5=11. In this case, the frequency f of the pulse signal corresponds to the descending branch of the dependence $N(f)$ at N=94.

Conclusion

We have proposed a method for estimating the amplitude and frequency of the pulse action applied to the spike neural network. Non-identical neuropodic Fitzhugh–Nagumo oscillators were used as network elements. It is shown that the spike activity of the studied neural network depends in a complex way on the parameters of the external periodic pulse action. Since such a network can generate the same number of spikes with different parameters of the pulse signal, simultaneous evaluation of all parameters of an external stimulus by the response of a neural network is generally impossible. Therefore, we solved the problems of estimating the amplitude and frequency of the pulse action separately.

To estimate the amplitude of the external pulse action, we detuned the oscillators of the network according to the parameter a, which is responsible for the excitation of the oscillator, and to estimate the frequency of the exciting pulses, we detuned the oscillators according to the parameter ε, on the value of which the natural oscillation frequency of the oscillator depends. The proposed method is implemented in both numerical and radiophysical experiments, showing good consistency of the results obtained. The influence of the shape and duration of external pulses on the generation of spike activity in the network has also been investigated.

The results obtained can be in demand in robotics when solving information processing problems related to controlling the movement of robots [19,42]. For example, spike neural networks can be used to evaluate the characteristics of external signals recorded by sensors of a mobile robot. Then, based on the values obtained, with the help of other control systems, the robot's movement mode can be changed if necessary.

References

1. Haykin S. Neural Networks and Learning Machines. Upper Saddle River, New Jersey: Prentice Hall; 2009. 906 p.
2. Ripley BD. Pattern Recognition and Neural Networks. Cambridge: Cambridge University Press; 1996. 403 p. DOI: 10.1017/CBO9780511812651.
3. Egmont-Petersen M, de Ridder D, Handels H. Image processing with neural networks — a review. Pattern Recognition. 2002;35(10):2279–2301. DOI: 10.1016/S0031-3203(01)00178-9.
4. Dwarakish GS, Rakshith S, Natesan U. Review on applications of neural network in coastal engineering. Artificial Intelligent Systems and Machine Learning. 2013;5(7):324–331.
5. Frolov N, Maksimenko V, Lüttjohann A, Koronovskii A, Hramov A. Feed-forward artificial neural network provides data-driven inference of functional connectivity. Chaos. 2019;29(9):091101. DOI: 10.1063/1.5117263.
6. Hramov AE, Maksimenko VA, Pisarchik AN. Physical principles of brain–computer interfaces and their applications for rehabilitation, robotics and control of human brain states. Phys. Rep. 2021;918:1–133. DOI: 10.1016/j.physrep.2021.03.002.
7. McCulloch WS, Pitts W. A logical calculus of the ideas immanent in nervous activity. Bulletin of Mathematical Biophysics. 1943;5(4):115–133. DOI: 10.1007/BF02478259.
8. Rabinovich MI, Varona P, Selverston AI, Abarbanel HDI. Dynamical principles in neuroscience. Rev. Mod. Phys. 2006;78(4):1213–1265. DOI: 10.1103/RevModPhys.78.1213.
9. Dmitrichev AS, Kasatkin DV, Klinshov VV, Kirillov SY, Maslennikov OV, Shchapin DS,

Nekorkin VI. Nonlinear dynamical models of neurons: Review. Izvestiya VUZ. Applied Nonlinear Dynamics. 2018;26(4):5–58 (in Russian). DOI: 10.18500/0869-6632-2018-26-4-5-58.
10. Quiroga RQ, Panzeri S. Principles of Neural Coding. Boca Raton: CRC Press; 2013. 663 p. DOI: 10.1201/b14756.
11. Yu D, Deng L. Automatic Speech Recognition: A Deep Learning Approach. London: Springer; 2015. 321 p. DOI: 10.1007/978-1-4471-5779-3.
12. Hossain MS, Muhammad G. Emotion recognition using deep learning approach from audio-visual emotional big data. Information Fusion. 2019;49:69–78. DOI: 10.1016/j.inffus.2018.09.008.
13. Kasabov NK. Evolving Connectionist Systems: The Knowledge Engineering Approach. London: Springer; 2007. 451 p. DOI: 10.1007/978-1-84628-347-5.
14. Lobov S, Mironov V, Kastalskiy I, Kazantsev V. A spiking neural network in sEMG feature extraction. Sensors. 2015;15(11):27894–27904. DOI: 10.3390/s151127894.
15. Virgilio CD, Sossa JH, Antelis JM, Falcón LE. Spiking Neural Networks applied to the classification of motor tasks in EEG signals. Neural Netw. 2020;122:130–143. DOI: 10.1016/j.neunet.2019.09.037.
16. Lobov SA, Chernyshov AV, Krilova NP, Shamshin MO, Kazantsev VB. Competitive learning in a spiking neural network: Towards an intelligent pattern classifier. Sensors. 2020;20(2):500. DOI: 10.3390/s20020500.
17. Wang X, Hou ZG, Lv F, Tan M, Wang Y. Mobile robots' modular navigation controller using spiking neural networks. Neurocomputing. 2014;134:230–238. DOI: 10.1016/j.neucom.2013.07.055.
18. Chou TS, Bucci LD, Krichmar JL. Learning touch preferences with a tactile robot using dopamine modulated STDP in a model of insular cortex. Front. Neurorobot. 2015;9:6. DOI: 10.3389/fnbot.2015.00006.
19. Bing Z, Meschede C, Röhrbein F, Huang K, Knoll AC. A survey of robotics control based on learning-inspired spiking neural networks. Front. Neurorobot. 2018;12:35. DOI: 10.3389/fnbot.2018.00035.
20. Lobov SA, Mikhaylov AN, Shamshin M, Makarov VA, Kazantsev VB. Spatial properties of STDP in a self-learning spiking neural network enable controlling a mobile robot. Front. Neurosci. 2020;14:88. DOI: 10.3389/fnins.2020.00088.
21. Croisier H, Dauby PC. Continuation and bifurcation analysis of a periodically forced excitable system. J. Theor. Biol. 2007;246(3):430–448. DOI: 10.1016/j.jtbi.2007.01.017.
22. Feoktistov AV, Anishchenko VS. Dynamics of the FitzHugh–Nagumo system under external periodic force. Izvestiya VUZ. Applied Nonlinear Dynamics. 2011;19(5):35–44 (in Russian). DOI: 10.18500/0869-6632-2011-19-5-35-44.
23. Novikov N, Gutkin B. Role of synaptic nonlinearity in persistent firing rate shifts caused by external periodic forcing. Phys. Rev. E. 2020;101(5):052408. DOI: 10.1103/PhysRevE.101.052408.
24. Eidum DM, Henriquez CS. Modeling the effects of sinusoidal stimulation and synaptic plasticity on linked neural oscillators. Chaos. 2020;30(3):033105. DOI: 10.1063/1.5126104.
25. Rooy M, Novikov NA, Zakharov DG, Gutkin BS. Interaction between PFC neural networks ultraslow fluctuations and brain oscillations. Izvestiya VUZ. Applied Nonlinear Dynamics. 2020;28(1):90–97. DOI: 10.18500/0869-6632-2020-28-1-90-97.
26. Andreev AV, Ivanchenko MV, Pisarchik AN, Hramov AE. Stimulus classification using chimera-like states in a spiking neural network. Chaos, Solitons & Fractals. 2020;139:110061. DOI: 10.1016/j.chaos.2020.110061.
27. Ponomarenko VI, Kulminskiy DD, Andreev AV, Prokhorov MD. Assessment of an external periodic force amplitude using a small spike neuron network in a radiophysical experiment. Tech. Phys. Lett. 2021;47(2):162–165. DOI: 10.1134/S1063785021020255.
28. Bezruchko BP, Smirnov DA. Constructing nonautonomous differential equations from experimental time series. Phys. Rev. E. 2001;63(1):016207. DOI: 10.1103/PhysRevE.63.016207.

29. Smirnov DA, Sysoev IV, Seleznev EP, Bezruchko BP. Reconstructing nonautonomous system models with discrete spectrum of external action. Tech. Phys. Lett. 2003;29(10):824–827. DOI: 10.1134/1.1623857.
30. Yakhno YV, Molkov JI, Mukhin DN, Loskutov EM, Feigin AM. Reconstruction of an evolution operator as a technique of analysis of epileptiform electric brain activity. Izvestiya VUZ. Applied Nonlinear Dynamics. 2011;19(6):156–172 (in Russian). DOI: 10.18500/0869-6632-2011-19-6-156-172.
31. Sysoeva MV, Ponomarenko VI, Prokhorov MD, Sysoev IV. Reconstruction of time-delay systems under external periodic driving. Russian Journal of Nonlinear Dynamics. 2013;9(4):613–625
(in Russian). DOI: 10.20537/nd1304001.
32. Dahlem MA, Hiller G, Panchuk A, Schöll E. Dynamics of delay-coupled excitable neural systems. Int. J. Bifurc. Chaos. 2009;19(2):745–753. DOI: 10.1142/S0218127409023111.
33. Plotnikov SA, Lehnert J, Fradkov AL, Schöll E. Adaptive control of synchronization in delay-coupled heterogeneous networks of FitzHugh–Nagumo nodes. Int. J. Bifurc. Chaos. 2016;26(4):
1650058. DOI: 10.1142/S0218127416500589.
34. Lindner B, García-Ojalvo J, Neiman A, Schimansky-Geier L. Effects of noise in excitable systems. Phys. Rep. 2004;392(6):321–424. DOI: 10.1016/j.physrep.2003.10.015.
35. Tass PA. Phase Resetting in Medicine and Biology: Stochastic Modelling and Data Analysis. Berlin: Springer; 1999. 329 p. DOI: 10.1007/978-3-540-38161-7.
36. Holt AB, Kormann E, Gulberti A, Pötter-Nerger M, McNamara CG, Cagnan H, Baaske MK, Little S, Köppen JA, Buhmann C, Westphal M, Gerloff C, Engel AK, Brown P, Hamel W, Moll CKE, Sharott A. Phase-dependent suppression of beta oscillations in Parkinson's disease patients. J. Neurosci. 2019;39(6):1119–1134. DOI: 10.1523/JNEUROSCI.1913-18.2018.
37. Mau ETK, Rosenblum M. Optimizing charge-balanced pulse stimulation for desynchronization. Chaos. 2022;32(1):013103. DOI: 10.1063/5.0070036.
38. Kulminskiy DD, Ponomarenko VI, Prokhorov MD, Hramov AE. Synchronization in ensembles of delay-coupled nonidentical neuronlike oscillators. Nonlinear Dyn. 2019;98(1):735–748. DOI: 10.1007/s11071-019-05224-x.
39. Kulminskiy DD, Ponomarenko VI, Sysoev IV, Prokhorov MD. A new approach to the experimental study of large ensembles of radioengineering oscillators with complex couplings. Tech. Phys. Lett. 2020;46(2):175–178. DOI: 10.1134/S1063785020020236.
40. Shchapin DS. Dynamics of two neuronlike elements with inhibitory feedback. J. Commun. Technol. Electron. 2009;54(2):175–184. DOI: 10.1134/S1064226909020089.
41. Sysoev IV, Prokhorov MD, Ponomarenko VI, Bezruchko BP. Reconstruction of ensembles of coupled time-delay systems from time series. Phys. Rev. E. 2014;89(6):062911. DOI: 10.1103/PhysRevE.89.062911.
42. Kurkin SA, Kulminskiy DD, Ponomarenko VI, Prokhorov MD, Astakhov SV, Hramov AE. Central pattern generator based on self-sustained oscillator coupled to a chain of oscillatory circuits. Chaos. 2022;32(3):033117. DOI: 10.1063/5.0077789.