



## STEEP UNIDIRECTIONAL WAVES: EXPERIMENTS AND MODELING

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We demonstrate the possibility to obtain experimentally a single steep unidirectional wave at a prescribed cross-section of the wave tank. It is shown that the evolution of wave groups with moderate to high maximum steepness is accompanied by notable nonlinearity. The spatial version of the Zakharov equation that was obtained by the authors recently was used as a theoretical model. The equation describes nonlinear spatial evolution of unidirectional wave groups with wide spectra. Good agreement was obtained between the experimental results and the model computations for wave groups with moderate steepness. For wave groups with very high steepness, a good qualitative agreement was observed. It is suggested that bound waves affect the wave envelope shape as well as the location of the focusing. Lack of the quantitative agreement between the numerical computations and the experiments for very steep wave groups is partially attributed to the inability of the conservative Zakharov model to describe the non-conservative effects like the wave breaking.

### Introduction

An ability to produce a single steep wave at a prescribed location in a laboratory wave tank of constant depth is often required for model testing in coastal and ocean engineering. Such waves can be generated by focusing a large number of waves at a given location and instant. Dispersive properties of deep or intermediate-depth surface gravity waves can be utilized for this purpose. Since longer gravity waves propagate faster, a wave group generated at the wave maker in which wave length increases from the front to the tail may be designed to focus the wave energy at a desired location. Such a wave sequence can be seen as a group that is modulated both in amplitude and in frequency. One-dimensional theory describing spatial and temporal focusing of various harmonics of dispersive gravity waves based on the linear Schrödinger equation was suggested by Pelinovsky & Kharif (2000). These authors suggested such a focusing as a possible mechanism responsible for generation of extremely steep singular waves, the so called freak, or rogue, waves. However, the experiments of Brown & Jensen (2001) demonstrated that nonlinear effects are essential in the evolution of those waves. An extensive review of field observations of those waves, as well as relevant theoretical, numerical and experimental studies was recently presented by Kharif and Pelinovsky (2003).

The essential nonlinear behavior of wave groups with high maximum wave steepness has been demonstrated in a number of studies. Attempts were made to describe the propagation of deep or intermediate depth gravity water-wave groups with a relatively narrow initial spectrum by a cubic Schrödinger equation (CSE), Shemer et al. (1998). It

was demonstrated in this study that while this equation is adequate for description of the global properties of the group envelope evolution, it is incapable to capture more subtle features such as the emerging front-tail asymmetry observed in experiments. For the weakly-dispersive wave groups in shallow water, application of the Korteweg - deVries equation provided results that were in very good agreement with the experiments (Kit et al. 2000). In the case of stronger dispersion in deeper water, more advanced models than the CSE are required. This stems from the fact that due to nonlinear interaction, considerable widening of the initially narrow spectrum is observed. The modified Schrödinger equation due to Dysthe (1979) is a higher (4<sup>th</sup>) order extension of the CSE. Application of this model indeed provided good agreement with experiments (Shemer et al. 2002). An alternative theoretical model that is free of band-width constraints is the Zakharov (1968) equation. Unidirectional spatial version of this equation was derived in Shemer et al. (2001) and applied successfully do describe the evolution of nonlinear wave groups in the tank. Kit & Shemer (2002) showed the relation between the spatial versions of the Dysthe and the Zakharov equations.

An attempt to check the limit of applicability of the Dysthe equation to describe evolution of wave groups with wider spectrum has been carried out in Shemer et al. (2002). Numerical solutions of the wave group evolution problem were performed out using both the Dysthe model and the Zakharov equation. Comparison of results obtained demonstrated that while the Dysthe model performed in a satisfactory fashion for not too wide spectra, it failed when initially very wide spectra were used.

Extremely steep (freak) wave can be seen as a wave group with a very narrow envelope and correspondingly wide spectrum. Kharif et al. (2001) applied the CSE model to simulate nonlinear freak wave generation by the focusing mechanism. For the reasons presented above, numerical simulation of this problem requires, however, application of a model without strong bandwidth limitations. In the current study we perform an experimental study of propagation of steep wave groups with wide spectrum in a laboratory tank, accompanied by numerical simulations based on the spatial version of the Zakharov equation.

## 1. Theory

The purpose of the present study is to obtain at a prescribed distance from the wave,  $x=x_0$ , steep unidirectional wave group with a narrow, Gaussian-shaped envelope with the surface elevation variation in time,  $\zeta(t)$ , given by

$$\zeta(t) = \zeta_0 \exp(- (t/mT_0)^2) \cos(\omega_0 t), \quad (1)$$

where  $\omega_0=2\pi/T_0$  is the carrier wave frequency, and  $\zeta_0$  is the maximum wave amplitude in the group. The small parameter representing the magnitude of nonlinearity  $\epsilon$  is the maximum wave steepness  $\epsilon=\zeta_0 k_0$ . The wave number is related to the frequency  $\omega$  by the finite depth dispersion relation

$$\omega^2 = kg \tanh(kh), \quad (2)$$

$g$  being the acceleration due to gravity. The parameter  $m$  determines the width of the group; higher values of  $m$  correspond to wider groups and consequently narrower spectra. The spectrum of the surface elevation given by (1) is also Gaussian.

To produce single steep wave at a desired location along the tank, we apply the «time-reversal» idea suggested by Pelinovsky & Kharif (2000). The wave field at earlier locations,  $x < x_0$  is obtained from the computed complex surface elevation frequency spectrum at this location. To this end, the unidirectional discretized spatial version of the Zakharov equation is used:

$$ic_g \partial B_j(x)/\partial x = \sum_{\omega_j + \omega_l = \omega_m + \omega_n} V(\omega_j, \omega_l, \omega_m, \omega_n) B_l^* B_m B_n \exp(-i(k_j + k_l - k_m - k_n)x), \quad (3)$$

where  $c_g$  is the group velocity and \* denotes complex conjugate. This equation describes the slow evolution along the tank of every free spectral component  $B_j = B(\omega_j)$  of the surface elevation spectrum in inviscid fluid of constant (infinite or finite) depth and accounts for Class I, or quartet, nonlinear interactions among various components. The procedure for computation of the interaction coefficients  $V$  in (3) is based on Krasitski (1994) and was developed by Annenkov (2002).

The dependent variables  $B(\omega_j, x)$  representing the free components in the wave field are related to the generalized complex «amplitudes»  $b(\omega_j, x)$ . These complex amplitudes are composed of the Fourier transforms of the surface elevation  $\hat{\zeta}(\omega_j, x)$  and of the velocity potential at the free surface  $\hat{\phi}^s(\omega_j, x)$ :

$$b(\omega, x) = (g/(2\omega))^{1/2} \hat{\zeta}(\omega, x) + i(\omega/(2g))^{1/2} \hat{\phi}^s(\omega, x). \quad (4)$$

The amplitudes  $b$  represents a sum of the free and the bound waves given by

$$b(\omega_j, x) = [\varepsilon B(\omega_j, x_2) + \varepsilon^2 B'(\omega_j, x, x_2) + \varepsilon^3 B''(\omega_j, x, x_2) \dots] \exp(ikx). \quad (5)$$

The bound higher order components  $B'$  and  $B''$  can be computed at each location once the free wave solution  $B_j(x)$  is known. The corresponding formulae, as well as the kernels necessary for their computations are given in Krasitski (1994). The scaled coordinate  $x_2 = \varepsilon^2 x$ . Inversion of (4) allows computing the Fourier components of the surface elevation  $\hat{\zeta}(\omega, x)$ . Inverse Fourier transform then yields the temporal variation for the surface elevation  $\zeta(x, t)$ .

The spectrum corresponding to (1) is integrated using (3) from the planned focusing location  $x_0$  backwards up to the wavemaker at  $x=0$ . The waveforms derived from the computed spectra serve as the wavemaker driving signals, with corrections that account for the actual wavemaker response. The study is carried out for various maximum wave amplitudes.

## 2. Experimental Facility and Procedure

Experiments are carried out in a wave tank which is 18m long, 1.2m wide and 0.6 m deep. A paddle-type wavemaker hinged near the floor is located at one end of the tank. The wavemaker consists of four vertical modules, which in the present experiments are adjusted to move in phase with identical amplitudes and frequencies. The wavemaker is driven by a computer-generated signal. The instantaneous surface elevation is measured simultaneously by four resistance-type wave gauges. The probes are mounted on a bar parallel to the side walls of the tank and fixed to a carriage which can be moved along the tank.

Probes are calibrated *in situ* using a stepping motor and a computerized static calibration procedure described in detail in Shemer et al. (1987). The calibration is performed at the beginning of each experimental run. The probe response is essentially linear for the range of surface elevations under consideration in the present study. The voltages of the four wave gauges, the signal driving the wavemaker and the wavemaker position potentiometers outputs are sampled using an A/D converter and stored at the computer hard disk for further processing.

The carrier wave period adopted in (1) is  $T_0 = 0.7s$ , corresponding to the wavenumber  $k_0 = 8.22m^{-1}$ , so that  $k_0 h = 4.93$  and thus deep-water dispersion relation is satisfied. The maximum driving amplitudes are selected so that at the focusing location, the resulting carrier wave has the maximum wave amplitudes  $\zeta_0$  corresponding to the

steepness  $\varepsilon=k_0\xi_0$  ranging from  $\varepsilon=0.1$  to  $\varepsilon=0.4$ . The location of the focusing point ranged from  $x_0=5\text{m}$  to  $x_0=8\text{m}$ .

The Gaussian energy spectrum of (1) has a shape with the relative width at the energy level of  $1/2$  of the spectrum maximum given by

$$\Delta\omega/\omega_0 = 1/(m\pi)^{1/2} (\ln 2)^{1/2}. \quad (6)$$

The value of the parameter  $m$  in the experiments was selected to be  $m=0.6$ , so that the relative spectrum width  $\Delta\omega/\omega_0=0.312$ , which is beyond the domain of applicability of the narrow spectrum assumption of the cubic Schrödinger and Dysthe models. Moreover, the lower harmonics of the spectrum, contrary to the carrier wave, do not satisfy the deep water condition anymore. Therefore, in all expressions for the interaction coefficients finite depth versions were used in this study.

For each set of selected variable parameters (maximum steepness  $\varepsilon$  and the distance of the focusing location from the wavemaker  $x_0$ ) the solution of the system of  $N$  ordinary differential equations (3),  $N$  being the total number of wave harmonics considered, was obtained for distances from the wavemaker in the range  $1\text{m} \leq x \leq 10\text{m}$ . The number of free wave harmonics considered in this study is  $N=100$ . The wavemaker driving signal was adjusted to get as good as possible agreement between the calculated and the measured wave field at  $x=1\text{m}$ . This value of  $x$  was chosen since at this distance from the wavemaker the evanescent modes decay and do not contaminate anymore the wave field.

### 3. Results

A representative selection of the accumulated in this study cases is discussed in this Section. In the first case considered the focusing point in the computations was selected at  $x_0=8\text{m}$ , and the maximum wave steepness at the focusing location  $\xi_0 k_0=0.2$ . The computed and the measured temporal variation of the surface elevation at different locations along the tank are presented in Figs. 1, *a*, *b*, respectively. From the computation results in Fig. 1, *a* one can see that the selected value of the group-width parameter  $m$  in (1) indeed yields a narrow wave group with a single steep wave at this location. Closer to the wavemaker the group becomes notably wider, and the maximum wave amplitudes decrease accordingly. The amplitude and the frequency modulation within the group are clearly seen. Note that the computed surface elevations at equal distances from the focusing point, i.e. at  $x=6\text{m}$  and at  $x=10\text{m}$ , are identical if time is reversed after the focusing point. Before the focusing, the group starts with slowly-propagating short waves, while longer waves appear later. After the focusing point the faster long waves appear first.

The experimental results presented in Fig. 1, *b* demonstrate a very good agreement with the computations. Residual noise is observed behind the group, and the wave shape at the focusing location is not exactly symmetric.

The computed and the measured spectra for the experimental parameters of Fig. 1 are presented in Fig. 2 at various locations along the tank. The variation of the spectral shape along the tank is evident and indicates that wave evolution is essentially nonlinear even at this relatively low amplitude of forcing. The agreement between experiments and computations is quite satisfactory and both the numerical simulations and the measurements exhibit similar features. The spectral shapes shown in Fig. 2, *b* indicate that the spectrum becomes wider with the distance from the wavemaker and at the prescribed distance of  $x_0=8\text{m}$  the spectrum approaches the Gaussian shape of the numerical simulations. The peak frequency at  $x=1\text{m}$  is shifted to the right relative to the carrier frequency  $f_0=1/T_0$ . The computed spectra (Fig. 2, *a*) at the equal distances from the focusing location, e.g. at  $x=6\text{m}$  and at  $x=10\text{m}$ , are identical, as expected. The low

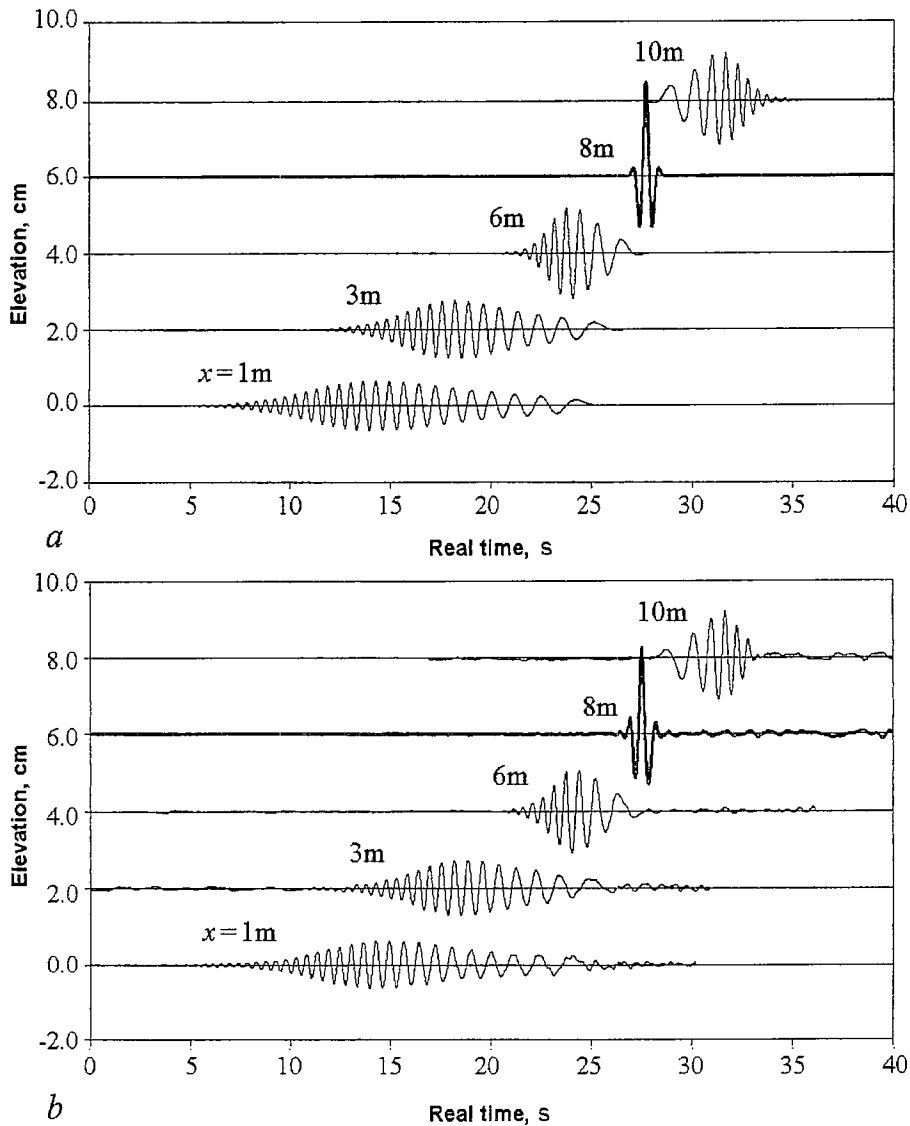


Fig. 1

frequency part of the computed spectrum remains unaffected during the evolution process.

When the maximum wave steepness is increased to  $\zeta_0 k_0 = 0.4$ , the agreement between the computed (Fig. 3, *a*) and the measured (Fig. 3, *b*) surface elevation along the tank remains quite good. The distortion of the envelope shape at  $x = 1\text{m}$  is visible in computations as well as in experiments. Note that the peak values within the group appear to be somewhat different in those figures. It should be stressed that the computed surface elevation is obtained taking into account the free modes only, while in the experiments the effect of the bound waves can be of significance.

Additional reason for certain disagreement between the measured results and the computations can be attributed to the difficulties to reproduce accurately in the experiments the computed spectrum at the vicinity of the wavemaker. The results on the wave amplitude spectra for the conditions of Fig. 3 are given in Fig. 4. Comparison of the variation of the computed spectral shapes at moderate maximum wave steepness in Fig. 2, *a*, with those in Fig. 4, *a* for very steep wave group, clearly indicates that the evolution process in the latter case is strongly affected by nonlinearity. The agreement between the

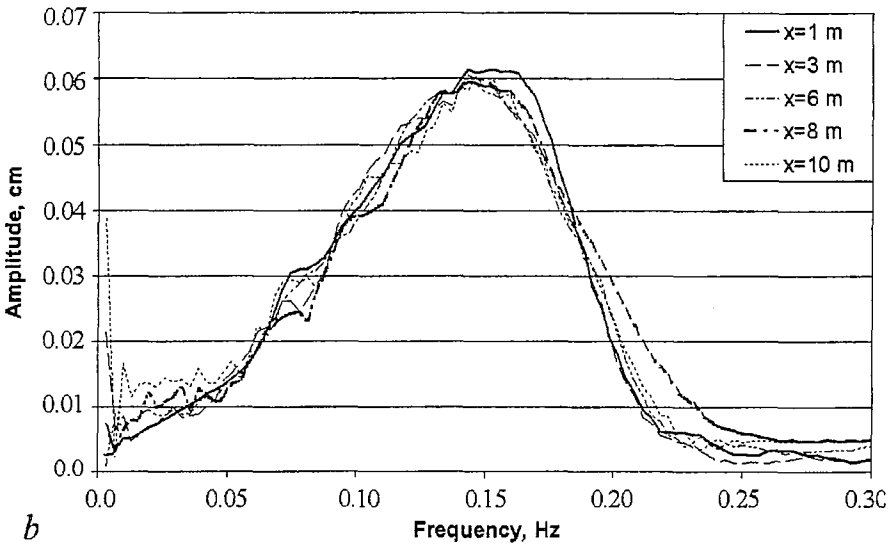
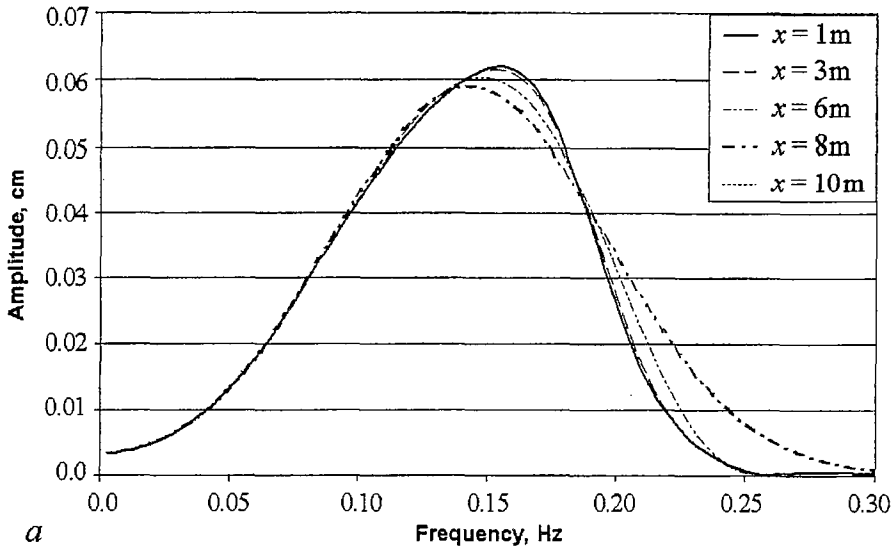


Fig. 2

measured and the computed spectra is now less impressive. This can be partially attributed to discrepancy between the actual measured spectral shape at  $x=1\text{m}$  (Fig. 4, *b*) and that computed at this location (Fig. 4, *a*). While the lower frequency parts of the spectra at all locations are quite similar in computations and in the experiments, the spectral amplitudes in the experiment also decay much faster than in the computations. This can possibly stem from the energy dissipation of the shorter waves due to both viscous effects and breaking.

It is customary in ocean engineering to determine wave height,  $H$ , as the difference between the consecutive minimum and maximum surface elevations. We adopt this definition here to study the wave height evolution along the tank. The evolution of the maximum wave height,  $H_{\max}$ , within the group along the tank for  $\epsilon=0.2$  is shown in Fig. 5, *a* for the focusing location of  $x_0=6\text{m}$ . Although there is a consistent shift between the measured and the computed values due to the difficulties in accurate generation of the required initial waveform at the wavemaker, the rate of increase of the maximum wave height during the focusing process and the following decrease in the maximum wave height during defocusing for  $x>x_0$  are practically identical. At an essentially higher

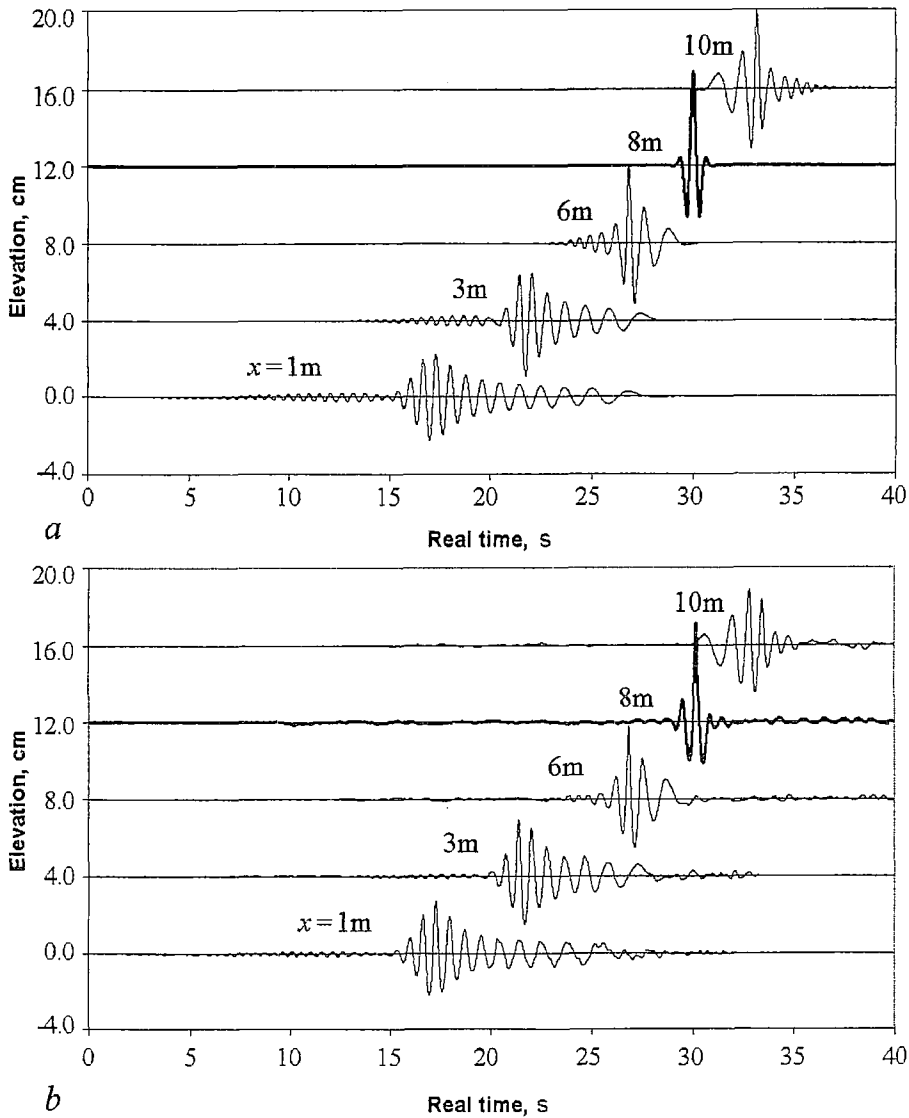


Fig. 3

amplitude ( $\epsilon=0.4$ ), the disagreement between the computational and the experimental results shown in Fig. 5, *b* for  $x_0=6\text{m}$  is clearly visible. In the theoretical computations the growing and the decaying branches remain symmetric relative to the focusing point  $x_0$ . In the experiments, the growth rate of the maximum wave steepness is somewhat lower than in simulations, suggesting stronger dissipation. The notable decay of the measured value of  $H_{\max}$  following the focusing location  $x_0$  is an indication that this very steep wave breaks.

### Concluding remarks

The ability to obtain focused steep waves at any desired location along the tank is demonstrated. It is shown that the focusing process is accompanied by a notable change of the spectral shape and is thus essentially nonlinear. The unidirectional spatial discrete version of the Zakharov equation is adequate to describe nonlinear evolution of wave groups with wide spectrum and moderate steepness. For very steep waves, the agreement

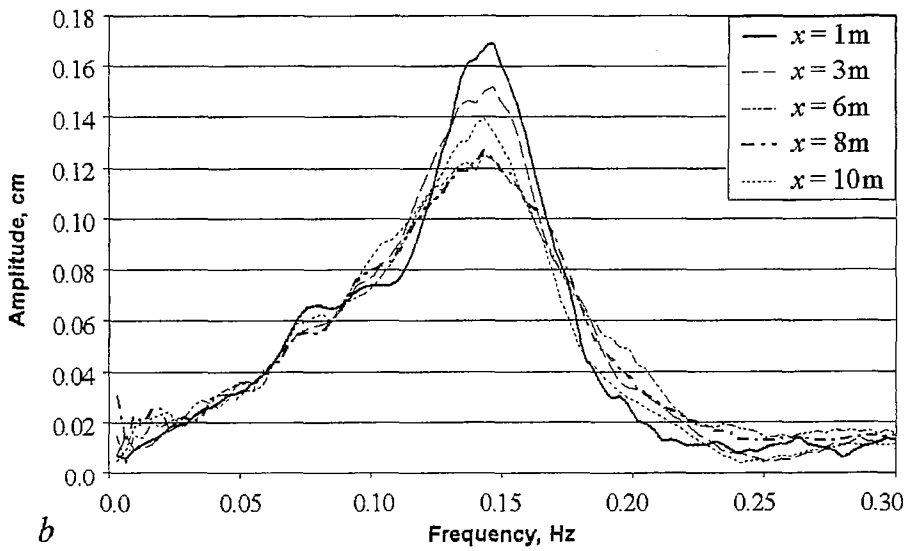
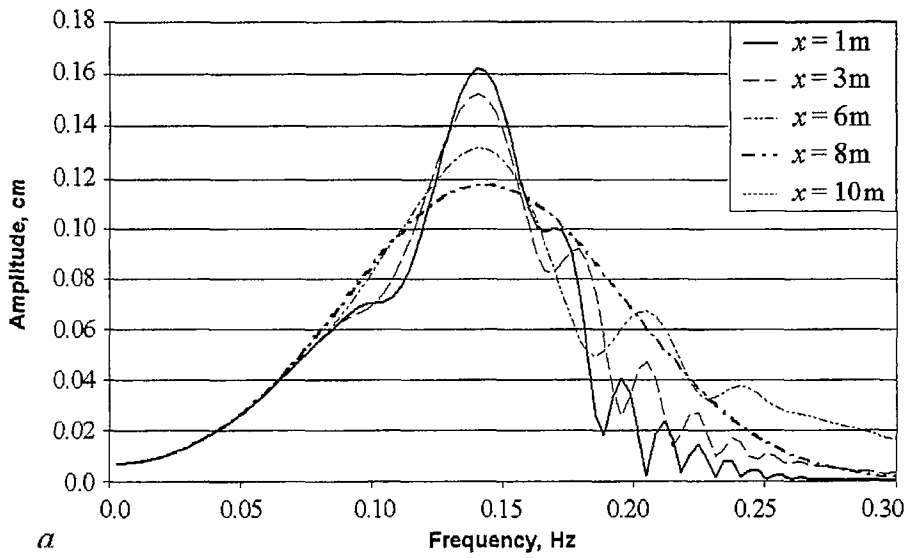
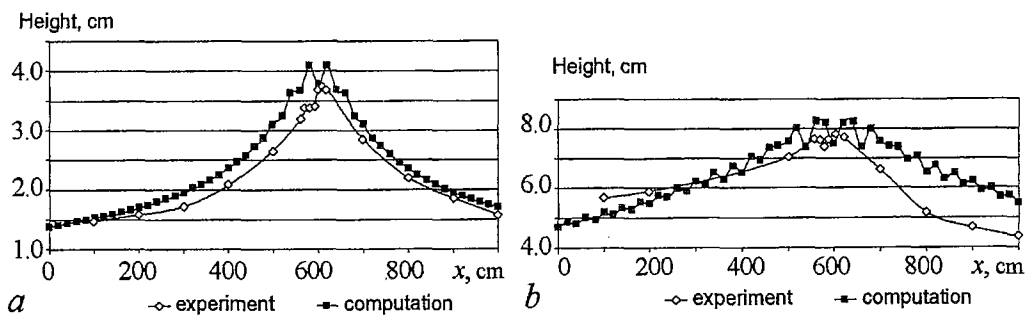


Fig. 4



b

Fig. 5



between the model predictions and the experiments is only qualitative. This is attributed to a number of effects. The reproduction in the experiments of the desired waveform at the wavemaker becomes more complicated as the amplitude increases. The effects related to the bound waves also strongly depend of the wave steepness. These effects in principle can be accounted for in the framework of the Zakharov equation. The corrections due to bound waves were not introduced in the current study. The most important factor, however, that causes the discrepancy between the experiments and the computations, is the wave breaking. The breaking effects and the resulting energy dissipation were clearly documented in the present study. The Hamiltonian Zakharov model is incapable of describing these non-conservative effects.

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## **ОДНОНАПРАВЛЕННАЯ ВОЛНА ВЫСОКОЙ КРУТИЗНЫ: ЭКСПЕРИМЕНТ И МОДЕЛИРОВАНИЕ**

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В работе представлена возможность получения однонаправленной волны высокой крутизны в предписанном сечении по длине канала. Показано, что процесс фокусирования волновой группы сопровождается заметным изменением формы спектра в результате существенной нелинейности. В качестве теоретической модели была использована полученная ранее авторами версия уравнения Захарова, описывающая нелинейную эволюцию в пространстве волновых групп с широким спектром. Получено хорошее согласие между численными и экспериментальными результатами для волновых групп с умеренной крутизной волны, а также хорошее качественное соответствие для волновых групп с высокой крутизной. Выдвинуто предположение о влиянии связанных волн на изменение ожидаемой формы волны и на местоположение места фокусирования волновой группы. Отсутствие согласия между экспериментом и численным счетом для волновых групп с очень высокой крутизной можно объяснить отчасти тем фактом, что модель Захарова, основанная на сохранении гамильтониана, неспособна описывать такие неконсервативные процессы, как разрушение волны.

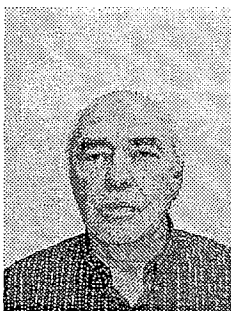


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