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Simulation of self-induced capillary break up of a viscous liquid jet

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Abstract. The *aim* of the study is to reveal the patterns of self-induced disintegration of a viscous liquid jet flowing out at low speed from a capillary hole under microgravity conditions. The research *method* is numerical modeling of the regularities of self-induced capillary decay using the methods of Lagrange mechanics. *Results.* A verified technique for numerical simulation of a capillary jet of a viscous liquid based on the methods of Lagrange mechanics. Identified patterns of self-induced decay of a viscous jet under microgravity conditions. Dependence of the length of the undisintegrated part of the jet on the viscosity of the liquid and the velocity of its outflow from the capillary nozzle. *Conclusion.* The developed numerical simulation technique allows one to correctly and efficiently (from the point of view of the computing resource used) simulate the dynamics of a capillary jet, taking into account complex nonlinear and boundary effects. A pronounced effect of viscosity on the regularities of the disintegration of a jet moving at low speed has been established. The obtained spectral characteristics of perturbations in the jet make it possible to raise the question of the possibility of developing an asymptotic theory of the self-induced decay of a viscous jet.

Keywords: capillary disintegration of a jet, capillary waves, global instability, liquid droplet-radiator.

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Introduction

The problem of modeling the capillary decay of a liquid jet has many technical applications. One of them is related to the creation of a new generation of low-potential heat removal systems in liquid droplet-radiator (LDR). The idea of the LDR is to use a drip shroud of an ultrahigh vacuum liquid working medium as a radiating surface [1–4]. The shroud is created by a droplet generator, cooled during free propagation in space and collected in a trap. The advantages of droplet radiators are high resistance to the effects of micrometeorites, as well as significantly lower weight compared to traditional panel radiators.

To increase the efficiency and manufacturability of the LDR, it is necessary to minimize the droplet velocity [4], limited by the capillary limit of the jet flow rate from the capillary nozzle:

$$V = \sqrt{\frac{\sigma}{\rho r_0}};$$

where σ and ρ are the surface tension and density of the liquid, and r_0 is the radius of the capillary channel from which the jet flows.

The experimental study of the decay patterns of a disintegration of a jet of a nozzle at a velocity close to V is difficult due to the action of gravity. Due to the low velocity of the inertial force, capillary and gravitational forces turn out to be comparable, and the capillary decay regime of the jet becomes chaotic. A small change in the defining parameters can lead to a change in the droplet formation mode. The patterns of chaotic jet decay were discussed in the works [5, 6].

The patterns of capillary disintegration of a jet without the influence of gravity have been experimentally studied by Japanese researchers using [7–10] drop towers. Experiments were carried out with jets generated by various capillary nozzles. The radius of the jet, the conditions for wetting the nozzle with the flowing liquid, the profile of the jet velocity at the outlet of the nozzle, etc., changed. It is shown that under any boundary conditions at the outlet of the nozzle, the capillary decay of a slow jet in microgravity occurs spontaneously, without introducing external disturbances into the system. Under the influence of various factors (separation of droplets, fluctuations of their nuclei, etc.), short-wave traveling capillary waves arise in the jet (Fig. 1). They move towards the capillary nozzle, reflect off it and, due to the Doppler effect, transform into long-wave growing perturbations. The development of long-wave disturbances leads to the disintegration of the jet, during which new disturbances are formed.

Experiments have shown that in self-induced capillary disintegration, there are several equilibrium values of the length of the unbroken part of the jet (a self-decaying jet is a multistable dynamic system). At the same time, only a state with a minimum jet length turns out to be resistant to external influences. The installation process can take a long time (up to several tens of seconds). Because of this, a full-fledged experimental study of the patterns of self-induced decay is possible only in space experiments. To date, only one such experiment has been conducted, its results are presented in [7]. Water was used as the working fluid, the diameter of the jet was about one millimeter. To describe the effect of viscosity on capillary waves, a dimensionless similarity criterion is used — Oh Onesorge number:

$$\mathrm{Oh} = \frac{\mu}{\sqrt{\sigma \rho r_0}};$$

where μ is the coefficient of dynamic viscosity of the liquid. In the conditions of the space experiment [7], the value of Oh is 10^{-3}. At the same time, in relation to the problem of creating LDR, it is of interest to study the patterns of self-induced disintegration of a jets of

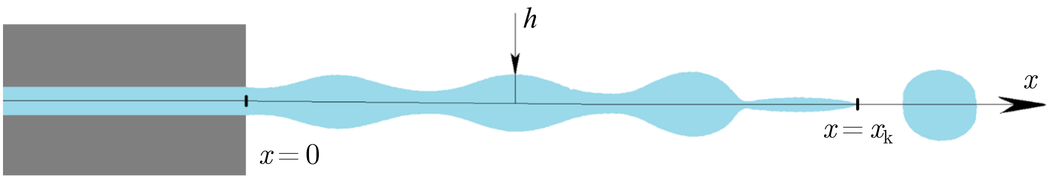

Fig 1. Outflow of a jet from a capillary nozzle

viscous liquids with a characteristic value of the Onesorge number $\mathrm{Oh} \quad 0.1$. This is due to the fact that all potentially suitable working fluids with low volatility have a high viscosity.

In relation to the inviscid case, two theoretical models have been developed that make it possible to describe the mechanism of self-induced disintegration of a slow jet. The first one was proposed by P. A. Yakubenko [11] and is based on the model of the phenomenon of global instability developed by A. G. Kulikovsky [12]. With its help, the regularities of the development of disturbances propagating in the jet upstream and upstream have been studied. The model makes it possible to explain the causes of spontaneous jet disintegration under any boundary conditions, as well as to calculate the spectrum of dominant disturbances. The disadvantage of the approach is the inability to calculate the amplitude of disturbances and the length of the disintegration jet.

The second — boundary instability model — was developed by A. Umemuru [7]. It is believed that disturbances in the jet are generated as a result of the separation of the droplet embryo from it. Using the Green's function, the spectrum of waves propagating against the flow of the jet is calculated. Taking into account the boundary conditions at the nozzle, the characteristics of the disturbance reflected from the boundary are calculated. The length of the decaying part of the jet is determined using the characteristics of the reflected disturbance. The approach allows us to calculate the length of the decaying jet. Using it, the existence of several metastable decay modes has been revealed. It is shown that the state with the minimum jet length is stable. The disadvantages of the technique include the use of an approximate ratio for calculating the Green's function, which does not take into account the presence of growing perturbation modes and large-amplitude caustic waves [13]. Another disadvantage is the use of linear theory, while in the vicinity of the edge of the jet, where traveling waves are formed, the amplitude of the disturbances is large. The reflection conditions on the capillary nozzle in some cases are based on approximate linear ratios. Finally, the developed model does not take into account dissipative effects (it is the short-wave traveling capillary waves that attenuate most intensively in the jet).

In the presented work, the patterns of self-induced decay of a viscous liquid jet flowing out of a capillary nozzle at a rate comparable to the capillary limit V in microgravity and vacuum are investigated by numerical modeling.

1. The technique of numerical modeling

As a rule, for modeling droplet formation during the decay of axisymmetric jets (see Fig. 1) asymptotic expansions of the Navier–Stokes [14] equation system are used. The ratio of the radius of the jet to its length, the amplitude of the initial perturbation of the jet, etc., are chosen as the small parameters by which the decomposition is performed. One of the most frequently used decompositions is proposed in [15]. After switching to dimensionless variables (the initial radius of the jet r_0 is chosen as the unit of length, velocity — value V, time— ratio r_0/V), the equations of perturbation development take the form

$$\begin{aligned} &\partial_t u + u\partial_x u = \partial_x \left(\frac{\partial_{xx} h}{(1+\partial_x h^2)^{3/2}} - \frac{1}{h\,(1+\partial_x h^2)^{1/2}} \right) + 3\mathrm{Oh}\frac{1}{h^2}\partial_x \left(h^2 \partial_x u\right); \\ &\partial_t h + u\partial_x h = -\frac{1}{2} h \partial_x u; \end{aligned} \tag{1}$$

where h is the radius of the jet, u is the average mass velocity of the jet substance.

The (1) system is derived from the Navier –Stokes equations the assumption that the characteristic wavelength of the jet perturbations significantly exceeds its radius. This allows us to use only the main terms of the Taylor series expansions of the dependence of the velocity of the liquid and the pressure in the jet on the radius. The minimum wavelength of the growing disturbance is $2\pi r_0$. Disturbances with a shorter wavelength are effectively attenuated by dissipative effects. Therefore, in practically interesting cases, the assumptions used in the output of (1) are fulfilled with high accuracy. This is confirmed by numerous comparisons of theoretically obtained results with experiments, for example [16, 17].

One of the difficulties in applying the system of equations (1) to model the patterns of self-induced decay is the need to describe the multiple separation of droplet nuclei from the jet. When it breaks, singularities [18] are formed in the solution of the problem, for the numerical resolution of which permanent adaptation of the computational grid is necessary. In addition, there are a number of difficulties associated with describing the edge of the jet. In this paper, the methods of Lagrangian mechanics were used to model the capillary jet. Previously, this method was used in [5] to simulate the chaotic dynamics of the inertial-gravitational regime of liquid pumping from a tap.

The model equations are obtained under the same assumptions as the (1) system: the constancy of the axial velocity and pressure along the radius of the jet, as well as the incompressibility of the liquid. A variable is being introduced ξ

$$\xi(x,t) = \int_{x}^{x_0} \pi h^2(z,t)\, dz,$$

where x_0 is the coordinate of the end of the jet, z is the axial coordinate of the jet (see Fig. 1). The axial velocity of the jet substance can be represented as $u = \partial_t x(\xi, t)$. If the cutoff coordinate of the capillary nozzle is $x = 0$, the expression for the kinetic energy of the jet will take the form

$$K = \frac{1}{2}\rho \int_{0}^{\xi(0,t)} u^2(\xi,t)\, d\xi.$$

If the jet is in microgravity, then the potential energy corresponds to the surface energy

$$\Pi = 2\pi\sigma \int_{0}^{x_0} h(z,t)\sqrt{1 + (\partial_z h)^2}\, dz.$$

The Lagrangian of the jet can be represented as

$$L = K - \Pi = \frac{1}{2}\rho \int_{0}^{\xi(0,t)} u^2(\xi,t)\, d\xi - 2\pi\sigma \int_{0}^{x_0} h(z,t)\sqrt{1 + (\partial_z h)^2}\, dz. \tag{2}$$

The dissipative function in the orthonormal coordinate system $\{y_i\}$, one of the axes of which coincides with the axis of the jet, has the following form [19]:

$$E = -\frac{1}{2}\mu \int_{0}^{\xi(0,t)} \sum_{i,j} \left(\frac{\partial v_j}{\partial y_i} + \frac{\partial v_i}{\partial y_j}\right)^2 d\xi,$$

where v_i is the projection of the velocity of the jet substance on the axis y_i. Taking into account the axial symmetry

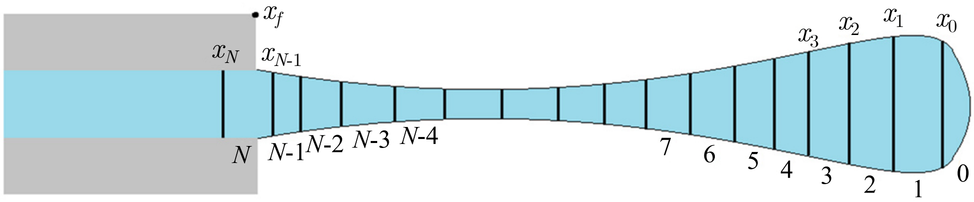

Fig 2. Scheme of splitting the jet into liquid particles

$$E = -3\mu \int_0^{\xi(0;t)} \left(\frac{\partial_\xi u}{\partial_\xi z}\right)^2 d\xi. \qquad (3)$$

A discretization of the Lagrangian was performed for the numerical solution. It was believed that the jet was divided into $N + 1$ liquid particles (Fig. 2). The zero particle corresponds to the edge of the jet, and N is a liquid particle that has not had time to completely exit the capillary opening. In this case, it is assumed that the kth particle is limited by coordinates x_{k-1} и x_{k+1}.

The volume and mass of the kth liquid particle are equal, respectively

$$\xi_k = \int_{x_{k-1}}^{x_k} \pi h^2(z;t)\,dz;$$
$$m_k = \rho\xi_k.$$

If we assume that the kth particle moves at the velocity of the point x_k, the kinetic energy of the jet will be

$$K = \frac{1}{2}\rho \int_0^{\xi(0;t)} u^2(\xi;t)\,d\xi = \frac{1}{2}\sum_k m_k (\dot{x}_k)^2 = \frac{1}{2}\sum_k m_k u_k^2.$$

In a discrete approximation, the dissipative function takes the form

$$E = -3\mu \int_0^{\xi(0;t)} \left(\frac{\partial_\xi u}{\partial_\xi z}\right)^2 d\xi = -3\mu \sum_k m_k \frac{(\dot{x}_k - \dot{x}_{k+1})^2}{(x_k - x_{k+1})^2}.$$

The jet radius is calculated using the formula

$$h_k = \sqrt{\frac{m_k}{\pi(x_k - x_{k+1})}}.$$

The surface area of the kth liquid particle is calculated on the assumption that its shape corresponds to a truncated cone

$$S_k = \pi(h_k + h_{k-1})\sqrt{\frac{1}{4}(x_k - x_{k-1})^2 + (h_k - h_{k-1})^2}.$$

Then the potential energy of the jet:

$$\Pi = 2\pi\sigma \int_0^{x_0} h(z;t)\sqrt{1 + (\partial_z h)^2}\,dz = \sigma\sum_k S_k = \sigma S_\Sigma.$$

Taking into account the above ratios

$$L = \frac{1}{2}\sum_{k} m_k u_k^2 - \sigma S_{\Sigma};$$

and the equation of motion of the kth liquid particle takes the form

$$\frac{d}{dt}\frac{\partial L}{\partial u_k} = \frac{\partial L}{\partial x_k} + \frac{1}{2}\frac{\partial \dot{E}}{\partial u_k}.$$

Substituting the expression for the Lagrangian and the dissipative function into the latter ratio, taking into account the relationship between the mass of a liquid particle and the radius of the jet, one can obtain:

$$\frac{d}{dt}(m_k u_k) = -\frac{\partial S_{\Sigma}}{\partial x_k} - 3\mathrm{Oh}\left(h_k^2 \frac{u_k - u_{k+1}}{x_k - x_{k+1}} - h_{k-1}^2 \frac{u_{k-1} - u_k}{x_{k-1} - x_k}\right). \quad (4)$$

The expression (4) is a difference analogue of the dynamic equation of the system(1).

To describe the behavior of the jet edge, a term characterizing the area of the boundary element must be added to the expression for calculating the total area of its surface. During the research, it was assumed that the surface of the jet edge has the shape of a paraboloid. Its characteristics were determined using information about the volume of the boundary element, as well as taking into account the dependence of the radius of the jet on the coordinate in the vicinity of the edge of the jet. The radius of the jet at the outlet of the nozzle was assumed to be equal to the radius of the capillary opening.

The numerical problem (4) is tough due to the presence of the third derivative of the jet radius in a nonlinear expression describing the action of capillary forces. To solve it, the "predictor - relaxator"scheme [20] was used. The calculation of the movement of particles at each time step was carried out in several stages.

At the predictor stage, it was assumed that the displacement of particles from the initial state occurs under the action of inertia and the viscosity force F_v. For each particle, the updated value of the velocity $u_k \leftarrow u_k + \Delta t \cdot F_v / m_k$ and the corresponding displacement of this velocity were calculated $x_k \leftarrow x_k + \Delta t \cdot u_k$.

At the relaxator stage, the position of the particles was clarified by taking into account the action of capillary forces F_{cap}. The F_{cap} value was calculated for the particle configuration obtained at the predictor stage. The position of the particles was specified in accordance with the formula $x_k \leftarrow x_k + F_{\mathrm{cap}}(\Delta t)^2/(2m_k)$.

At the third stage, the velocity of liquid particles was calculated by dividing the displacement vector by the time step value.

The value of the time step for the next stage of the calculation of Δt was determined after calculating the particle velocity. To do this, the distance between neighboring particles was divided by the value of their relative velocity. The minimum obtained "collision time"of t_{col} liquid particles was selected. The time step value is calculated using this time. Numerical experiments have shown that the convergence of the numerical algorithm is achieved in the case when Δt is about 15 times less than t_{col}. When making calculations, Δt was considered 100 times less t_{col}.

A special feature of the problem is the occurrence of self-similar solutions in the solution of zones describing the patterns of jet rupture when droplet nuclei are separated from it (Fig. 3). In this case, the nuclei of large-radius droplets are connected to each other by a thin constriction. To describe such phenomena, a dynamic adaptation of the splitting of the jet into liquid elements was carried out. Numerical experiments have shown that the calculation result ceases to depend on the splitting frequency in the case when the distance between adjacent liquid particles is less

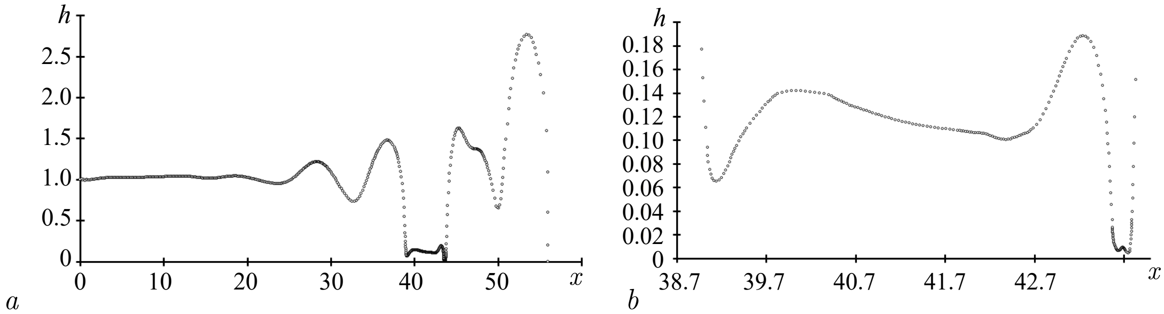

Fig 3. Dependence of the jet radius on the coordinate

than 1/3 of the local radius of the jet. The calculation algorithm automatically separated or combined the particles so that the distance between them was in the range from 1/15 to 1/5 of the local radius of the jet. Other reasons for the change in the number of liquid particles in the calculation are the outflow of new portions of liquid from the capillary nozzle, as well as the separation of droplet nuclei from the jet.

Since the process of jet rupture is not hydrodynamic in nature (within the framework of hydrodynamic laws, the liquid isthmus is thinned to the moment when it breaks due to thermodynamic fluctuations), when performing numerical calculations, it was believed that the rupture occurs at the moment when the radius of the jet becomes less than some critical value, the choice of a specific value of which was determined by the objectives of the calculation.

In Fig. 3 the results of calculating the jet with $\mathrm{Oh} = 0.15$ flowing out of the nozzle with an average velocity of $u_0 = 2$ (the velocity was modulated according to the law $u = u_0(1 + 0.1 \cdot \sin(2\pi t \cdot 0.7/u_0))$ at time $t = 37$ (fig. 3, *a* — the entire jet; fig. 3, *b* —isthmus connecting droplet germs). In the vicinity of the point with the coordinate $x = 43.7$, the process of formation of a secondary zone of a self-similar solution in the area of separation of a thin isthmus from the droplet embryo is visible.

Convergence verification and verification of the developed computer code was carried out by numerically calculating the increment of growth of long-wave disturbances (wave number $k = 2\pi r_0/\lambda$, where λ is the wavelength of the disturbance). It follows from the system of equations (1) that small perturbations with $k < 1$ grow with an increment of ω, determined by the formula [13]:

$$\omega = \sqrt{\frac{1}{2}k^2\left(1 - k^2\right) - \frac{9}{4}\mathrm{Oh}^2k^4} - \frac{3}{2}\mathrm{Oh}k^2.$$

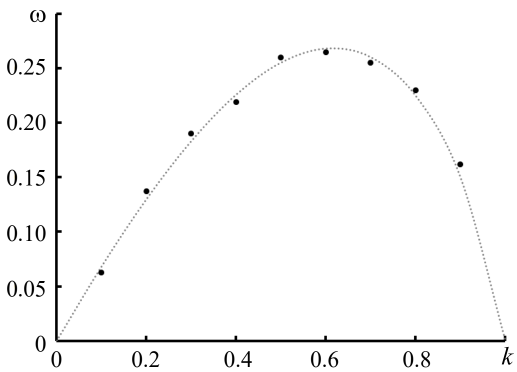

Fig 4. Comparison of the results of numerical calculation (points) of the disturbance growth factor ω with the analytically obtained value (dotted line) at $\mathrm{Oh} = 0.15$

In Fig. 4 the results of comparing the dependence of $w(k)$ at $\mathrm{Oh} = 0{:}15$ with the results of a numerical calculation carried out to study the development of disturbances in a liquid cylinder with a length of $11l$ are presented. The position of the boundary liquid elements did not change. The amplitude of the sinusoidal initial disturbance of the jet radius was 1% of its average value. The dependence of the amplitude of the disturbance in the central wave on time was analyzed. After the transients, the mode of exponential growth of the disturbance amplitude was established, and when it exceeded the value of the order of 0.1 of the average radius of the jet, changes in the magnitude of the growth factor caused by the development of nonlinearity were observed. The value of w was calculated from the data obtained during the time interval between the initial process of establishing and the development of nonlinearity.

2. Numerical simulation results

The simulation of self-induced capillary decay was carried out under the assumption of the presence at the initial moment of time at the outlet of the nozzle of a stationary capillary meniscus consisting of ten liquid calculation elements. The fluid flow rate at the beginning of the calculation was u. In order for a stable decay regime to quickly form in the jet, corresponding to the minimum length of the unbroken part of the jet, disturbances were introduced into the system, that is, the jet outflow velocity was modulated with an amplitude of 0.1 and a period of $2p \cdot 0{:}7{=}u$ (the value of $k = 0{:}7$ is close to the maximum dependence of the growth factor on k). Thin isthmuses practically do not affect the dynamics of the main part of the jet. Therefore, it was assumed that the rupture occurs if the radius of the jet is less than $0{:}05r_0$. In some cases, an extended fragment containing pronounced nuclei of several droplets was observed to detach from the jet; a similar phenomenon was observed in field experiments [21]. When separating the droplet embryo from the jet for subsequent analysis, the following were preserved: the time of the jet rupture, the coordinate of the center of mass of the separated jet segment, as well as its mass and average mass velocity.

The difficulty of determining the length of the unbroken part of the jet l is due to oscillatory changes in the position of its edge. The dependence of the coordinate of the center of mass of the separated jet segment on the separation time was constructed. An example of a graph of this dependence is shown in Fig. 5, *a*; the calculation was performed for the values $\mathrm{Oh} = 0{:}05$, $u = 1$. It can be seen that the position of the jet edge fluctuates relative to the average value with an amplitude of approximately 10% of the jet length. Similar fluctuations were observed in

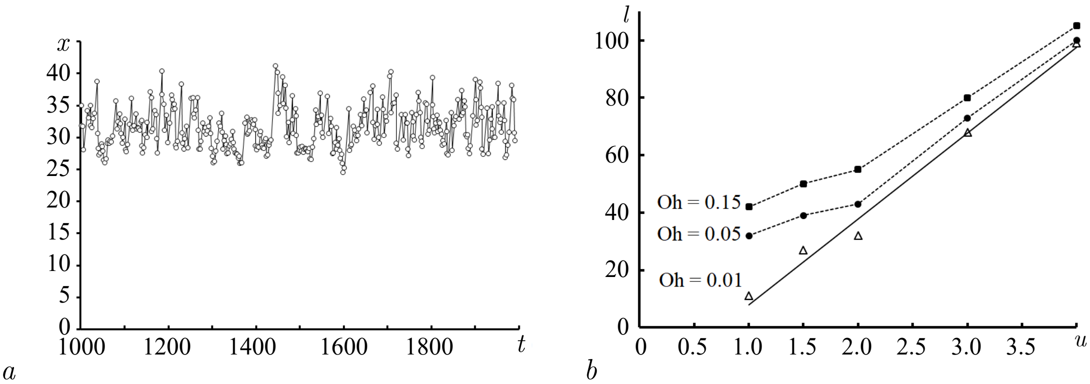

Fig 5. *a* — Time dependence of the coordinate of the center of mass of a drop detached from the jet ($u = 1$, $\mathrm{Oh} = 0{:}05$). *b* — Dependence $l(u)$ for various Oh (solid line — space experiment; calculation: △ — $\mathrm{Oh} = 0{:}01$; • — $\mathrm{Oh} = 0{:}05$; ■ — $\mathrm{Oh} = 0{:}15$)

the experiments [7–10]. The length of the non-formed part of the jet was considered to be the average value of the coordinate of the center of mass of the separating droplet.

In Fig. 5, *b* shows the dependence of l on the jet outflow velocity for various Oh. A solid line shows the dependence experimentally obtained in the space experiment [7]. To verify the numerical modeling technique, in the first series of calculations (the results are indicated by triangles on the graph), the dependence of $l(u)$ for $\mathrm{Oh} = 0.01$ was studied. The results are in good agreement with the space experiment [7]. The dependence of $l(u)$ was also calculated for $\mathrm{Oh} = 0.05$ (circles on the graph) and $\mathrm{Oh} = 0.15$ (squares). At values of u 1, the Onesorge number significantly affects the length of the remaining part of the jet: with an increase in viscosity to $\mathrm{Oh} = 0.15$ l increases several times. However, as the speed increases, this effect weakens. U u approximately equal to 4, the relative elongation of the unbroken part of the jet due to the action of viscous forces is of the order of 10%.

Additionally, the Fourier spectrum of perturbations in the jet $a(k)$ was calculated. In order to prevent the influence of patterns of rupture of constrictions between droplet nuclei on the perturbation spectrum (short waves are formed in a thin constriction), the moment when there were no constrictions was chosen for calculating $a(k)$. The system has a finite length, the spectrum is "sampled"in increments of Δk $2\pi/l$. The magnitude of the peak values in the a spectrum was squared to analyze the spectral distribution of energy.

In Fig. 6 shows the dependence of the peak values of $a^2(k)$ for jets with $u = 1$ and $u = 4$ at $\mathrm{Oh} = 0.05$ and $\mathrm{Oh} = 0.15$. At a low flow rate, a wide range of disturbances is observed. As the expiration rate increases, the spectrum concentrates near $k = 1$. In addition to this main maximum at high velocity u, an additional maximum is observed in the spectrum near $k = 1.25$ — the value of the wave number close to the Airy wavelength [13].

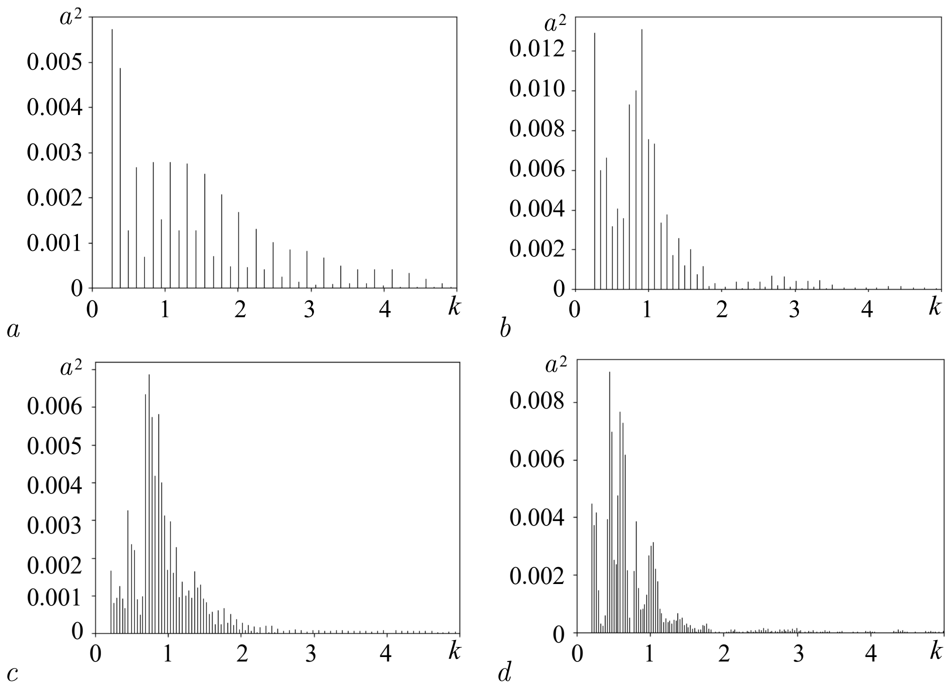

Fig 6. Spectrum of perturbations in the jet: *a* — $u = 1$, $\mathrm{Oh} = 0.05$; *b* — $u = 1$, $\mathrm{Oh} = 0.15$; *c* — $u = 4$, $\mathrm{Oh} = 0.05$; *d* — $u = 4$, $\mathrm{Oh} = 0.15$

Conclusion

The proposed technique makes it possible to correctly and effectively simulate the dynamics of a capillary jet with the possibility of taking into account complex boundary effects.

The self-induced disintegration of a viscous liquid jet under microgravity conditions has been simulated. A pronounced effect of viscosity on the patterns of decay of a jet moving at low speed has been established.

Currently, there is no analytical model for the phenomenon of self-induced disintegration of a viscous liquid jet. The results of the performed study of the disturbance spectrum in a self-decaying jet indicate that at $\mathrm{Oh} \quad 0:1$, the spectrum of traveling waves is concentrated in a small neighborhood of a dimensionless wavenumber $k = 1$. The question of the possibility of developing an asymptotic theory of self-induced disintegration of a viscous liquid jet requires further study.

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