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INFORMATION CONCERNING THE STATES OF OPEN SYSTEMS

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The concept of the Shannon information in the theory of communication is well-known. The information is thus connected with entropy which is the most complete measure of degree of uncertainty states at the statistical description of macroscopic objects.

In open systems the sequences of nonequilibrium phase transition are possible. They can lead both to degradation, and to self-organization. In these cases entropy serves as the measure of relative degree of chaoticity states of open systems. In the paper for the first time the definition of information concerning the states of open systems is proposed.

The concrete definition of the information for processes in Boltzmann gas, for nonlinear Brownian motion in autooscillatory systems - generators are presented. The connection of concept of information with the criterion of a relative degree chaoticity - with S-theorem, as well as with definition of Lyapunov functionals for open systems is established.

The definition of the information for the medico-biological systems, where one of the main concepts is the «norm of chaoticity», is considered as well.

1. Introduction

The aim of this paper is to consider the new notion «Information concerning the states of open systems».

In the theory of communications two definitions of the information are in the use. Both were introduced by C.Shannon [1].

For one of them the formula for the information coincides with the Boltzmann formula for entropy. For this reason it is called S-information.

Alongside with it there is another definition. The second definition of information was offered by C. Shannon [1-3]. This time the information was defined via difference of unconditional and conditional entropies.

In both cases the information is connected with entropy, which serves the best measure of a degree of uncertainty under statistical description of macroscopical systems.

Scientific significance and level of C. Shannon writings on information theory were excellently summarized by A.N. Kolmogorov in his preface to Russian edition of the collected works by C. Shannon «Selecta on theory of information and cybernetics» (Moscow: Foreign Literature Publishers, 1963): «The significance of Shannon work for pure mathematics was not appreciated at once. I remember that even at the International congress of mathematicians in Amsterdam (1951) my American colleagues, specialists in information theory, considered my interest to Shannon work somewhat exaggerated, for it was more technics rather than mathematics. Now such opinions hardly need refutation. True, Shannon left to his successors strict mathematical «justification» of his ideas in the cases of any difficulty. However, his mathematical intuition is amazingly precise».

Entropy can play another roles as well when we deal with open systems. In particular, entropy can play the role of relative degree of order (or chaos) for the states of open systems. As a consequence, the new roles occur for the information.

Now we can answer the question: What does the notion «Physics of open systems» mean ?

2. Information concerning the states of open systems

«Physics of open systems» is an interdisciplinary field of science. Here we can only give a brief list of key words and notions to characterize it:

Chaos and Order; Entropy and Information; Lyapunov functionals for open systems; Criteria for the relative degree of order in open systems; Norm of chaoticity; Degradation and Self-organizing; Diagnostics of open systems; Constructive role of dynamic instability of atomic motion; Transition from reversible to irreversible equations; Structure of continuous medium; Unified kinetic and hydrodynamic description of nonequilibrium processes; Dissipative structures; Nonequilibrium process in active media; Equilibrium and nonequilibrium phase transitions; Unified kinetic description of laminar and turbulent motions; Quantum open systems.

Certainly, many of these concepts are not at all «new». The purpose of the «Physics of open systems» is to elaborate ideas and methods for the integrated description of this broad class of problems.

Open systems can exchange energy, matter, and (last but not the least) information with the environment. We shall consider only open macroscopic systems. They consist of many objects, involve structural elements of different nature.

Due to the complexity of open systems, they may demonstrate a variety of structures. Dissipation plays a constructive role in the formation of these structures. To emphasize this, I. Prigogine has coined the term *«dissipative structures»* [4,5]. This comprehensive and exact term covers all sorts of structures: temporal, spatial and temporal-spatial structures. The latter are exemplified by autowaves.

The complexity of open systems provides an ample opportunity for cooperative phenomena to occur. In order to emphasize the role of collective interactions in the formation of dissipative structures, H. Haken has introduced the term *«synergetics»*, that means joint action [6-9].

All systems considered here are macroscopic. This means that they consist of a large number of elements. This allows in many cases to treat such systems as a continuous medium. Such convention changes dramatically the nature of the system. In order to avoid potential problems, it is necessary to use a physical definition of continuous medium rather than a formal mathematical definition. This requires a concrete definition of physically infinitesimal time and length scales in term of characteristic parameters of the system. The corresponding physically infinitesimal volume is the equivalent to a physical «point» [10,11]. The indicated brief list of main concepts of physics of open systems shows, that absence of sharp and clear definition of concept «information» for states of open systems is one of gaps of this theory. The wiping out of this gap is the main purpose of the present article.

3. Entropy and Information

A. Boltzmann Information (S-information). There are two different statistical definitions of Shannon information. The first one coincides by form with Boltzmann definition of entropy. If $f(X)$ is any dimensionless distribution function of dimensionless variable X, then the Boltzmann entropy and information («S- information») is [1,8,12]

$$I[X] = S[X] = -\int f(X) \ln f(X) dX \qquad (1)$$

or for discrete variables

$$I[n] = S[n] = -\Sigma_n f_n \ln f_n. \qquad (2)$$

Although in many cases the calculation of S-information is certainly useful, it does not reflect the existence of self-organization in open systems [10,11].

In [12,13] the arguments are given for the existence of the conservation law of information and entropy

$$I[X] + S[X] = \text{const.} \qquad (3)$$

This statement, however, is in the contradiction with the definition of S-information.

B. Shannon Information. To define the changing of information in processes in open systems it is more effective to use more general definition of information [1-3]

$$I[X|Y] = S[X] - S[X|Y]. \qquad (4)$$

Here $S[X]$ is the ordinary Boltzmann - Shannon entropy

$$S[X] = -\int f(X)\ln f(X)dX \qquad (5)$$

and $S[X|Y]$ is the conditional entropy. It is connected with the conditional distribution function $f\,[X|Y]$ $(f\,(X,Y)=f\,[X|Y]f\,(Y))$ by relation

$$S[X|Y] = -\int f\,(X,Y)\ln f\,(X|Y)dXdY. \qquad (6)$$

The expression (5) for the information can be presented in more symmetrical form

$$I[X|Y] = \int \ln f\,(X,Y)/\{f(X)f(Y)\}f(X,Y)dXdY \geq 0. \qquad (7)$$

We see that the information is defined in such way is positive. The equality corresponds to the case when the quantities X,Y are statistically independent.

Let the distribution function be defined completely by the first moment

$$f(Y) = \delta(Y - a), \qquad (8)$$

a is any characteristic parameter. In open systems it can play a role of control (governing or ruling) parameter. After integrating in (6), (7) over Y we obtain the following expression for the information

$$I[X|a] = S[X] - S[X|a] \equiv S[X] + \int f(X|a)\ln f(X|a)dX. \qquad (9)$$

It is necessary, however, to remark that in the general case the expression (9) has lost the most important property of the Shannon information - it is not positive. To ensure the positiveness of the expression we must use the corresponding additional conditions.

Thus it is necessary to establish the conditions at which the information $I[X|a]\geq 0$. We shall do this for concrete examples.

4. Boltzmann's H-theorem

A. Lyapunov functional Λ_S. The name of H-theorem (where H stands for «*heat*») was introduced by British physicist Burbury in 1894, several years after Ludwig Boltzmann had proved this statement.

Textbooks in molecular and statistical physics maintain that Boltzmann's H - theorem holds for closed systems. This assertion needs a certain refinement which will be very important for the formulation of criteria of self-organization «S-theorem».

Boltzmann's H-theorem states that the entropy of a closed system increases in the course of the evolution to the equilibrium state, and remains constant in the state of equilibrium:

$$dS/dt \geq 0. \qquad (10)$$

It is important, however, that the mean energy of rarefied Boltzmann gas remains constant in the course of evolution to the equilibrium state:

$$\langle E\rangle \equiv \langle p^2/2m\rangle = \text{const.} \tag{11}$$

So, the conserved quantity is not the energy E, but the mean energy $\langle E\rangle$, therefore the fluctuations of energy are allowed. This shows that there exists the internal non-closedness of the Boltzmann gas system, which is due not to the energy exchange with the environment, but to the existence of an infinite (in the thermodynamic limit) buffer of internal degree of freedom. Boltzmann's kinetic equation only contains a tiny fraction of total information about the system.

The inclusion of condition (11) allows us to formulate Boltzmann's H-theorem in terms of Lyapunov functional

$$\Lambda_S = S_0\text{-}S(t) = k_B\int \ln[f(r,p,t)/f_0(r,p)]f(r,p,t)drdp/(2\pi\hbar)^3 \geq 0, \tag{12}$$

$$d\Lambda_S/dt = d/dt[S_0\text{-}S(t)] \leq 0. \tag{13}$$

The second inequality follows from the H-theorem.

These two inequalities show that $\Lambda_S = S_0\text{-}S(t)$ is the Lyapunov functional [10,11].

B. Information and Lyapunov functional Λ_S. Conservation law of entropy and information for Boltzmann gas. Let us return to the expression (9). Let the values of the control parameter a be positive and nonconditional entropy $S[X]$ corresponding to the zero value of the control parameter

$$a \geq 0 \quad \text{and} \quad S[X] = S[X|a{=}0], \tag{14}$$

therefore the information is

$$I[X|a{=}0] = 0. \tag{15}$$

Thus, the nonconditional entropy $S[X]$ does not depend on values of the control parameter a and corresponds therefore to the equilibrium state.

We can use the expression (9) for the Boltzmann gas. In this case time t can play the role of the control parameter $0 \leq t \leq \infty$. The value $t{=}\infty$ corresponds to the equilibrium state. The information is defined as

$$I[r,p,t] = \Lambda_S = S_0\text{-}S(t) = k_B\int \ln[f(r,p,t)\ /f_0(r,p)]f(r,p,t)drdp/(2\pi\hbar)^3 \geq 0, \tag{16}$$

$$I[r,p|t] = \Lambda_S(t) = S_0\text{-}S(t) = k_B\int \ln[f(r,p,t)\ /f_0(r,p)]f(r,p,t)drdp/(2\pi\hbar)^3 \geq 0. \tag{17}$$

We see that in the course of the time evolution to the equilibrium state the conservation law for the sum of information and entropy is valid:

$$I[r,p|t] + S(t) = S_0 = \text{const.} \tag{18}$$

The constant is defined by the equilibrium value of entropy. For the equilibrium state the information and the Lyapunov functional are the zero:

$$I[r,p|t{=}\infty] = \Lambda_S(t{=}\infty) = 0. \tag{19}$$

We see that for the Boltzmann gas the positiveness of information is natural property of the system. To ensure the fulfillment of the condition $I[X|a] \geq 0$ in a general case we must use the additional condition according to which the mean value of the energy does not depend on the value of the control parameter.

We will show this in the next section.

5. S-theorem and conservation law of information and entropy

A. S-theorem as a criterion for the relative degree of order. Among all thermodynamic functions, only the entropy S possesses a combination of properties that allow it to be used as a measure of uncertainty (chaoticity) in the statistical description of processes in macroscopic systems.

It is naturally to think that the criterion for the relative degree of order should be universal. There is no reason why it should be applicable only to a class of systems where average energy is conserved during evolution. Which route may lead to the solution of this problem?

Entropy being the sole function with the properties of a measure of chaos, there is but one option. It is necessary to redefine entropy so that the average energy remains constant in the course of evolution.

We shall consider for simple illustration the evolution of stationary states in the Van der Pol generator at different values of the feedback parameter a. For this system the criterion of a relative degree of chaoticity was formulated for the first time in [14,15] and called the «S-theorem».

Let us assume the existence of two states with the feedback parameter a=0 and $a=a_1$.

Let us denote the macroscopic characteristic of the stationary state as X. The role of X for a generator can be played by the oscillation energy E. Let us further denote the distribution functions of two distinguished states as f_0, f_1 and the corresponding entropy values as S_0, S_1.

The renormalization to given value of mean energy is carried out by substituting the new value $\tilde{T}$ for temperature T. The new temperature is determined from the solution of the equation $(X \to E)$

$$k_B\tilde{T} = \int E\tilde{f}_0(E,a=0)dE = \int Ef_1(E,a=a_1)dE. \tag{20}$$

Giving the correct choice of the «physical chaos state», the solution of this equation has the form:

$$\tilde{T}(a) \geq T. \tag{21}$$

The sign of equality is relevant at $a=a_0=0$, i.e. for the state of physical chaos. Evidently, the state «0» should be «heated» to equalize average energies. As the comparison is now made at identical values of the average effective energy, the entropy difference $\tilde{S}_0$, S_1 can serve as a measure of the relative degree of order in the distinguished states:

$$\tilde{S}_0\text{-}S_1 = \int (\ln[f_1(E)/\tilde{f}_0(E)]\, f_1(E)dE \geq 0. \tag{22}$$

at the condition that

$$\langle E \rangle = \text{const.} \tag{23}$$

To summarize, the result of computing the relative degree of order in the two distinguished nonequilibrium states is represented by two inequalities. One of them (21) confirms the correct choice of the «0» state as presenting physical chaos. The formula (22) provides a quantitative measure of the relative degree of order in the distinguished states.

Using the general formula (9) we can define the information $\tilde{I}(E)$ for the state of generation

$$\tilde{I}(E) = \tilde{S}_0\text{-}S_1 = \int \ln[f_1(E)/\tilde{f}_0(E)]f_1(E)dE \geq 0. \tag{24}$$

We see that the information is the zero for the equilibrium state, when the value of the feedback parameter equals zero.

It is possible to use the criterion «S-theorem» to estimate the relative degree of order for the transition from laminar to turbulent flow [15].

B. Estimation of information and of the relative degree of order by experimental data. Practical application of the S-theorem implies that the effective Hamilton function is known. In many cases, however, there are no adequate mathematical models for open physical systems. This problem is even more complicated as far as biological, social, and economic entities are concerned.

Therefore, it is sometimes necessary to be able to determine the relative degree of order in open systems directly from experimental data. This can be achieved in the following way:

1. By selecting control parameters for a given system, e.g. two states of the system with control parameters a_0 and $a_0+\Delta a$.

2. By experimentally obtaining sufficiently long temporal realizations for the chosen values of the governing parameters.

$$X_0(t,a_0), \qquad X(t,a_0+\Delta a). \tag{25}$$

These data are loaded into a computer and used to construct the corresponding distribution functions:

$$f_0(X,a_0), \qquad f(X,a_0+\Delta a). \tag{26}$$

The two distributions are normalized to unity.

Further operations are as above.

C. Diagnostics of medico-biological objects. Let us consider some applications of the S-theorem for the purpose of medico-biological diagnostics. Investigations of this problem were initiated in Kiev and Moscow in 1990, using both mathematical models and experimental data. In 1994, the first results of the analysis of cardiograms based on the S-theorem were obtained by the joint efforts of biologists and clinicians in the Laboratories of Nonlinear Dynamics at the Saratov and Potsdam Universities (see in [11,16]).

Biological experiments reported by T. G. Anishchenko from Saratov university revealed significant differences in the responsiveness of male and female rats to the noise stress. Biochemical studies have demonstrated opposite changes in the behavior of the two sexes. This finding provided the basis for a study of men and women's behavior in response to stress. The evaluation was also made using the S-theorem.

Two cardiograms were obtained from each patient included in the study, one before and the other after identical stress impact (a shrilly acoustic signal).

Two cardiograms being available from each subject, this allowed a change in the relative degree of order to be individually estimated using the S-theorem. The experiment has demonstrated opposite changes in the degree of order in men and women, the former showing a decreased degree of chaos, while in the latter it increased.

In both cases, there was a deviation from the «norm of chaos» suggesting «pathology». It is for physicians to decide which «disease» is more dangerous.

The return to the «norm of chaos» may be spontaneous. Then, the «recovery» occurs unaided, with time serving as the control parameter. If the patient's conditions are normalized by drug therapy, its efficacy is possible to evaluate using the same criterion.

The investigation of cardiograms before and after stress gives possibility to find the corresponding change of information. Let us return for this to the general formula (9).

We mark by $\tilde{I}_W$ and $\tilde{I}_M$ the renormalized information obtained on women's and men's cardiograms, correspondingly. From the experiments described it follows that for women the more chaotic state is the state after stress. In opposite for men the state before stress, the normal state, is more chaotic.

Therefore, for women $\tilde{S}_{\text{after}}^{(W)}$ is the renormalized entropy for the state after stress, and $\tilde{S}_{\text{before}}^{(M)}$ is the renormalized entropy for normal state - the state before the stress. With taking into account this definitions we obtain two following definitions of information

$$\tilde{I}_W = \tilde{S}_{\text{after}}^{(W)} - S_{\text{before}}^{(W)} \geq 0, \qquad \tilde{I}_M = \tilde{S}_{\text{before}}^{(M)} - S_{\text{after}}^{(M)} \geq 0. \tag{27}$$

We see that for women as a result of stress the quantity of information obtained by cardiogram increases. On contrary, for men the corresponding information after stress decreases.

6. What is self-organization?

A. Starting from equilibrium. Two classes of systems were outlined in a previous Section.

One of them includes many physical systems exemplified in the foregoing discussion by two cases. To begin with, it is a Van der Pol generator in which losses (of electrical resistance) are first compensated as the feedback parameter grows while its further rise results in the transition to the developed generation region. According to the *S*-theorem, this is a case of self-organization. This process starts from the equilibrium, as in the absence of feedback only thermal fluctuations are in an electrical contour. This leads to the conclusion that the process of self-organization may be defined as the transition from the most chaotic (equilibrium) state to a more ordered one (generation).

The situation is similar to the transition from laminar to turbulent flow in a pipe with increasing pressure difference (a higher Reynolds number).

Here, the reference point for the degree of chaos is also the equilibrium state of a fluid in the absence of pressure difference, that is at the zero control parameter. In this case, hydrodynamic motion is lacking and only chaotic motion of molecules occurs. Evidently, this state is the most chaotic.

Again, the process of self-organization is the transition from more chaotic state to a less chaotic one. Is this the universal definition of self-organization? It can be inferred from the previous section that the process of self-organization is not necessarily associated with an increase of the degree of order.

Indeed, there is a broad class of systems (in the first place, biological systems) for which neither the state of complete chaos (thermodynamic equilibrium) nor that of ideal order can be realized. Biological systems would not function under such conditions.

A more fundamental notion for such systems is the «norm of chaos» which has been used more than once in the previous discussion. This notion is compatible with that of «health». Then, self-organization is the process of recovery [11,16].

Now, let us turn back to the studies on the responsiveness of men and women to stress. Earlier, we have agreed to regard post-stress conditions as «pathology». This means that the transitions to the «norm of chaos» in women is actually the «recovery» referred to above as self-organization, i.e. the transition from a more chaotic to less chaotic state. Conversely, the stress-induced state for men is «illness» which corresponds to a more ordered state. Hence, the «recovery» (self-organization) for men is the transition from an ordered state to a more chaotic one.

Thus, the concepts of self-organization and degradation in biological systems cannot be unequivocally related to an enhanced (self-organization) or impaired (degradation) degree of order respectively. A more fundamental notion for such systems is the «norm of chaos» which can be estimated from empirical data using the criterion «*S*-theorem» [11,16].

To summarize, it appears from the above analysis that in certain cases self-organization is easy to observe, e.g. the generation developing in a Van der Pol system with an increasing feedback parameter. Other well-known examples are the appearance of a new structure (Benard cells) at the liquid surface heated from below and Taylor vortices between rotating coaxial cylinders. Using the most fortunate term «dissipative structures» coined by I. Prigogine [5,17], the self-organization process may be described as the spontaneous occurrence of structures in nonlinear dissipative open systems, e.g. temporal dissipative structures in the Van der Pol generator and spatial dissipative structures exemplified by the Benard cells and Taylor vortices. Elimination of the control parameter (feedback, temperature gradient, etc.) in all these cases results in a «system at rest», i.e. one in the state of thermodynamic equilibrium.

Such understanding of the term «self-organization» underlies the theory of formation of dissipative structures. The first systematic exposition of this range of problems has been given in the well-known book by G. Nicolis and I. Prigogine [4]. The starting point was Prigogine's ideas on thermodynamics of irreversible nonequilibrium processes.

Haken's theory of self-organization is based on the appearance of structures due to collective interactions [6]. In other words, cooperative processes are posited as being of primary importance. This prompted H. Haken to use the term «synergetics» for this new interdisciplinary field of research.

B. Norm of chaoticity. Two kinds of self-organization. In more complicated cases such as transition from one turbulent motion to another, in biological systems, it is possible to distinguish between the processes of degradation and self-organization based on the criterion «*S*-theorem». In such cases, the understanding of self-organization as the appearance of new structures or the transition from less to more ordered states becomes insufficient.

This inference is valid for all systems in which the equilibrium state can not serve as the reference point for the relative degree of chaos (or order). Here, the «norm of chaos» concept is of greater importance and, in the general case, certainly applies to the nonequilibrium state, with the transition from «pathology» to «health» corresponding to self-organization. Since deviation from the norm is possible in two directions (towards a greater or smaller degree of chaos), the self-organization process may in general case also proceed in two directions [11,16].

Therefore, the traditional definition of self-organization as the spontaneous formation of structures in dynamic nonlinear dissipative open systems is too «narrow». A more comprehensive description of self-organization processes, even their mere identification, is feasible by the methods of the statistical theory of open systems.

The term «self-organization» is actually rooted deep in ancient thought. This is a very interesting question worthy of illustration by the following facts.

In 1966, the book on «Principles of Self-Organization» was published in the Russian language. It is a collection of reports delivered to a Symposium at the Illinois State University, USA, in 1961. Here is a quotation from the Preface to the Russian edition by A. Lerner, the editor:

«Despite the marked prevalence of self-organizing systems and persistent attempts of scientists to understand the phenomena occurring in such systems, self-organization has in a way remained for many centuries perhaps the most mysterious phenomenon, the most intimate of nature's secrets». The Preface goes on to state: «...the reader will hardly find here a report which would not claim to disclose the mystery of self-organization».

Heinz von Foerster, the editor of the American publication, writes in the Introduction with reference to a story by Plato, a famous Greek philosopher:

«The house of Agathon was the place where the first memorable symposium was held on the problems lying at the junction of different sciences, attended by philosophers, statesmen, dramatists, poets, sociologists, linguists, doctors and students learning various trades».

The report by Y. Eshby, a known expert in the field, contains a statement to the effect that the word «self-organization» can also mean «transition from bad to good organization», even though the author does not explain how to distinguish between «bad» and «good». An approach to this problem is illustrated by the above-mentioned analysis of cardiograms which allowed to differentiate between «health» and «pathology». Such a distinction is also possible based on the above criterion for the relative degree of chaos in different states of open systems.

Naturally, there are more diagnostic criteria to evaluate the state of biological systems. However, the comparison of different diagnostic tools is a matter which requires special attention.

C. Information in processes of self-organization. So, we divide the open systems into two classes. For the first ones for self-organization's processes exists the most chaotic start point - the equilibrium state. For such systems the self-organization process may be defined as the spontaneous occurrence of structures in nonlinear dissipative open systems in course from the equilibrium states. In this cases the further one gets from the equilibrium states the information increase.

However we saw that, as example, for biological systems the understanding of self-organization as the appearance of new structures or the transition from less to more ordered states becomes insufficient. Here, the «norm of chaos» concept is of greater importance and, in the general case, certainly applies to the nonequilibrium state, with the transition from «pathology» to «health» corresponding to self-organization. Since deviation from the norm is possible in two directions (towards a greater or smaller degree of chaos), the self-organization process may in the general case also proceed in two directions.

As consequence for such systems there exist two roots for the appearance of information. In process of recovery for women (from «after» to «before») when process goes from more chaotic to less chaotic state - to norm of chaoticity, in the process of self-organization the information is obtained by cardiogram increases. On the contrary, for men the recovery (the self-organization's process) is the route from more ordered to less ordered state. As a result, for men the information in the process of self-organization decreases.

Now we'll show how to use the criterion «*S*-theorem» the diagnostic of quantum systems.

7. Statistical presentation of Heisenberg uncertainty principle

A. The oscillatory form of Heisenberg relation. As well known from the text books on quantum mechanics, the Heisenberg uncertainty principle follows from a inequality

$$\int |(x/L)\psi + L d\psi/dx|^2 dx/L \geq 0, \qquad \int |\psi|^2 dx/L = 1. \tag{28}$$

Here L is any length parameter.

Let $f(x,p,t)$ be a quantum distribution function - Wigner function, then the last inequality we can present in the following form:

$$\int (x^2/L^2 + L^2p^2/\hbar^2) f(x,p,t)\, dx dp/(2\pi\hbar) \geq 1. \tag{29}$$

The left side of this inequality we can present as mean value of energy for harmonic oscillator with the proper frequency defined by relation

$$\omega_0 = \hbar/(mL^2), \qquad \hbar^2/(2mL^2) = {}^1/_2\hbar\omega_0. \tag{30}$$

Thus,

$$\int [m\omega_0^2x^2/2 + p^2/(2m)] f(x,p,t)\, dx dp/(2\pi\hbar) \geq {}^1/_2\hbar\omega_0. \tag{31}$$

This means that the mean value of the harmonic oscillators can not be less than the corresponding zero energy

$$m\omega_0^2\langle x^2\rangle/2 + \langle p^2\rangle/(2m) \geq {}^1/_2\hbar\omega_0. \tag{32}$$

The inequality presented here can be written as

$$L^4 - \hbar^2L^2/\langle p^2\rangle + \hbar^2\langle x^2\rangle/\langle p^2\rangle \geq 0 \tag{33}$$

and

$$\omega_0^2 - \hbar\omega_0/(m\langle x^2\rangle) + \langle p^2\rangle/(m^2\langle x^2\rangle) \geq 0. \tag{34}$$

From this inequality the Heisenberg uncertainty relation follows

$$\langle x^2\rangle\langle p^2\rangle \geq \hbar^2/4. \tag{35}$$

In general, parameters L and ω_0 have arbitrary values. For the sign «=» these parameters do not fit - there are some restrictions:

$$L^2 = \hbar/(m\omega_0) = 2\langle x^2\rangle = \hbar^2/(2\langle p^2\rangle), \tag{36}$$

or in the other form

$$\langle p^2\rangle/m = m\omega_0^2\langle x^2\rangle = \hbar^2/(2mL^2) = {}^1/_2\hbar\omega_0. \tag{37}$$

B. The sign «=». Distribution functions. For the sign «=» the equation (28) has the following solution

$$|\psi(x)|^2 = 1/(2\pi\langle x^2\rangle)^{1/2}\exp[-x^2/(2\langle x^2\rangle)]. \quad (38)$$

The corresponding solution of the equation (29) can be presented as the Wigner distribution for the harmonic oscillator with the proper frequency ω_0

$$f(x,p) = \hbar/(\langle x^2\rangle\langle p^2\rangle)^{1/2}\exp[-x^2/(2\langle x^2\rangle) - p^2/(2\langle p^2\rangle)]. \quad (39)$$

The dispersions $\langle x^2\rangle,\langle p^2\rangle$ are defined by relations (37).

8. *S*-theorem for quantum systems. Relative ordering of states «=», «>»

A. S-theorem. Of all macroscopic functions, only entropy S possesses a combination of properties that allow it to be used as a measure of uncertainty in the statistical description of processes in miscroscopic systems.

Entropy being the sole function with properties of a measure of chaos, there is but one option. It is necessary to redefine entropy so that the average energy remains constant in the course of evolution.

But evolution in time it is equally possible to consider the evolution of stationary states in open systems at slowly changing of control (governing) parameters. It is for this type of evolution, that criterion «S-theorem» was introduced [14,15].

Here we shall consider the evolution of quantum states corresponding, accordingly, to the sign «=» and «>» in Heisenberg relation.

In general, the degree of order of the distinguished state differs, which account for one of them being more chaotic than the other.

Let, by assumption, the quantum state corresponding to the sign «=» be the most chaotic. For this state the quantum distribution function $f_0(x,p)$ is determined by (39). The corresponding entropy

$$S_0[x,p] = -\int f_0(x,p)\ln f_0(x,p)dxdp/(2\pi\hbar) =$$

$$= -\int f_0(x)\ln f_0(x)dx/L - \int f_0(p)\ln f_0(p)Ldp/(2\pi\hbar) \equiv S_0[x] + S_0[p]. \quad (40)$$

The mean energy for this state is defined by the zero energy

$$\langle E\rangle = m\omega_0^2\langle x^2\rangle/2 + \langle p^2\rangle/(2m) = {}^1/_2\hbar\omega_0. \quad (41)$$

It follows from the inequality (32) that the mean energy is higher than this value. But according to the S-theorem to determine the relative degree of order it is necessary to compare the states at equal values of the mean energy. To satisfy this condition, it is necessary to replace the distribution function by the renormalized one

$$f_0(x,p) \rightarrow \tilde{f}_0(x,p). \quad (42)$$

The renormalized distribution function is Gaussian as well but with renormalized values $\widetilde{\langle x^2\rangle}$, $\widetilde{\langle p^2\rangle}$ for dispersions:

$$\tilde{f}_0(x,p) = [\hbar/(\widetilde{\langle x^2\rangle}\widetilde{\langle p^2\rangle})^{1/2}]\exp[-x^2/(2\widetilde{\langle x^2\rangle}) - p^2/(2\widetilde{\langle p^2\rangle})] \geq 0. \quad (43)$$

To change the mean value of energy we introduce some non zero temperature T

$$\widetilde{\langle p^2\rangle}/m = m\omega_0^2\widetilde{\langle x^2\rangle} = k_BT_{\omega 0} = {}^1/_2\hbar\omega_0\coth[\hbar\omega_0/(2k_BT)] \geq {}^1/_2\hbar\omega_0. \quad (44)$$

Let the quantum distribution function $f(x,p,t)$ charactcrize any nonstationary state with the sign «>» in the Heisenberg uncertainty relation. The quantum distribution function $f(x,p,t)$ may have negative values as well, but the corresponding distribution functions separately for coordinates and momenta in any cases are positive

$$\int f(x,p,t)Ldp/(2\pi\hbar) = f(x,t) \geq 0, \qquad \int f(x,p,t)dx/L = f(p,t) \geq 0. \tag{45}$$

The corresponding entropy is

$$S[x,p] = -\int f_0(x,p)\ln f_0(x,p)dxdp/(2\pi\hbar) =$$

$$= -\int f(x)\ln f(x)dx/L - \int f(p)\ln f(p)Ldp/(2\pi\hbar) \equiv S[x] + S[p]. \tag{46}$$

To find the necessary value of temperature T we must solve the equation

$$\int[m\omega_0^2x^2/2 + p^2/(2m)]\tilde{f}_0(x,p)dxdp/(2\pi\hbar) = \int[m\omega_0^2x^2/2 + p^2/(2m)]f(x,p)dxdp/(2\pi\hbar) \equiv$$

$$\equiv \int[m\omega_0^2x^2/2]f(x)dx/L + \int[p^2/(2m)]\, f(p)Ldp/(2\pi\hbar). \tag{47}$$

The solution of this equation is

$$T(t) \geq 0, \tag{48}$$

therefore the choice of the state with the sign «=» in the Heisenberg uncertainty relation is correct as more chaotic state. In (47) the variable t for nonequilibrium states plays the role of parameter.

Using the expression (43) for the renormalized distribution function $\tilde{f}_0(x,p)$ and the constancy condition (47) for the average energy, the expression for the entropy difference of states with signs «=», «>» can be presented as inequality

$$\tilde{S}[x,p] - S[x,p] \equiv \tilde{S}[x] - S[x] + \tilde{S}[p] - S[p]=$$

$$= \int f(x,t)\ln[f(x,t)/\tilde{f}(x)]dx/L + \int f(p,t)\ln[f(p,t)/\tilde{f}(p)]Ldp/(2\pi\hbar) \geq 0. \tag{49}$$

Thus, the state with the sign «=» in the Heisenberg relation is the most chaotic. The last expression serves as the quantitative measure for relative degree of order any quantum state - stationary or nonstationary - and the most chaotic state which corresponds to sign «=» in the Heisenberg uncertainty relation.

It is necessary to remember that the oscillatory model was considered above only in the special case of real physical oscillator. The model was considered as well in much more general context. The parameter L in the previous formulas is some general length parameter. If L is the size of the system then the relation L and ω_0 allows to use «as example» the oscillatory model for description of a free particle motion [11,12].

9. Entropy and information for quantum systems

Let us return to the expression (9) for the information and consider the corresponding expression for quantum system. Let $S[X]\rightarrow S_0[x,p]$ be the nonconditional entropy for the state with the sign «=» in the Heisenberg uncertainty relation. It corresponds to the ground state of the system at the temperature T=0. Let also $\tilde{S}_0[x,p]$ be the renormalized entropy for the state with the sign «=» in the Heisenberg uncertainty relation, but for temperature T>0 and, at last, $S[x,p,t]=S[x,t]+S[p,t]$ is the entropy of any nonequilibrium or stationary states. The corresponding notation will be used for conditional information, too. Then, under the renormalization to some mean value energy of oscillator we have the relation

$$I[z,p,t] = \tilde{S}[x,p] - S[x,p,t] \geq 0. \tag{50}$$

Thus, according to S-theorem the transition from the ground state to any exited (nonequilibrium) state is the transition from more chaotic state to more ordered one. From the last expression it follows that any exited state is more informative than the ground state. For the ground state the Shannon entropy is equal to zero.

Evidently that the Boltzmann information (S-information) has the nonzero value for ground state of quantum system as well. The value of entropy S_0 defines the corresponding constant in the Nernst law for the entropy at the zero value of temperature.

10. Information, free energy and Lyapunov functional Λ_F for Brownian motion

The Fokker - Planck equation for energy distribution function $f(E,t)$ is

$$\partial f/\partial t = (D\partial/\partial E)(E\partial f/\partial E) + (\partial/\partial E)[(-a+bE)Ef]. \tag{51}$$

Here D is the noise intensity; $a=a_f-\gamma$, a_f is the feedback parameter, γ and b are the coefficients of linear and nonlinear friction. Let $f_0(E)$ be the stationary solution of this equation with the effective Hamilton function $H(E)$:

$$f_0(E) = \exp[(F_0-H(E))/D], \qquad H(E) = -aE + {}^1/_2E^2. \tag{52}$$

F_0,S_0 are corresponding free energy and entropy

$$F_0 = \langle H(E)\rangle_0 - DS_0, \tag{53}$$

D plays the role of effective temperature. We define the corresponding nonequilibrium free energy via nonequilibrium distribution function $f(E,t)$ ($\int fdE=\int f_0dE=1$)

$$F(t) = \langle H(E)\rangle_t - DS(t). \tag{54}$$

Then the difference between the free energy $F(t)$, F_0 we can present in the form

$$\Lambda_F = F(t) - F_0 = D\int_0^\infty \ln[f(E,t)/F_0]f(E,t)dE \geq 0. \tag{55}$$

During the evolution towards stationary state the free energy steadily decreases

$$d\Lambda_F/dt = d(F(t) - F_0)/dt \leq 0. \tag{56}$$

We come to a result similar to Boltzmann H-theorem in the form (12), (13). This time, however, the Lyapunov functional is not the difference of entropies $\Lambda_S=S_0-S(t)$, but rather is defined by the difference in the free energy $\Lambda_F=F(t)-F_0$. This is the consequence of the fact that during the time evolution in the Fokker - Planck equation the mean value of the energy is not constant.

The last result is less informative than H-theorem since the free energy does not posses the set of properties which would have made it useful as a measure of indeterminacy.

In this situation we can, instead of the Shannon information, determine the information via difference of nonconditional and conditional free energies

$$I_F[E|t] = F[E|t] - F_0[E] = D\int_0^\infty \ln(f(E,t)/F_0)f(E,t)dE \geq 0. \tag{57}$$

From this two last expressions it follows that the information for the Brownian motion is equal to the corresponding Lyapunov functional

$$I_F[E|t] = \Lambda_F = F(t) - F_0 \geq 0. \tag{58}$$

Thus in the course of the time evolution to the stationary state the quantity of information decreases and remains equal to zero in the stationary state.

11. Self-organization in Van der Pol generator

Now we shall apply the criterion «S-theorem» for the particular case of evolution of stationary states of Van der Pol generator as the feedback (the control) parameter is varied [10,11]. By this example it will be possible to demonstrate the principal difference between Boltzmann entropy (S-entropy) and Shannon entropy.

So, let us return to the stationary solution (52) and write it out for three selected states:

1. Feedback parameter is zero, $a_f=0$. In this case we have the Boltzmann equilibrium distribution.
2. Generation threshold, $a_f=\gamma$.
3. Regime of well-developed generation, $a_f>>\gamma$. In this case we may use the Gaussian distribution

$$f_0^{(3)} = [1/(2\pi\langle\delta E\rangle^2)]^{1/2}\exp[(E-a/b)^2/(2\langle\delta E\rangle^2)], \qquad \langle\delta E\rangle^2 = D/b. \qquad (59)$$

For this cases we find mean energy values, and the corresponding entropy values.

We note that, under realistic assumption $k_BTb/\gamma<<1$, the entropy increases as the system moves from the equilibrium state to regime of well-known generation:

$$S_{(1)} < S_{(2)} < S_{(3)}. \qquad (60)$$

This could be interpreted as a decrease of order of state as the generation develops. Intuitively it is clear, however, that the degree of order should increase. What is wrong?

To clear up the situation we shall compare the values of mean energy for the three selected states. From the stationary distribution function it follows that

$$\langle E\rangle_{(1)} < \langle E\rangle_{(2)} < \langle E\rangle_{(3)}. \qquad (61)$$

We see that the mean energy also increases as the generation develops. At the same time, we know that S-theorem must be put in correspondence with the states which mean energy is the same. Accordingly, we have to use an appropriate procedure of renormalization.

In our current example it would be natural to choose state «1» (the state of equilibrium) as the reference state. Then the effective Hamilton function coincides with the energy E and the additional condition (20) we can present in the form

$$k_B\tilde{T}(a_f) = \int_0^\infty E\tilde{f}_0(E,a_f=0)dE = \int_0^\infty Ef_1(E,a_f)dE. \qquad (62)$$

This equation allows us to find the effective temperature as the control parameter a_f.

As result for the free selected states it follows that the effective temperature steadily grows, and the renormalized entropy steadily decreases as the feedback parameter a_f is increased.

This allows us to conclude that the evolution towards well-developed generation is a process of self-organization. Moreover, these results confirm that our choice of the feedback parameter a_f as the controlling parameter is correct.

12. Shannon entropy and «S-information» for Van der Pol generator

A. Shannon entropy. Let $\tilde{S}[E]$ be the renormalized nonconditional entropy for the state «1» - the equilibrium state with the renormalized temperature $\tilde{T}$, and $S[E|a_f]$ be the conditional entropy - the entropy for nonequilibrium state with the feedback parameter $a_f>0$. Then the corresponding Shannon entropy is defined by the expression

$$I(E,a_f) = \tilde{S}[E]-S[E|a_f] = \int \ln[f(E,a_f)/\tilde{f}_{(0)}]f(E,a_f)dE \geq 0. \qquad (63)$$

For the two selected states «1»,«3» this expression for the Shannon entropy is

$$I(E,a_f) = \tilde{S}_{(1)}[E] - S_{(3)}[E|a_f] \approx \ln[(1/(2\pi\varepsilon))\ (\gamma+a)/\gamma]^{1/2} > 0. \quad (64)$$

Here we introduce the notation for the small parameter in the regime of well-developed generation

$$\varepsilon = Db/a^2 << 1, \qquad a = a_f - \gamma \approx a_f. \quad (65)$$

Thus, in the equilibrium state (for a_f=0) the Shannon information is equal to zero. In the course of the developing of generation, when dimensionless parameter ε decreases, the Shannon information is increased.

B. «S-information». Let us return to expression (1) which defines the values of «S-information». For the Van der Pol generator it can be presented as

$$I_S[E] = S[E] = -\int f(E)\ln f(E)dE. \quad (66)$$

From three states selected above, we shall choose now two states «2» and «3». Then the differences of «S-information» can be written as follows:

$$I_S^{(3)}[E] - I_S^{(2)}[E] = S_{(3)}[E] - S_{(2)}[E] = \ln[2(\gamma+a)/\gamma]^{1/2} \geq 0. \quad (67)$$

This expression implies the entropy and the information increase as the generation develops. If this result is to be interpreted as the increase of chaoticity, we challenge both our physical intuition and the criterion «S-theorem».

It is possible, however, to interpret this result in the spirit of information theory, as the information gain: the information about the system increases upon transition to the regime of well-developed generation [8]. The state of well-developed generation in thus regarded as more informative.

13. Conclusion

The role of entropy and information, as we have seen, enlarges considerably when we come to the problem of the relative order for states of open systems. It is possible to show that entropy can play another role as well, acting as measure of diversity of state of open systems for natural selection, which, according to Darwin, is the heart of biological evolution. The simple illustration was done in [10]. There the local entropy is a measure of diversity.

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ИНФОРМАЦИЯ ОТКРЫТЫХ СИСТЕМ

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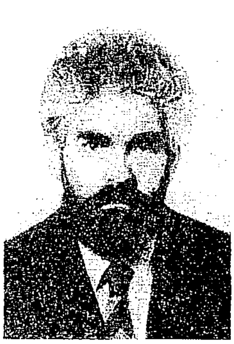
В теории связи используются два определения Шеннона понятия «информация». Одно из них совпадает с энтропией Больцмана и является, фактически, мерой неопределенности при статистическом описании. Второе выражается через разность значений бузусловной и условной энтропий. Конкретизация второго определения позволяет ввести меру информации открытых систем в зависимости от значений управляющих параметров.

Выделен класс систем, для которых возможно равновесное состояние и имеет место закон сохранения суммы информации и энтропии. Для систем этого класса в равновесном состоянии информация равна нулю, а энтроия максимальна.В процессах самоорганизации по мере удаления от равновесного состояния информация возрастает. Выделен другой класс систем, для которых равновесное состояние невозможно. Для них вводится понятие «норма хаотичности» и рассматриваются два рода процессов самоорганизации. Дается соответствующее определение информации.

В качестве примеров дано определение информации для процессов в газе Больцмана, для нелинейного броуновского движения в автоколебательных системах - генераторах. Установлена связь концепции информации с критерием относительной степени хаотичности - S-теоремой, равно как с определением ляпуновского функционала для открытых систем. Рассмотрено определение информации для медико-биологических систем, где одним из основных является понятие «норма хоатичности».



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