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RELAXATION OSCILLATIONS IN SOLID-STATE LASERS WITH DERIVATIVE FEEDBACK

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The feedback proportional to the derivative of laser intensity can affect only the system stability without changing the system steady state.

In the present work by the example of a two-mode solid-state laser we show the possibility to excite quasi-sinusoidal self-modulation regime using only negative feedback which is proportional in this case to the derivative of the strong mode intensity I_1. The region of parameters at which the stationary regime becomes unstable is found.

Modulation of pump parameter being introduced in the laser equations as $A=A_0(1+ + \zeta(\tau))$ and feedback as $-K(dI_1/dt)$. K is the feedback coefficient which we change to get the unstability region.

At approaching the bifurcation boundary the effect of regenerative amplification of noise in the vicinity of low-frequency relaxation oscillation must be observed. In the transfer functions (in the presence of the feedback) it looks like disappearing of high frequency peak while low frequency peak grows and narrows. The effect of regenerative amplification of noise (increase in the amplitude of the low frequency peak) is accompanied by the effect of total suppression of the high frequency relaxation oscillation and by a significant decrease of the system sensitivity to the external perturbations.

1. Introduction

Investigations of the possibilities for controlling relaxation oscillations and decreasing the level of technical fluctuations in solid-state lasers have been conducted for a fairly long time. Conventional approaches (e.g., stabilizing the power supply of flash lamps and decreasing the temperature variations of the cooling liquid, and thereby, of the laser rod) have been used successfully to reduce technical fluctuations [1]. However, for frequencies at or above 10 kHz, the noise level approached its natural limit, although a noise resonance peak in the range of 50 - 100 kHz remained, reflecting the specific dynamical nature of solid-state lasers.

The recent development of monolithic ring lasers with high stability and semiconductor diode laser pumping has aroused new interest in the creation of low- noise lasers to be used as master oscillators for various precision systems. Harb et al. [2] demonstrated experimentally the suppression of relaxation oscillations in a unidirectional ring laser using feedback. The theoretical and experimental possibility of manipulating relaxation oscillations in a multimode laser using a combination of both positive and negative derivative feedback has also been shown [3,4]. The use of feedback proportional to the derivative of the laser intensity is more preferable than other types of feedback, since it does not alter the steady state affecting only the transient process [3].

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The present work considers a method for suppression of relaxation oscillations in solid-state class B lasers by using feedback proportional either to the derivative of the total intensity or to the derivative of intensity of an individual selected mode. We have studied the relaxation oscillations in lasers with negative derivative feedback proportional either to the total intensity or to the intensity of a selected mode. At some feedback value the in-phase relaxation oscillations disappear causing a decrease in the noise modulation depth of laser radiation. This phenomenon is studied for the example of a two-mode solid-state laser with a Fabry - Perot cavity.

2. Two-mode laser with derivative feedback

The dynamics of multimode class B lasers is described by the rate equations [2,4]:

$$
\begin{aligned}
dI_j/d\tau &= GI_j(N_0 + N_j - 1 - C_j),\\
dN_0/d\tau &= A - N_0(1 + \Sigma I_k) - \Sigma N_k I_k, \qquad (1)\\
dN_j/d\tau &= -N_j(1 + \Sigma I_k) - 0.5N_0 I_j,
\end{aligned}
$$

where I_j - is the jth mode intensity, N_j - is the jth amplitude of the population inversion gratings, N_0 - is the spatially uniform part of inversion, j=1,2 in the case of a two-mode laser, and $G(1+C_j)$ is the loss rate for mode j.

The optoelectronic derivative feedback is introduced by controlling the power of a pump laser

$$A = A_0 - K_i(dI_i/dt), \qquad \text{where } i = tot, 1, 2,$$

according to the scheme shown in Fig.1. Using this scheme we can choose either the total intensity or the selected mode intensity for the negative derivative feedback loop. The additional losses we have chosen for the modes are:

$$C_1 = 0; \qquad 0 \le C_2 \equiv C \le 0.3.$$

By controlling additional losses in the second mode (C), we can vary the steady state intensities of lasing modes I_1 and I_2 .

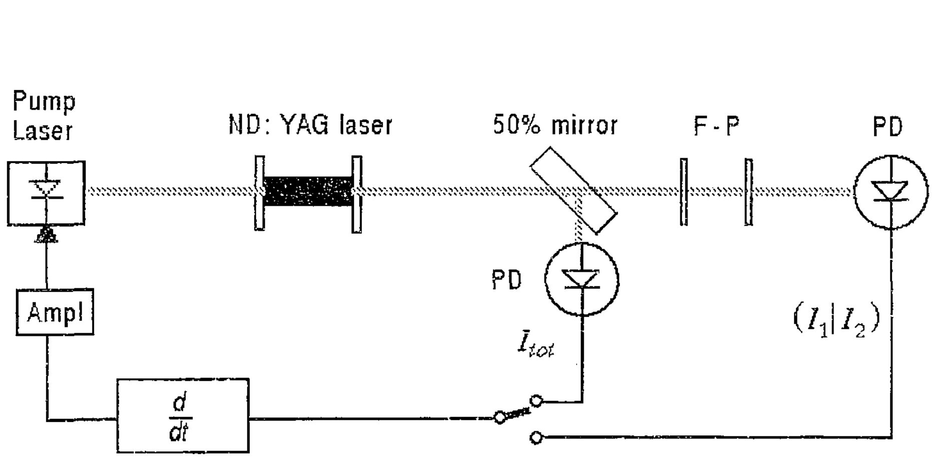

Fig. 1. The feedback application scheme: F-P is Fabry - Perot etalon; PD is Photo diode

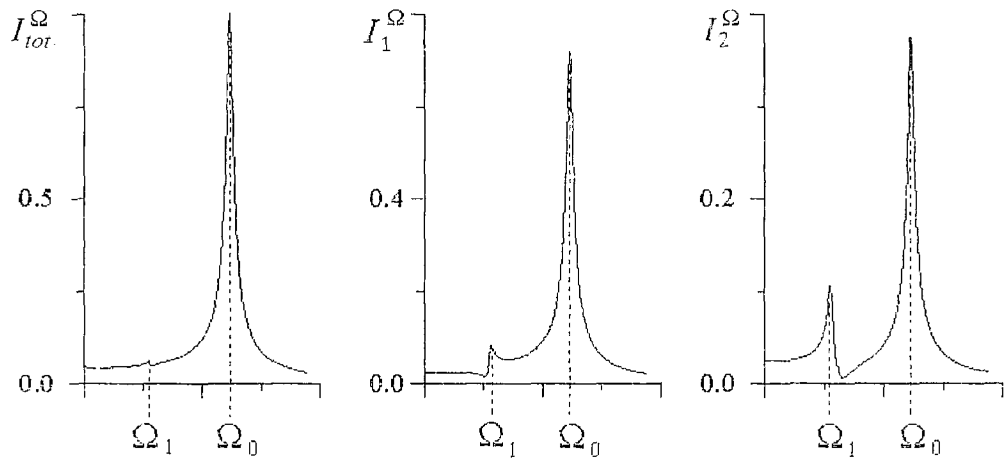

Fig. 2. Transfer functions of the total intensity and of the individual modes (without feedback)

There is only one steady state solution around which the linearized dynamics are governed by the following characteristic equation:

$$||A_{\mu\nu} - \lambda\delta_{\mu\nu}|| = 0. \quad (2)$$

Here $A_{\mu\nu}$ are the coefficients of characteristic matrix of the system, $\delta_{\mu\nu}$ is the Kronecker delta. In the absence of the feedback the system possesses 2 pairs of complex-conjugate eigenvalues with negative real part plus 1 negative real root:

$$\lambda_{1,2} = \delta_0 \pm i\Omega_0, \qquad \lambda_{3,4} = \delta_1 \pm i\Omega_1, \qquad \lambda_5 = \delta_5 \quad (3)$$

with δ_0, δ_1, $\delta_5<0$. Relaxation oscillations appear as resonance peaks at Ω_0 and Ω_1 in transfer functions and power spectra (Fig.2). Eigenvalues $\lambda_{1,2}$ correspond to in-phase relaxation oscillations of the laser mode intensities at frequency Ω_0 with damping rate δ_0. The low-frequency relaxation oscillations at Ω_1 indicate the competitive interaction of the modes in a laser which appears only in transfer functions and power spectra of individual modes.

Introduction of the negative feedback leads to a sharp increase in the damping rate of in-phase relaxation oscillations δ_0. A pair of complex-conjugate roots transforms into a pair of negative real roots at a certain critical value of negative feedback. This corresponds to a disappearance of the in-phase resonance peak at frequency Ω_0 from the transfer function (see Fig.3). However, in each case (with derivative feedback proportional to the individual modes or the total intensity) the main relaxation oscillation is suppressed at different values of feedback coefficients K_{tot}^{cr}, K_1^{cr}, K_2^{cr}. The following relation is fulfilled here $(K_{tot}^{cr})^{-1}=(K_1^{cr})^{-1}+ (K_2^{cr})^{-1}$, reflecting the simple fact that $I_{tot}=I_1+I_2$.

Qualitative differences in feedbacks of different types are seen by the difference in their effect on low-frequency relaxation oscillation. The total intensity feedback does not in any way affect the low-frequency oscillation while the feedback proportional to the strong and weak modes affect this oscillation in quite opposite ways. This is seen in Fig. 3. The main difference is in the dependences of $\delta_1(K_1)$ and $\delta_1(K_2)$. The figure shows that there is a domain of values of the feedback coefficient K_1 where the damping rate $\delta_1(K_1)$ approaches zero so that there is a probability that the sign of $\delta_1(K_1)$ will change. We found a domain of parameters (A, C, K_1) where the steady state regime of two-mode lasing becomes unstable ($\delta_1(K_1)>0$). Fig.4 shows the domain of laser instability in the

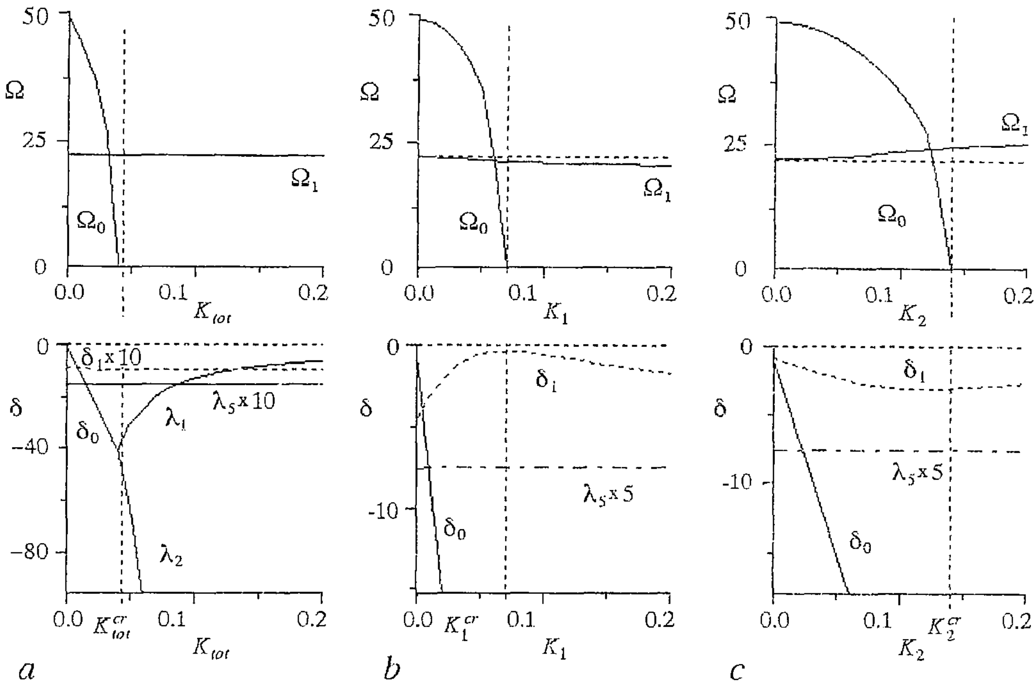

Fig. 3. Eigenvalues (damping rates and oscillation frequencies) versus the strength of feedback coefficients for different types of negative derivative feedback: from modulating total intensity (a), a strong mode (b) and a weak mode (c)

plane of parameters (C, K_1) for fixed pumping rate A=2.0. An increase in pumping starting from this level (A) leads to a gradual decrease of the area of the instability domain down to zero with a simultaneous shift of the domain to larger values of the loss difference and the feedback coefficient (point A=2.4). A decrease in the pumping rate below A=2.0 also leads to a gradual disappearance of the instability domain, but this is accompanied by a shift of the domain to smaller values of the loss difference and the feedback coefficient (point A=1.4).

Inside the shaded area the intensities are spontaneously modulated quasi-sinusoidally at a frequency close to Ω_1. Fig.5 presents the modulation depth of each mode

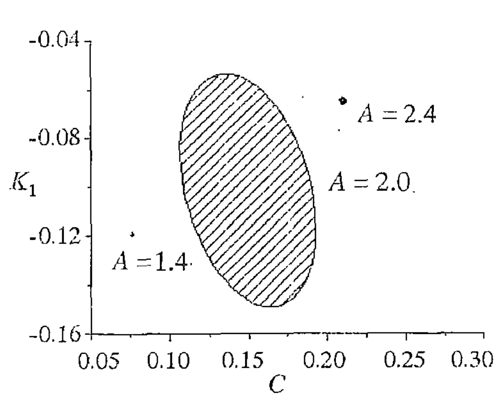

Fig. 4. Laser instability domain (shaded) in the (C, K_1) parameter plane for A=2.0

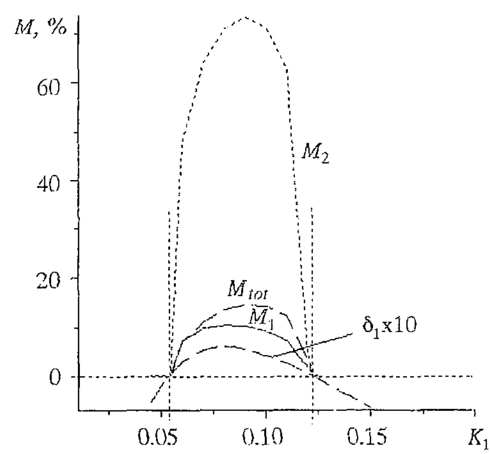

Fig. 5. The modulation depth of the intensities for each mode $M_{1,2}$ and the total intensity M_{tot} versus feedback coefficient

($M_{1,2}$) and of the total intensity (M_{tot}) versus the feedback coefficient. In the center of the instability domain the modulation depth is: about $M_2 \approx 75\%$ (for a weak mode), $M_1 \approx 10\%$ (for a strong mode) and about $M_{tot} \approx 15\%$ for the total intensity. The dashed curve shows the behavior of dependence $\delta_1(K_1) \cdot 10$ which clearly delineates the instability domain. On the boundary of the instability the modulation depth is close to zero indicating that there is a *supercritical Hopf bifurcation* at the frequency of the low-frequency relaxation oscillations.

How can it be explained from a physical point of view that a laser instability appears for negative feedback derived from only the strong mode? The reasons are evident from the Fig. 6, where the transfer function for the strong mode is given in a cylindrical coordinates in the absence of a feedback. The amplitude of laser response is plotted along the radius, the azimuthal angle represents the phase difference between the laser response and the external perturbations while the axial indicates the oscillation frequency. The bottom figure (Fig. 6,*b*) shows the projection of this curve on the plane (amplitude - phase) in the polar coordinates. It is seen from the figures that the response at the low-frequency resonance peak in the transfer function for a strong mode is antiphased with respect to the response at the high-frequency resonance peak. Hence negative derivative feedback (in general) becomes positive feedback in the vicinity of the low-frequency resonance peak. That is why suppression of the high frequency relaxation oscillation at certain parameters of the system leads to excitation of oscillations at relaxation frequency Ω_1. For comparison Fig. 7 shows the transfer function for a weak mode. It is obvious from this figure that the response at the low-frequency resonance

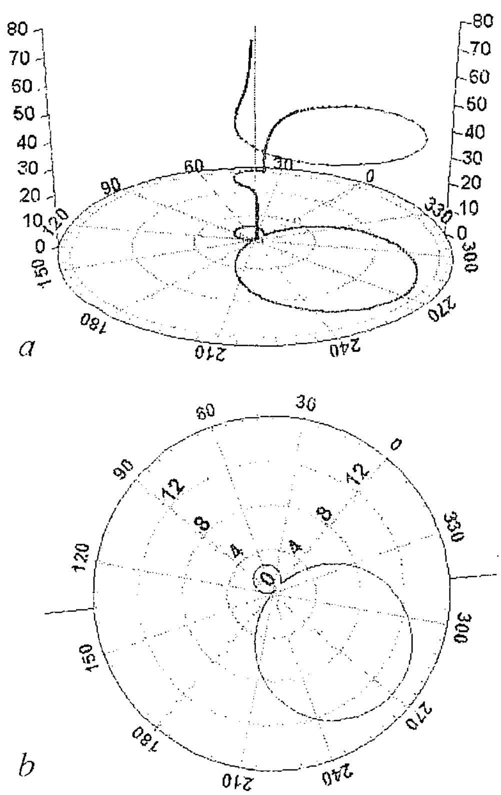

Fig. 6. The transfer function for a strong mode (without feedback) in a cylindrical coordinate system (radial coordinate: modulation depth; axial coordinate: modulation frequency; azimuthal angle: relative phase of modulation and response)

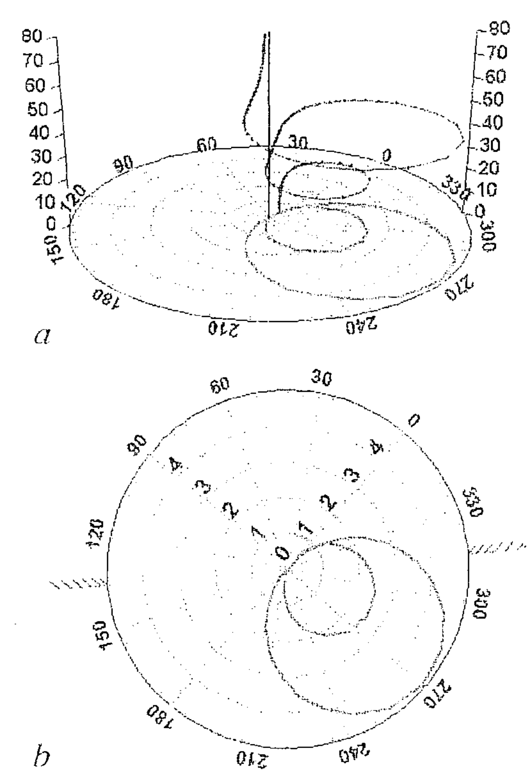

Fig. 7. The transfer function for a weak mode (without feedback) in a cylindrical coordinate system. Coordinates as in Fig. 6

peak in the transfer function for a weak mode is in-phase with the response at the high-frequency resonance peak.

3. Suppression of the noise by negative derivative feedback

Introduction of white noise modulation of the pump $A=A_0(1+\zeta(\tau))$ to the linear differential equations obtained by linearizing the system (1) around the steady state with further Fourier transformation leads to linear algebraic equations of the following form:

$$\Sigma^{5}_{\nu=1}(A_{\mu\nu} - i\omega\delta_{\mu\nu})\, \xi_\nu(\omega) = \beta_\mu, \qquad \mu = 1,2,...,5. \tag{4}$$

Here ω is the modulation frequency, β_μ is the normalized vector of the right-hand sides of the system, which describes the pump modulation $\beta_\mu=[0, 0, m^{1/2}, 0, 0]$, m is the intensity of the technical noise source $\zeta(\tau)$, and $\xi_\nu(\omega)$ is the complex transfer function (amplitude spectrum) which describes the laser response to modulation.

We have numerically calculated the power spectra for the total intensity and the intensities of each mode with different feedbacks (see Fig. 8).

The dotted lines show noise spectra in the absence of the feedback. The solid curves indicate the system's response to external pump modulation of large levels of the feedback ($K_i >> K_i^{cr}$). It is seen from this figure that external perturbations are most effectively suppressed by using the total intensity feedback.

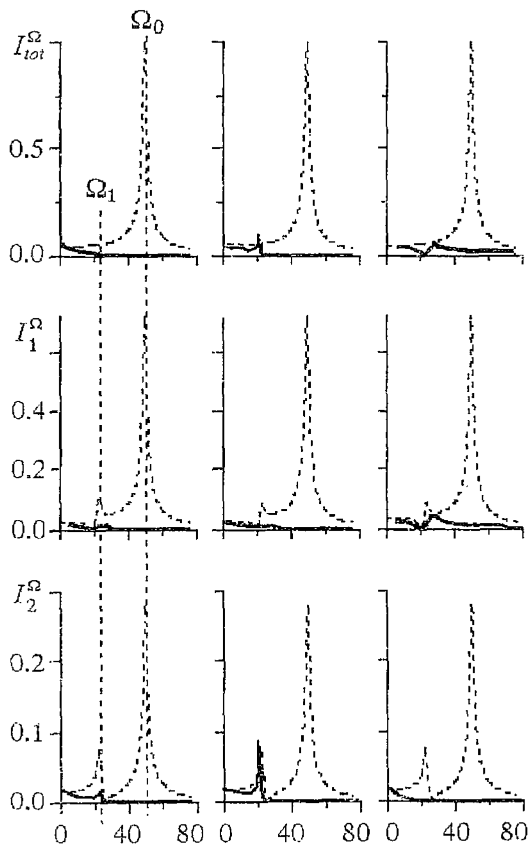

Fig. 8. The power spectra for the total intensity and the intensities of each mode with different negative derivative feedback from: total intensity (*a*); strong mode (*b*); and weak mode (*c*)

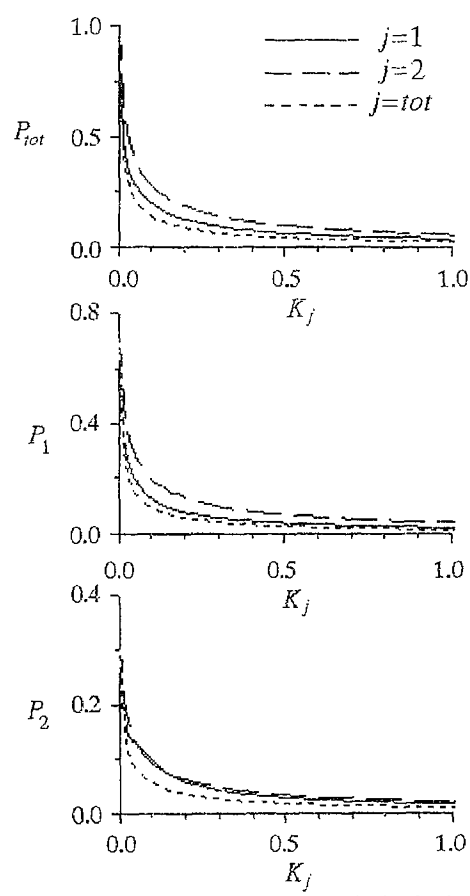

Fig. 9. Fractional modulation depth in both the individual modes and the total intensity versus the values of different feedback coefficients

The area below the curves has the physical meaning of modulation depth of the laser intensities. It is convenient to use this parameter to evaluate quantitatively the effectiveness of the feedback used. We calculated the dependences on the feedback value and type of the modulation depth using the formula

$$p_k = (1/I_k^0)\int_0^\infty I_k^\Omega d\Omega. \qquad (5)$$

The calculation showed that the feedback significantly decreases the modulation depth in both the individual modes and the total intensity (Fig. 9). It is obvious from this figure that the external influence is most effectively suppressed by the total intensity feedback. Our estimates show that in such a way it is possible to suppress technical fluctuations by 30 - 40 decibels.

4. Conclusion

These results allow us to conclude that negative derivative feedback proportional to the total intensity can significantly decrease the system response to external perturbations and can therefore reduce the level of technical noise.

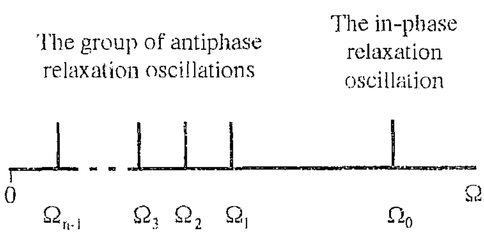

Fig. 10. Schematic dessign of the spectrum of multimode laser relaxation oscillations

As a result of observing a Hopf bifurcation in the case of multimode lasing, we can conclude that if a strong mode is used to form a signal in the circuit of the optoelectronic differential feedback it is always possible to find a domain of parameters where excitation of undamping quasi-sinusoidal oscillations is observed with a frequency of the highest frequency of the family of relaxation oscillations in the group of antiphase relaxation oscillations (see Fig. 10).

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РЕЛАКСАЦИОННЫЕ КОЛЕБАНИЯ В ТВЕРДОТЕЛЬНЫХ ЛАЗЕРАХ С ОБРАТНОЙ СВЯЗЬЮ

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Обратная связь, пропорциональная производной от излучения лазера, может воздействовать на устойчивость системы, не меняя ее стационарных состояний. В данной работе на примере двухмодового лазера показана возможность возбуждения квазисинусоидального режима самомодуляции при использовании только отрицательной обратной связи, пропорциональной производной от интенсивности сильной моды. Найдена область параметров, при которой стационарный режим становится нестабильным.

Модуляция накачки представляется в системе уравнений TSDM как $A=A_0(1+\zeta(\tau))$, а обратная связь вводится по формуле $K(dI_1/dt)$. Здесь K - коэффициент при обратной связи. При определенных соотношениях параметров K, A и C (межмодовой разности потерь) система попадает в область неустойчивости - наблюдается эффект регенеративного усиления шумов в области низкочастотного колебания. В передаточных функциях это проявляется как исчезновение высокочастотного пика и одновременное увеличение и сужение низкочастотного.

Также происходит существенное уменьшение чувствительности системы к внешним воздействиям (уменьшается интегральная глубина модуляции).

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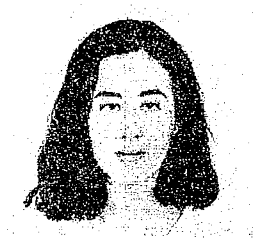

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