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A NEW MODEL OF BROWNIAN TRANSPORT

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A new model of the Brownian particle transport on the base of a deterministic modulation of ratchet potential is proposed. This model leads to the directed constant drift of the Brownian particles rather than to their diffusion.

1. Recently a great attention in a theory of the nonlinear Brownian motion is paid to so-called molecular, or Brownian, motors (or stochastic ratchets), which essentially come down to a directed diffusion current of the Brownian particles due to the asymmetrical conditions for the diffusion. These conditions are achieved via time modulation of a space periodic asymmetrical potential profile, taking into account diffusion times in the potential profile scales (see., e.g., [1-7] and the references cited there). In this work a new model of the Brownian motors, different from the known ones, is proposed. In this model the directed current of the Brownian particles arises due to the drift of the Brownian particles and not due to the diffusion.

2. Let us consider the Brownian particle motion in a viscous medium along the tilted potential profile $\Phi(x)=-ax$, where $a>0$, from the initial delta-shaped probability distribution $W(x,0)=\delta(x)$ (Fig. 1, *a*).

The Langevin equation for coordinates of the Brownian particles $x(t)$ in overdamped regime reads

$$dx(t)/dt = -(1/h)d\Phi(x)/dx + \xi(t) = \\ = a/h + \xi(t), \qquad (1)$$

where $\xi(t)$ is Gaussian thermal fluctuations with $\langle\xi(t)\rangle=0$, $\langle\xi(t)\xi(t+\tau)\rangle=D\delta(\tau)$. Here $D=2kT/h$ is the thermal fluctuations intensity, k is Boltzmann constant, h and T are the equivalent medium viscosity and the temperature.

The linearity of equation (1) implies that the probability density $W(x,t)$ of the Brownian particles is Gaussian for every t, and it is easy to find that

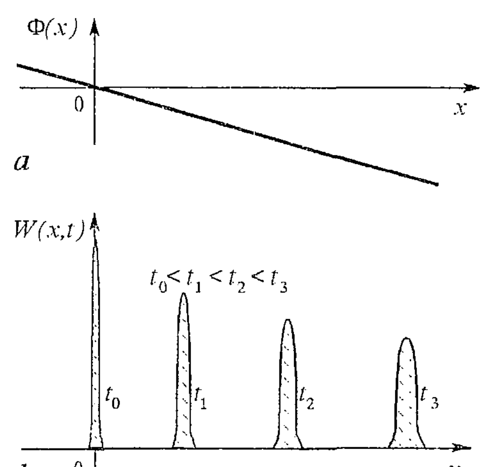

Fig. 1

$$W_G(x,t) = [1/(2\pi Dt)^{1/2}]\exp[-(x-at/h)^2/(2Dt)]. \qquad (2)$$

Hence, the probability density, on the one hand, travels in direction of positive x so that $\langle x\rangle=at/h$ and, on the other hand, spreads according the diffusion law: $\sigma^2_x= Dt$ (Fig. 1, *b*), where σ_x^2 is the variance.

Thus, the tilted potential profile may exert directed transport of Brownian particles in the direction of decreasing potential. At the same time, if this model is to be realistic, it is necessary to bound the potential.

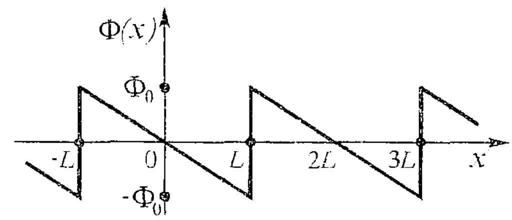

Fig. 2

3. Let us choose the ratchet potential with period $2L$ (Fig.2). The Brownian particle movement from x=0 to the right will take place according (2) as long as $\langle x\rangle+3\sigma_x<L$.

We suppose that the process of the diffusive spreading inside the potential period is slower than the drift, i.e. $(Dt)^{1/2} \ll L$, where $t=Lh/a$. It is easy to show that this condition means that thermal fluctuations are much less than the height of the potential barrier:

$$kT \ll \Phi_0/2 = La/2. \qquad (3)$$

This condition is taken, as a rule, in all studied models of stochastic ratchets, in order to prevent the Brownian particles from passing over potential barriers and breaking the symmetry of the diffusion.

4. Thus, under condition (3), by the time $t_L=Lh/a$ the probability density $W_G(x,t)$ moves to the point L practically without any spread.

The further movement of the probability density is stopped by the potential jump $2\Phi_0$. In order to overcome this situation, it is necessary to use appropriate modulation of the ratchet potential and in result to place the Brownian particles again on the potential profile slope, e.g. in the middle of the potential period.

It can be done, for example, as follows.

a) The most simple and effective method is to move the ratchet potential with the velocity a/h so, that the Brownian particles to be always in the middle of the slope. This modulation can be named as the *running potential* or the *surfing potential*. The shape of the running potential can be different. It will be sufficient to move to the right single slope only with the velocity a/h (Fig. 3).

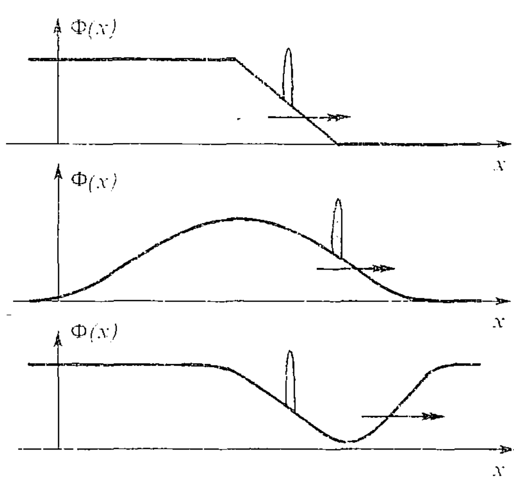

Fig. 3

It is obvious that an artificial creating of such a running potential with velocity depending on the slope steepness is a rather difficult, if not hopeless, problem. Howerver, in biological structures of a cell, where the directed Brownian motion is supposed to be instead of ordinary noneffective diffusion processes (see, e.g., [1]), this running potential can be materialized via the interaction of active macromolecules moving along a polymer with the electrical field of the polymer. It is possible that the running macromolecule by itself «makes» appropriate down-slope of the potential profile which is fully synchronized in a velocity with directed movement of macromolecules.

b) The second evident method of the deterministic modulation is step-wise displacement of the ratchet potential to and fro in a half of period when the Brownian particles are transported up to the potential jump. So, translation of the ratchet potential forth in half of period puts the Brownian particles again in the slope middle and gives possibility for them to be transported up to coordinate $x=2L$. The back displacement of the ratchet potential in initial position will bring the Brownian particles to the point $x=3L$, etc. Thus, if the ratchet potential is regularly translated in halves of periods to and fro, at interval of $\Delta t=Lh/a$, the mean value of the Brownian particles coordinate shifts to the right with constant velocity $\langle x\rangle=at/h$ (Fig. 4).

In order to exclude difficulties connected with an interaction of the Brownian particles with potential jump $2\Phi_0$ at the reverse displacement of the ratchet potential from position B to position A, it is sufficient to switch off the potential itself before translation from position B to position A (i.e. to set $\Phi(x)\equiv 0$) and then switch on the potential in position B.

5. Now let us take into account the spreading of the probability distribution of the Brownian particles drifting to the right, which may be detectable after N switching, when instead of $\sigma_x(L)=(Dt)^{1/2}=(2kTL/a)^{1/2}$ we shall have $\sigma_x(NL)=(2kTNL/a)^{1/2}$. At enough large N (i.e. large t) the probability density $W(x,t)$ will be against the potential jump and will take the non-Gaussian shape (Fig. 5) which at $kT<<\Phi_0/2$ is

$$W_0(x)=\begin{cases} 2a/(hD)\exp[2ax/(hD)], & x<0, \\ 0, & x>0. \end{cases} \tag{4}$$

After replacing of this probability density on the slope middle (at next switching) it will drift and spread according to

$$W(x,t)=\int_{-\infty}^{\infty} W_0(u)W_G(x-u,t)du, \tag{5}$$

where $W_G(z,t)$ is given by (2). Under condition $kT<<\Phi_0/2$ the spreading of the probability density (5) will be practically negligible in process of its movement down along the slope

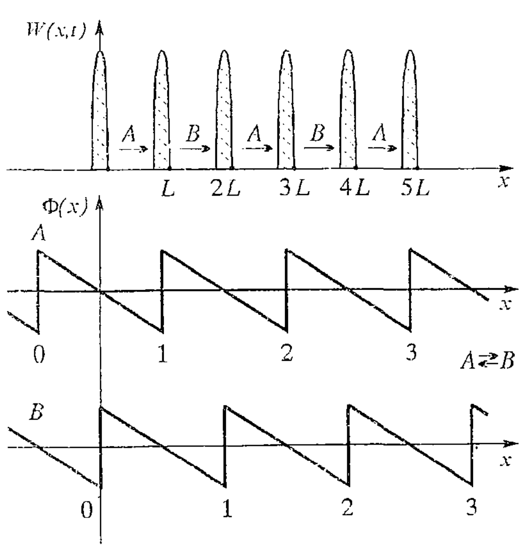

Fig. 4

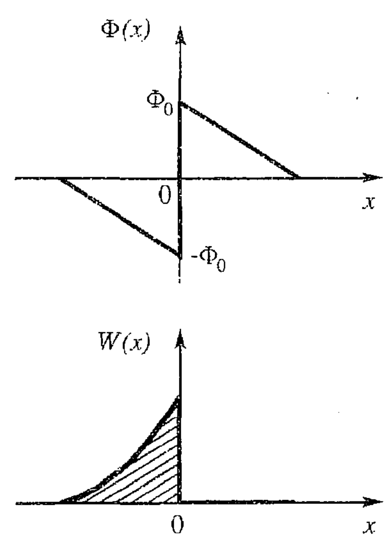

Fig. 5

till the next potential jump, and near the potential jump the probability density again takes the same shape (4).

So, at enough large number of the switching of the ratchet potential the probability density of the Brownian particles equals

$$W(x,t) = \begin{cases} 2a/(hD)\exp[2a/(hD)(x\text{-}at/h)], & x < at/h, \\ 0, & x > at/h \end{cases}$$

and in course of time it has practically constant shape.

6. Thus, the proposed model of the Brownian motor describes the directed transport of the Brownian particles with constant velocity and permits to translate the Brownian particles $\sim t$, and not $\sim t^{1/2}$, as it takes place for other known models of stochastic ratchets, because we have here deterministic drift of the probability density instead of drift associated with the asymmetrical diffusive spreading.

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НОВАЯ МОДЕЛЬ БРОУНОВСКОГО ПЕРЕНОСА

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Предложена новая модель переноса броуновских частиц с помощью детерминированной модуляции «ratchet» потенциала. Модель приводит, главным образом, к направленному постоянному дрейфу броуновских частиц, сопровождающемуся ограниченным расплыванием частиц.

Malakhov Askold Nikolaevich was born in 1926 in Gorky , Russia. In 1950 he graduated from the Gorky State University as physicist. In 1953 he got the candidate's degree (Ph.D.) and in 1968 he got the doctor's degree (D.Sc., Phys. & Math.). From 1963 to 1994 he held the chair of statistical radiophysics, from 1968 up to present full professor of statistical radiophysics at the Radiophysical Department of the Nizhniy Novgorod State University. He published more than 200 research works including 4 monographs, during the last five years - 23 scientific papers. He has trained 35 Ph.D. and 4 D.Sc., professors. Leader of one of the prominent Russian scientific schools. He is Honoured Scientist of Russia (1987) and Russian Federation State prize winner in Physics (1997). His fields of main scientific interest are the statistical physics and statistical radiophysics: the behaviour of nonlinear dynamical systems under fluctuations: nonlinear Brownian motion; the kinetics of phase transitions; time scales of relaxation processes in nonequilibrium systems; the decay time of the metastable states; the life time of nonequilibrium states of different physical systems. E-mail: askmal@rf.unn.runnet.ru