Applied Problems of Nonlinear Oscillation and Wave Theory

Izvestiya Vysshikh Uchebnykh Zavedeniy. Applied Nonlinear Dynamics. 2024;32(1)

Article DOI: 10.18500/0869-6632-003078

Comparative analysis of the secure communication schemes based on the generators of hyperbolic strange attractor and strange nonchaotic attractor

O. B. Isaeva[1,2]✉, *D. O. Lubchenko*[1,3]

[1]Saratov Branch of Kotelnikov Institute of Radioengineering and Electronics of the RAS, Russia
[2]Saratov State University, Russia
[3]Kotelnikov Institute of Radioengineering and Electronics of the RAS, Moscow, Russia
E-mail: ✉isaevao@rambler.ru, dima4398lub@mail.ru
Received 4.05.2023, accepted 15.07.2023, available online 29.11.2023, published 31.01.2024

Abstract. The *purpose* of this work is to analyse qualitative features of the information transmission process via several communication schemes based on the synchronization of transmitter and receiver, both being complex signal generators. For this purpose generators of the hyperbolic chaos and generators with the strange nonchaotic attractor are employed. Evaluation of advantages and disadvantages of such schemes is made comparing themselves with each other as well as with schemes based on the nonhyperbolic chaotic generators. *Methods.* The power spectra and the distributions of the largest finite-time Lyapunov exponent are used to confirm the complexity of the dynamics of the generators in use and to verify the wide-bandness, robustness and stochasticity of their signals. Confidentiality of the informational signal transmission is achieved using its nonlinear mixing to the dynamics of the transmitter. The special phase mixing is used since the model generators employed for the research demonstrate nontrivial dynamics for the angular variable — oscillations phase shift. The digital image is used as an information for transmission. Visual control during the transmission process allows to carry out the qualitative analysis of the success of the signal coding and its detecting by the receiver. *Results.* Successful transmission and decoding of information for all schemes under investigation are demonstrated for the case of identical transmitter and receiver. Parameter detuning of these generators leads to difficulties in separation of the informational signal from the chaotic/complex carrier due to loss of the full synchronization. For the nonhyperbolic chaos detuning of the parameter responsible for the amplitude of the signal leads to the bad quality of the detection while frequency detuning makes detection absolutely impossible. Schemes with the hyperbolic chaos and strange nonchaotic dynamics appear to demonstrate much better results. The information detection is much better in this case because of the robustness of the generalized synchronization. *Conclusion.* Robust chaotic and complex nonchaotic generators appear to have significant advantages for communication systems comparing to the chaotic generators of nonhyperbolic type.

Keywords: chaotic communication, hyperbolic chaos, strange nonchaotic attractor.

Acknowledgements. Supported by Russian Science Foundation, Grant No. 21-12-00121. We acknowledge PhD D. V. Savin for usefull discussion.

For citation: Isaeva OB, Lubchenko DO. Comparative analysis of the secure communication schemes based on the generators of hyperbolic strange attractor and strange nonchaotic attractor. Izvestiya VUZ. Applied Nonlinear Dynamics. 2024;32(1):31–41. DOI: 10.18500/0869-6632-003078

Introduction

Previously, the actual problem of chaotic communication quickly lost popularity due to its inherent intractable disadvantages [1, 2]. One of these disadvantages is the instability of communication circuits based on synchronization of the receiver with the transmitter. Synchronization is necessary to extract information from the signal that is transmitted over the communication signal. To achieve complete synchronization, the complete identity of the receiver and transmitter is required. This requirement can be considered from the point of view of confidentiality of the transfer. Not only the specific type of generator-transmitter, but also the exact values of the parameters are the key to decoding the transmitted information. On the other hand, unlike the mathematical model, it is quite difficult to achieve the absolute identity of the generator-transmitter and the generator-receiver even when creating a laboratory radio engineering layout. The effect of distortion and interference on the signal in the communication channel can make full synchronization unattainable. In the case of moderate non-identity , generalized synchronization [3] may occur, when the dynamic implementations of two related generators do not coincide, but correlate with each other. This work is devoted to the creation of a communication system in which generalized synchronization would be a sufficient condition for functional consistency. (The problem was identified earlier in the works of [2,4]).

The method of manipulating the phases of self-oscillating systems, proposed in 2005 by S.P.Kuznetsov, allowed us to obtain the dynamics of an artificial mathematical model, which previously existed only in the form of a display, in a realistic physical system [5]. We are talking about a mapping whose attractor is the Smale-Williams solenoid. In subsequent works, the method was developed and made it possible to implement a number of other mathematical artifacts. For example, the Hunt–Ott [6] mapping. These mappings are associated with chaotic hyperbolic dynamics and a strange non-chaotic attractor. The first type of behavior has the property of rudeness - insensitivity to disturbances [7], demonstrates rough synchronization modes [8,9]. Therefore, it seems promising for schemes based on synchronization of communication [10]. The second type of dynamics is non-chaotic. The achieved synchronization mode of coupled generators with such behavior should be rough in the absence of disturbances that remove the system from the class of quasi-periodically excited ones. The complex fractal structure of the attractor [11] should provide some privacy [12–14]. In this paper, communication schemes with generators with hyperbolic [5] and strange non-chaotic [6] dynamics with non-identical receiver and transmitter are analyzed, their advantages compared to conventional chaotic communication are shown.

1. Communication schemes

We will consider the following two mathematical models of communication schemes. The first model:

$$\begin{cases} \ddot{x}_1 - (A_1 \cos(\omega_1 t/N) - x_1^2)\dot{x}_1 + \omega_1^2 x_1 = \varepsilon y_1 \sin(\omega_1 t + \rho(t)), \\ \ddot{y}_1 - (-A_1 \cos(\omega_1 t/N) - y_1^2)\dot{y}_1 + (2\omega_1)^2 y_1 = \varepsilon x_1^2, \end{cases} \tag{1}$$

$$s(t) = y_1 \sin(\omega_1 t + \rho(t)), \tag{2}$$

$$\begin{cases} \ddot{x}_2 - (A_2 \cos(\omega_2 t/N) - x_2^2)\dot{x}_2 + \omega_2^2 x_2 = \varepsilon s(t), \\ \ddot{y}_2 - (-A_2 \cos(\omega_2 t/N) - y_2^2)\dot{y}_2 + (2\omega_2)^2 y_2 = \varepsilon x_2^2. \end{cases} \tag{3}$$

Here (1) — transmitter, (2) — signal in the communication channel, (3) — receiver. The second model:

$$\begin{cases} \ddot{x_1} - (A_1 \cos(\omega_1 t/N) - x_1^2)\dot{x_1} + \omega_1^2 x_1 = \varepsilon y_1 \sin(\omega_1 t + \xi\omega_1 t/N + \rho(t)), \\ \ddot{y_1} - (-A_1 \cos(\omega_1 t/N) - y_1^2)\dot{y_1} + (2\omega_1)^2 y_1 = \varepsilon x_1 \sin\omega_1 t, \end{cases} \quad (4)$$

$$s(t) = y_1 \sin(\omega_1 t + \xi\omega_1 t/N + \rho(t)), \quad (5)$$

$$\begin{cases} \ddot{x_2} - (A_2 \cos(\omega_2 t/N) - x_2^2)\dot{x_2} + \omega_2^2 x_2 = \varepsilon s(t), \\ \ddot{y_2} - (-A_2 \cos(\omega_2 t/N) - y_2^2)\dot{y_2} + (2\omega_2)^2 y_2 = \varepsilon x_2 \sin\omega_1 t, \end{cases} \quad (6)$$

Similarly, the equations (4), (5) and (6) are the transmitter, the signal in the communication channel and the receiver. In both circuits, the transmitter and receiver are identical generators connected to each other via the function $s(t)$ — the signal in the channel. Transmitted information signal $\rho(t)$ is non-linearly mixed with the dynamics of the transmitter. Such mixing has advantages in terms of increasing confidentiality compared to the usual additive [15–17]. Information is mixed into the phase fluctuations of the signal generated by the transmitter. Let's explain how this happens.

In the absence of mixing $\rho = 0$, the transmitters (1) and (4) are a hyperbolic chaos generator [5] and a generator with a strange non-chaotic attractor [6]. In this study, the same parameter values for both generators were used, but different from those indicated in the original works

$$A_1 = 8.0, \qquad \omega_1 = 2\pi, \qquad N = 6, \qquad \varepsilon = 0.5. \quad (7)$$

This is done to maximize the similarity between them. Both generators operate on the principle of phase manipulation. They consist of two van der Pol oscillators, which are alternately and counterphase slowly excited. The natural frequencies of the oscillators differ twice. Specially selected communication functions in the right-hand sides of the oscillator equations carry out the transmission in a resonant manner the transformed phase of oscillation between the oscillators. In the hyperbolic chaos generator (1), a transformation occurs in a stroboscopic section with a period $T = \omega_1/N$ with a phase $\varphi_n = \varphi(nT)$ $\varphi_{n+1} = 2\varphi_n$. This corresponds to the Bernoulli mapping. For the generator (4) in the same stroboscopic section, the phase undergoes a transformation $\varphi_{n+1} = \varphi_n + \theta_n + F(\varphi_n, \theta_n)$, where $\theta_{n+1} = \theta_n + \xi\omega/N$ — quasi-periodic (with irrational $\xi = (\sqrt{5}-1)/2$) impact, and F —some function of nonlinearity. This transformation corresponds to the Hunt-Ott mapping. The chosen method of introducing an information signal allows you to mix it directly with a non-trivially behaving oscillation phase produced by signal generators.

With an alternative set of parameters

$$A_1 = 3.79981, \qquad \omega_1 = 2\pi, \qquad N = 6, \qquad \varepsilon = 0.5 \quad (8)$$

the generator (1) can produce chaotic dynamics of a non-hyperbolic type. This can be seen from Fig. 1. The fragment *a* shows a hyperbolic attractor that is topologically equivalent to the Smale-Williams solenoid. On the fragment *b*, the attractor is quite complex, but does not have the characteristic structure of a solenoid. The *c* fragment demonstrates a strange non-chaotic attractor. The diagrams for the angular variables are shown below. Fig. 1, *d* and *e* — iterative diagrams, equivalent to the dynamics of the Bernoulli mapping and destroyed. Fragment *f* — dependence of the oscillator oscillation phase (4) on a quasi-periodic impact variable, which is clearly fractal in nature, as it should be for a strange non-chaotic attractor.

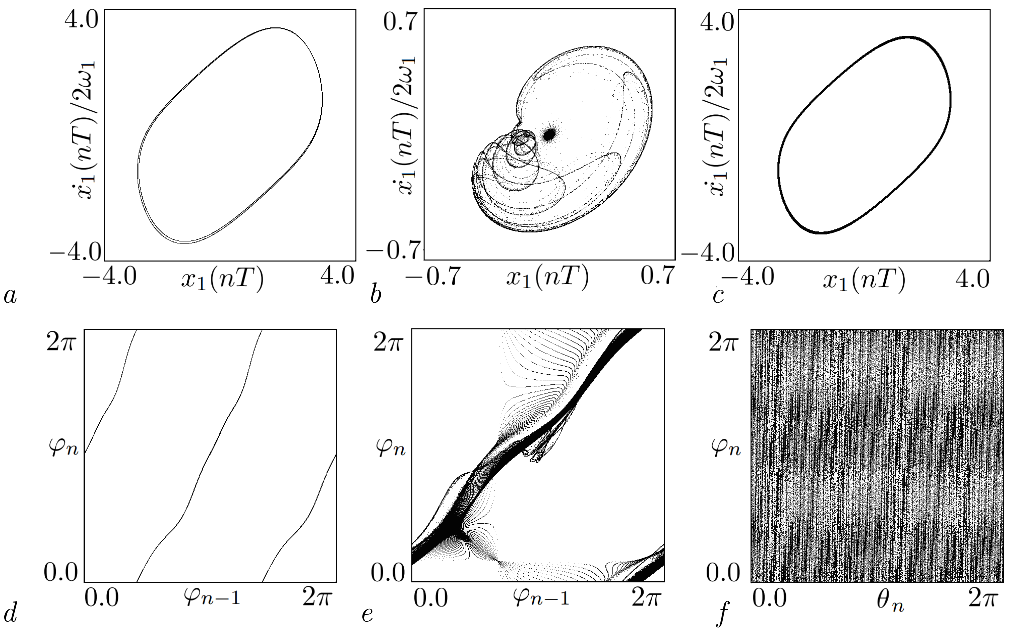

Fig 1. Phase portraits of the stroboscopic map with the period $T = 2\pi N/\omega_1$: for the generator (1) at the parameter values (7) in the hyperbolic chaos regime (*a*); for the same generator at the parameter values (8) in the nonhyperbolic chaos regime (*b*); for the generator with the strange nonchaotic attractor (4) at the parameter values (7) (*c*). Iteration diagrams for the angular variable $\varphi_n = \mathrm{Arg}(x_1(nT) + i\dot{x}_1(nT)/\omega_1)$ for hyperbolic (*d*) and nonhyperbolic (*e*) attractor. Diagram of angular variable φ_n versus the variable of quasi-periodic forcing $\theta_n = n\omega(\sqrt{5}-1)/2$ for the strange nonchaotic attractor (*f*). Figures are obtained in the absence of the informational signal $\rho(t) = 0$

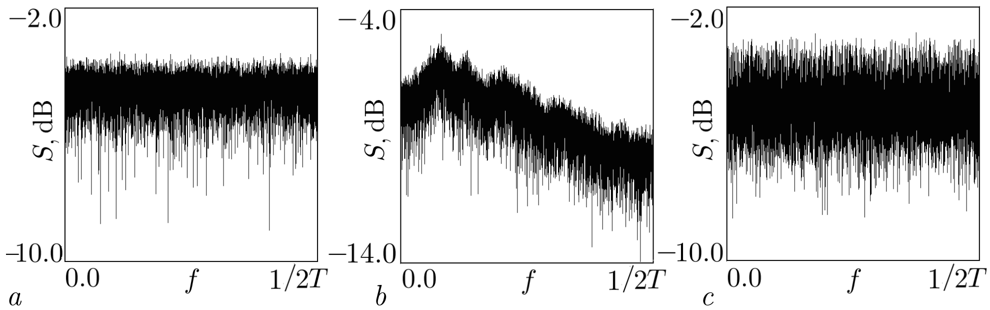

Fig 2. Power spectrum of the stroboscopic map for generation associating with the hyperbolic chaotic attractor (*a*), nonhyperbolic chaotic attractor (*b*) and strange nonchaotic attractor (*c*)

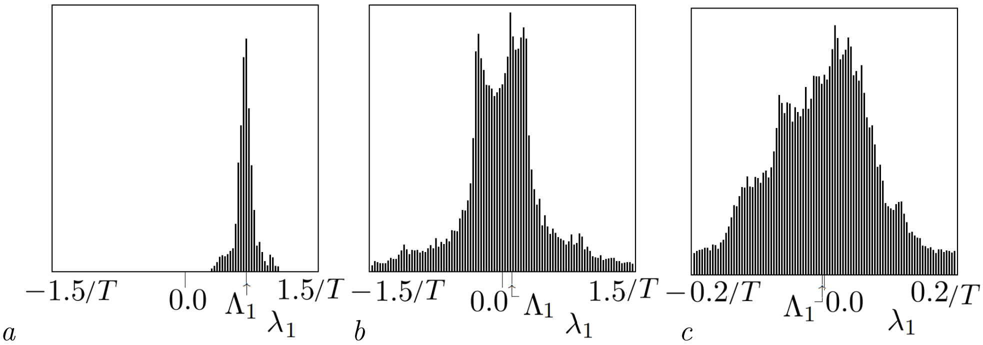

Fig 3. Distribution of the values of the largest finite-time Lyapunov exponent estimated on the period $\tau = 5T$ of the stroboscopic map for the generation associating with the hyperbolic chaotic attractor (a), nonhyperbolic chaotic attractor (b) and strange nonchaotic attractor (c)

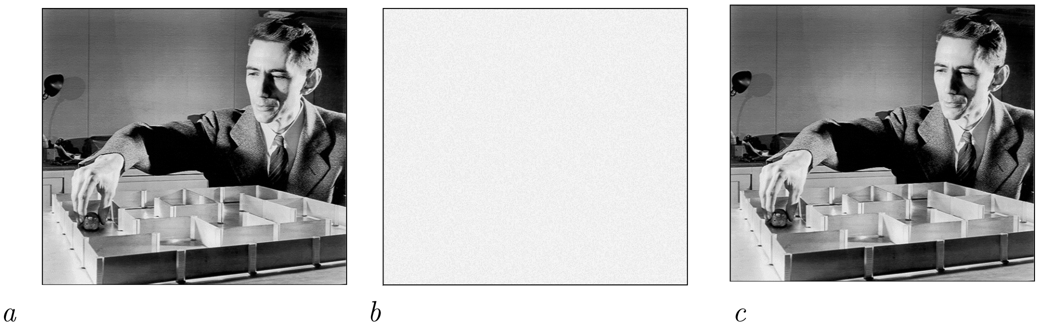

Fig 4. The transmitted informational signal — Claude Shannon photograph with resolution 571 × 630 pixels and "256 gray color tones" codiing (a), the signal in the communication channel after low-frequency filtration (b) and the decoded signal (c). The communication scheme (1–3) is used at the parameter values (7). The transmitter and the receiver are identical $A_1 = A_2$, $\omega_1 = \omega_2$

A convenient demonstration and confirmation of the specific properties of the dynamic modes used by us can be the power spectrum and the distribution of the largest finite-time Lyapunov exponent. (By distribution we mean the probability density - a function characterizing the comparative probability of the realization of certain values of the local Lyapunov exponent.) In Fig. 2 it can be seen that the spectra of all three selected dynamic modes are visually comparable to noise-like. For hyperbolic chaos, the spectrum is closest to the white noise spectrum, for non-hyperbolic chaos, the spectrum looks solid enough, but decreases, for non-chaotic mode, the spectrum may well be singularly continuous (advanced statistical spectral analysis is needed to confirm this fact). All three spectra look quite broadband, which is a useful property for a communication system. The distribution of the values of the local Lyapunov exponent λ_1 for the considered generations also correspond to known patterns (Fig. 3). For hyperbolic chaos, the local indicators are well grouped and are in the positive region. For non-hyperbolic chaos, the distribution has two distinct maxima— in the positive and negative regions. However, the full Lyapunov exponent Λ_1 is positive. The third distribution, with a small (in absolute value) but negative total indicator, captures both negative and positive regions. This corresponds to the presence of an inhomogeneous local 1)instability on the attractor, leading to its fractalization.

2. Demonstration of the functioning of communication schemes

Next, we present a demonstration of numerical modeling of the schemes under consideration. A graphic image was selected as the transmitted information message (Fig. 4, *a*). The information signal was a stepwise function of time. Each step had a length of τ and a height corresponding to one of the 256 grayscale colors of a single pixel in the image. The scope of this step function ranged from 0 to $\pi/2$.

In Fig. 4, *b*, *c* a demonstration of communication via a scheme (1–3) is shown with parameter values (7) and $A_2 = A_1$, $\omega_2 = \omega_1$. The results of the communication schemes with a nonhyperbolic chaotic and a strange nonchaotic carrier do not visually differ and therefore are not given. On the fragment *b* you can see the signal entering the communication channel, on the fragment *c* you can see perfectly detected information. The detection was performed according to the following method:

$$\sin\rho' = \langle 2s(t)\cos(\omega_2 t)/y_2(t)\rangle_\tau, \tag{9}$$

where the angle brackets mean the averaging over the period τ.

This period should be at least several periods of slow modulation of the carrier signal. For the results presented in the paper, $\tau = 5T$. This averaging removes the high-frequency components from the result of multiplying by the cosine of the ratio of the receiver and transmitter variables corresponding to each other $(y_1/y_2)\sin(\omega_1 t + \rho(t))\cos(\omega_2 t)$ for a $s(t)$ signal of the form (2). These variables are the same in case of full synchronization. With matching frequencies $\omega_1 = \omega_2$, according to as a rule, the product of trigonometric functions decomposes into the sum of the high-frequency component and the slowly changing $\sin\rho$: $\sin(\omega_1 t + \rho(t))\cos(\omega_1 t) = (1/2)(\sin\rho + \sin(2\omega_1 t + \rho))$. Thus, for an identical receiver and transmitter, the result is obvious $\sin\rho' = \sin\rho$. Similar reasoning can be done for the case of a signal in a communication channel (5).

Full synchronization with the same parameter values in the communication circuits (1–3) and (4–6) is achieved due to a sufficiently strong one-way communication via the $s(t)$ signal. When the parameters are disrupted, full synchronization is disrupted. As you can see in Fig. 5, this greatly affects the result. When the parameters A_1 and A_2, which are responsible for the amplitude of generation, are detuned, the detected image turns out to be quite noisy by five percent. At the same time, to a greater extent this happens for nonhyperbolic chaos. Setting the frequency parameters of the receiver and transmitter makes it completely impossible to recognize the transmitted message in this case. For a hyperbolic chaotic carrier and a strange nonchaotic carrier, part of the message is approximately restored. This is possible during the time intervals of information transmission, when the oscillators of the transmitter and receiver are pumped in the same phase.

3. An alternative method of information detection

The more successful results of restoring the transmitted information (Fig. 5) in the case of hyperbolic chaotic and strange nonchaotic carrier are explained, in our opinion, by the occurrence of coarse generalized synchronization for such modes. In generalized synchronization, there should be a non-zero correlation between the variables. Its time-local values probably behave more smoothly for coarse hyperbolic and nonchaotic strange coupled systems. Based on this assumption, an alternative method of information detection is proposed

$$\sin\rho' = \langle 2s(t)\cos(\omega_2 t)y_2(t)\rangle_\tau. \tag{10}$$

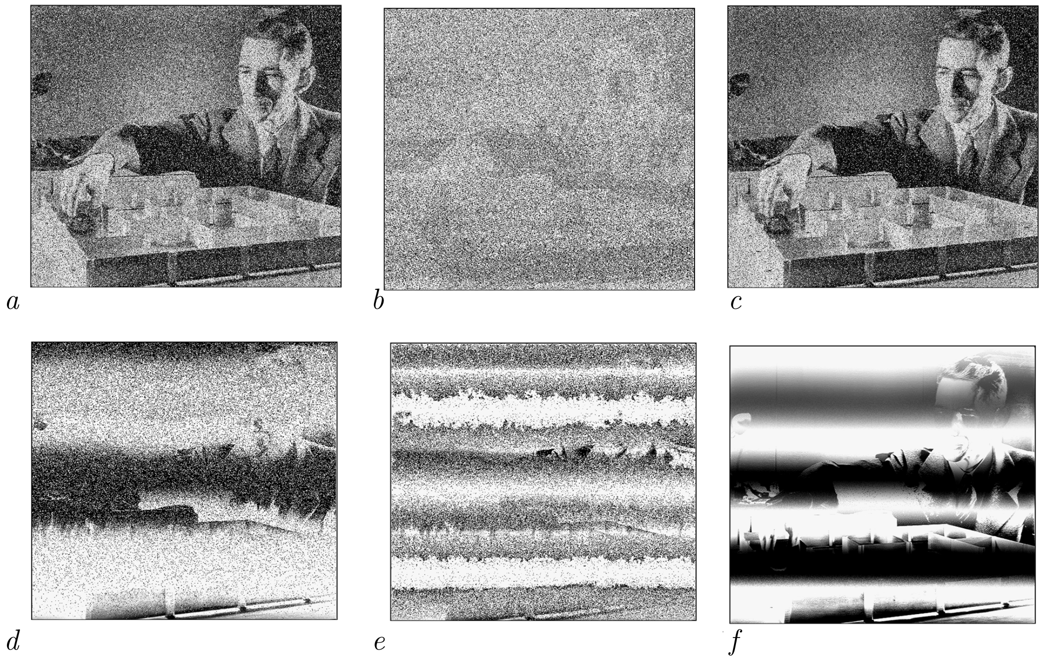

Fig 5. The image detected via the method (9) at nonidentical transmitter and receiver with the detuning $A_2 = 1.05A_1$ (a–c) and $\omega_2 = \omega_1 + 10^{-7}$ (d–f). Figures a, d are for the communication with the hyperbolic chaotic carrier, b, e — nonhyperbolic, c, f — strange nonchaotic

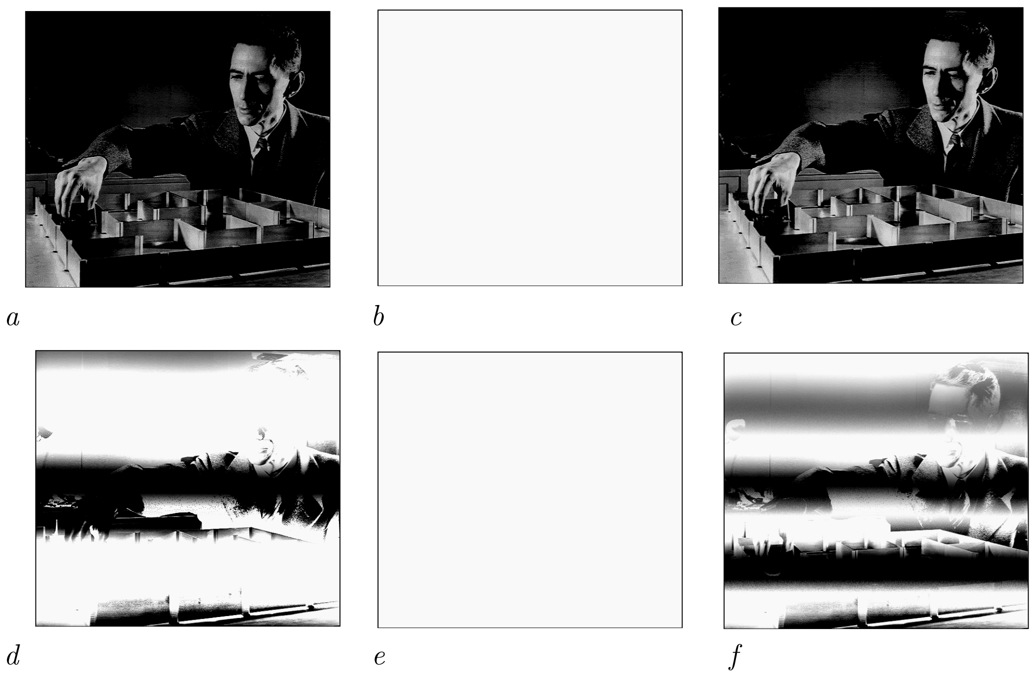

Fig 6. The image detected via the method (10) at nonidentical transmitter and receiver with the detuning $A_2 = 1.05A_1$ (a–c) and $\omega_2 = \omega_1 + 10^{-7}$ (d–f). Figures a, d are for the communication with the hyperbolic chaotic carrier, b, e — nonhyperbolic, c, f — strange nonchaotic

Here, unlike (9), it is not the ratio that is averaged, but the product of the variables y_1 and y_2. Averaging gives a local correlation value.

From the results of alternative detection in Fig. 6, a and c it can be seen that the local correlation $C_\tau(t) = \langle y_1(t) y_2(t) \rangle_\tau$ it is constant in time. Reconstructed image for hyperbolic and nonchaotic cases it is practically not noisy. However, its tone is slightly darker than that of the original image. This is explained by multiplying by the correlation constant: $\sin\rho' = C_\tau \sin\rho$. Images detected using an alternative method also look a little better with frequency detuning. The results of applying the method (10) are radically different for any nonidentical parameters for nonhyperbolic chaos. There is no local correlation, and recovery is absolutely impossible.

Conclusion

The use of a hyperbolic chaos generator and crude strange nonchaotic dynamics opens up new prospects for the development of confidential and broadband communication systems. In recent years, interest in this area of the technical application of dynamic chaos has declined. This is due to the extreme difficulties in achieving complete synchronization between the transmitter and receiver in a real experiment, which is necessary for successful detection of the transmitted information. In this paper, it is shown that coarse systems with complex dynamics can provide coarse generalized synchronization even in the case of nonidentical subsystems. Taking into account this fact makes it possible to successfully isolate the information component from the signal that came through the communication channel.

Communication schemes based on generalized synchronization have already been described in the literature and even embodied in the radio engineering experiment [2, 4]. These circuits work on the principle of switching the transmitter between two states corresponding to different synchronization modes of a receiver that is notidentical to it. Such schemes allow the transmission of only a binary digital signal: “1”, if generalized synchronization takes place; “0” - absent. The transmission demonstrated in this paper is carried out for a digital signal at a speed of 8 bits per count (that is, 8 times faster than in previously known circuits). And, moreover, it can be generalized to completely analog information. The advantage of the proposed scheme is the absence of transients that occur when switching the transmitter between two states. Taking into account these transients slows down transmission and complicates communication schemes in [2, 4].

References

1. Dmitriev AS, Panas AI. Dynamical Chaos: New Information Carriers for Communication Systems. Moscow: Fizmatlit; 2002. 252 p. (in Russian).
2. Koronovskii AA, Moskalenko OI, Hramov AE. On the use of chaotic synchronization for secure communication. Physics-Uspekhi. 2009;52(12):1213–1238. DOI: 10.3367/UFNe.0179.200912c.1281.
3. Pikovsky A, Rosenblum M, Kurths J. Synchronization: A Universal Concept in Nonlinear Sciences. New York: Cambridge University Press; 2001. 432 p. DOI: 10.1017/CBO9780511755743.
4. Prokhorov MD, Ponomarenko VI, Kulminskiy DD, Koronovskii AA, Moskalenko OI, Hramov AE. Resistant to noise chaotic communication scheme exploiting the regime of generalized synchronization. Nonlinear Dynamics. 2017;87(3):2039–2050. DOI: 10.1007/s11071-016-3174-6.
5. Kuznetsov SP. Example of a physical system with a hyperbolic attractor of the Smale-Williams type. Phys. Rev. Lett. 2005;95(14):144101. DOI: 10.1103/PhysRevLett.95.144101.
6. Jalnine AY, Kuznetsov SP. On the realization of the Hunt-Ott strange nonchaotic attractor in a physical system. Tech. Phys. 2007;52(4):401–408. DOI: 10.1134/S1063784207040020.
7. Kuznetsov SP. Hyperbolic Chaos: A Physicist’s View. Berlin, Heidelberg: Springer; 2012. 320 p. DOI: 10.1007/978-3-642-23666-2.

8. Kuptsov PV, Kuznetsov SP. Transition to a synchronous chaos regime in a system of coupled non-autonomous oscillators presented in terms of amplitude equations. Russian Journal of Nonlinear Dynamics. 2006;2(3):307–331 (in Russian). DOI: 10.20537/nd0603005.
9. Pikovsky AS. Synchronization of oscillators with hyperbolic chaotic phases. Izvestiya VUZ. Applied Nonlinear Dynamics. 2021;29(1):78–87. DOI: 10.18500/0869-6632-2021-29-1-78-87.
10. Isaeva OB, Jalnine AY, Kuznetsov SP. Chaotic communication with robust hyperbolic transmitter and receiver. In: 2017 Progress In Electromagnetics Research Symposium - Spring (PIERS). 22-25 May 2017, St. Petersburg, Russia. IEEE; 2017. P. 3129–3136. DOI: 10.1109/PIERS.2017.8262295.
11. Feudel U, Kuznetsov S, Pikovsky A. Strange Nonchaotic Attractors: Dynamics between Order and Chaos in Quasiperiodically Forced Systems. Singapore: World Scientific; 2006. 228 p. DOI: 10.1142/6006.
12. Ramaswamy R. Synchronization of strange nonchaotic attractors. Phys. Rev. E. 1997;56(6): 7294–7296. DOI: 10.1103/PhysRevE.56.7294.
13. Zhou CS, Chen TL. Robust communication via synchronization between nonchaotic strange attractors. Europhys. Lett. 1997;38(4):261–265. DOI: 10.1209/epl/i1997-00235-7.
14. Rizwana R, Raja Mohamed I. Applicability of strange nonchaotic Wien-bridge oscillators for secure communication. Pramana. 2018;91(1):10. DOI: 10.1007/s12043-018-1582-5.
15. Volkovskii AR, Rulkov NF. Synchronous chaotic response of a nonlinear oscillator system as a principle for the detection of the information component of chaos. Tech. Phys. Lett. 1993; 19(2):97–99.
16. Behnia S, Akhshani A, Mahmodi H, Akhavan A. A novel algorithm for image encryption based on mixture of chaotic maps. Chaos, Solitons & Fractals. 2008;35(2):408–419. DOI: 10.1016/j.chaos.2006.05.011.
17. Jalnine AY. A new information transfer scheme based on phase modulation of a carrier chaotic signal. Izvestiya VUZ. Applied Nonlinear Dynamics. 2014;22(5):3–12 (in Russian). DOI: 10.18500/0869-6632-2014-22-5-3-12.