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Article

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## Propagation of spin waves in a lattice of laterally and vertically coupled YIG microwaveguides by changing the magnetization angle in linear and nonlinear modes

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**Abstract.** *Purpose.* Investigation of the joint manifestation of the effects of anisotropic signal propagation, coupling, and nonlinear power dependence of the medium parameters in a lattice of laterally and vertically coupled spin-wave (SW) microwaveguides. Consideration of the case of the influence of the rotation of the magnetization angle and the change of the lateral gap between microwaveguides located on the same substrate on the transverse profile of the spin-wave beam and the spatial localization of the SW amplitude. *Methods.* The method of micromagnetic modeling based on the numerical solution of the Landau–Lifshitz–Hilbert equation shows the possibility of controlling the direction of propagation of SW in an ensemble of laterally and vertically coupled iron yttrium garnet (YIG) microwaveguides by changing the magnetization angle. By the method of numerical integration of the system of coupled discrete nonlinear Schrödinger equations, the possibility of changing the transverse profile of the spin-wave beam by changing the level of the initial signal amplitude is shown. *Results.* The spatial distributions of the components of the dynamic magnetization of the SW excited in two microwaveguides located on the same substrate obtained in micromagnetic simulations indicate a change in the character of localization of the SW power in the output sections of the microwaveguides. At variation of the lattice magnetization angle, a shift of the threshold power value is observed, at which a characteristic curbing of the transverse width of the spin-wave beam in the nonlinear mode appears. *Conclusion.* When excitation of surface magnetostatic SW in a lattice of laterally and vertically coupled microwaveguides, a transformation of the transverse profile of the wave is observed at a deviation of the magnetization angle of the structure by  $15^\circ$ , which is manifested in the change of the SW length and its localization in each of the microwaveguides. The combined effects of dipole coupling, gyrotropy, and nonlinearity of the medium make it possible to control the value of the threshold power of the SW, at which the mode of diffractionless propagation of the spin-wave beam is realized in a single layer of the structure.

**Keywords:** spin wave, micromagnetic modeling, system of coupled wave equations, spin-wave beam, discrete diffraction.

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## Introduction

Currently, methods for constructing information signal processing systems based on the effects of transfer of magnetic moments or electron spins without charge transfer are being actively studied [1]. In such devices, based on magnon principles, the information signal is encoded in the phase or amplitude of spin waves (SW), and logical operations are implemented based on the principles of spin-wave interference [2]. Finite-width planar ferrite waveguide microstructures based on yttrium iron garnet (Iron-Yttrium-Garnet — YIG) films can be used as basic elements of magnonic networks (MN) to create various signal processing devices: delay lines, filters, interferometers, switches and multiplexers [3–6].

Magnonic networks consisting of spin-wave microwave guides based on YIG films can be used for information processing and at the same time provide technological integration with the existing architecture based on complementary metal-oxide-semiconductor technologies [7]. The simplest element of magnonic networks can be a strip of ferromagnet limited in two directions and representing a waveguide channel for a spin wave. The use of YIG in the creation of spin-waveguide structures is due to the record low attenuation of the spin wave [2]. It was experimentally demonstrated that the creation of multilayer topologies of three-dimensional structures with breaking of translational symmetry allows considering the created elements as interconnect nodes for vertically integrated MN [9]. The operation of such magnonic elements as a spin-wave signal coupler based on laterally [3, 10] or vertically [11, 12] coupled microwaves can be based on nonlinear propagation modes of spin waves [13, 14], implemented by changing the saturation magnetization of YIG with an increase in the input signal level, and a subsequent change in the coupling length of the spin wave [3]. In this case, two types of coupled magnonic waveguide structures were investigated: multilayer structures [15, 16] and micro- and nanoscale waveguides located in planar geometry [3, 7].

In this paper, we consider methods of dual control of spin-wave signal characteristics based on changing the amplitude of spin waves and the magnetization angle of a two-layer structure, in each layer of which waveguide channels are formed. Using micromagnetic modeling, we investigated the modes of spin wave propagation in a YIG microwave guide array. The study of nonlinear effects is carried out based on a system of discrete coupled nonlinear Schrödinger equations. Using a comparison of the maps of spatial distribution of magnetization in a two-layer structure and in a lattice consisting of two layers, each of which has a lattice of lateral microwave guides, obtained in micromagnetic modeling, we show the possibility of controlling the transverse structure of the spin wave field. At the same time, the modes of control of the spin-wave signal transmission with a change in the magnetization angle of the structure are revealed. The considered structure based on a lattice of YIG microwave guides can be used to create systems for multichannel demultiplexing of an information signal.

## 1. Structure

Micromagnetic modeling [17] was carried out for an array of laterally and vertically coupled ferrite microwave guides (Fig. 1). YIG was chosen as the material. The microwave guides are made in the form of elongated strips with a length of  $L = 4$  mm, a width of  $c = 300$   $\mu\text{m}$  and a thickness of  $d = 10$   $\mu\text{m}$ . The structure is two layers, each containing six parallel-oriented microwave guides separated by an air gap. The saturation magnetization of YIG is  $M = 139$  G. The magnitude of the external magnetic field directed along the  $y$  axis is  $H_0 = 1200$  Oe. The numerical study was carried out at a frequency of 5.21 GHz for all cases considered in this paper. Moreover, this magnetization configuration provides efficient excitation of surface

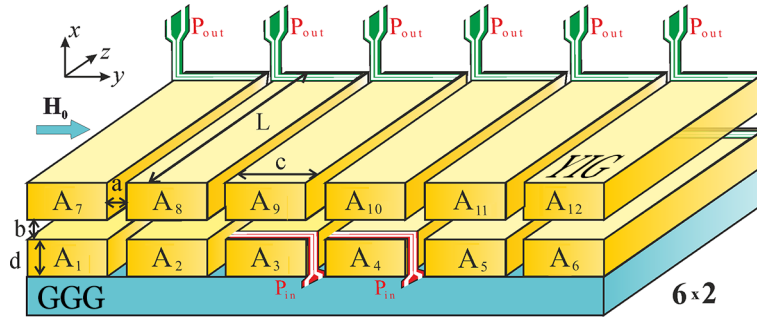


Fig 1. Schematic diagram of the microwaveguides array. The following notations are introduced in the figure:  $a$  — horizontal gap;  $b$  — vertical gap;  $c$  — width;  $d$  — thickness;  $L$  — length of microwaveguide;  $P_{in}$  and  $P_{out}$  — microstrip antennas for SW excitation and reception respectively (color online)

spin waves. Coupled ferrite structures expand the functionality of microwave devices due to an additional control parameter, which is the coupling between microwaves propagating in individual ferromagnetic films [13–16, 18–20]. In the experimental study of such structures, YIG waveguides are formed on a gadolinium-gallium garnet (GGG) substrate [10].

## 2. Micromagnetic modeling

Let us consider the principle of operation of the structure under study: a microwave signal is fed to the input antenna  $P_{in}$ , the frequency range of which depends on the magnitude of the constant external magnetic field. As the spin waves propagate, the spin-wave signal will be pumped into all microwave guides depending on the deflection angle of the external magnetic field, the frequency and phase difference of the microwave signal fed to the input antennas. Using micromagnetic modeling based on solving the Landau–Lifshitz–Gilbert (LLG) equation [17], a study was made of the propagation modes of spin waves in the YIG microwave guide array. To reduce signal reflections from the boundaries of the computational domain, regions with an exponentially increasing attenuation parameter were introduced in the numerical modeling. The initial value of the attenuation parameter was chosen to be  $\alpha = 10^{-5}$ , which corresponded to the attenuation parameter value for epitaxially grown YIG/GGG films [2]. The structure was excited by creating a localized region with an alternating external magnetic field on two central microwave guides in one of the layers. Due to the coupling between the spin waves propagating in individual layers and channels of the bilayer structure, the dynamic properties of the wave processes change. This is taken into account in the numerical simulation when calculating the magnitude of the dynamic demagnetization field. The numerical model of the structure formed by two lateral arrays of magnetic microwave guides located one above the other leads to a non-uniform distribution of the internal magnetic field along the width of the films.

In Fig. 2,  $a, b$  the source of spin waves is shown by black rectangles located in the left part of microwave guides  $A_3$  and  $A_4$ . The length of these microwave guides was chosen to be 6 mm, which is 2 mm longer than the length of the other waveguides in the array. Propagation in the waveguides of the lower layer  $A_1$ – $A_6$  is shown in Fig. 2,  $a, c$ , and in the waveguides of the upper layer  $A_7$ – $A_{12}$  — in Fig. 2,  $b, d$ . This excitation method allows one to implement the excitation of both symmetric and antisymmetric modes of a coupled structure consisting of two coupled microwave guides. In this paper, we considered the case of excitation of the symmetric mode [10]. We investigated the dynamics of propagation in a lattice of laterally and vertically coupled YIG microwave guides by constructing maps of the spatial distribution of the dynamic

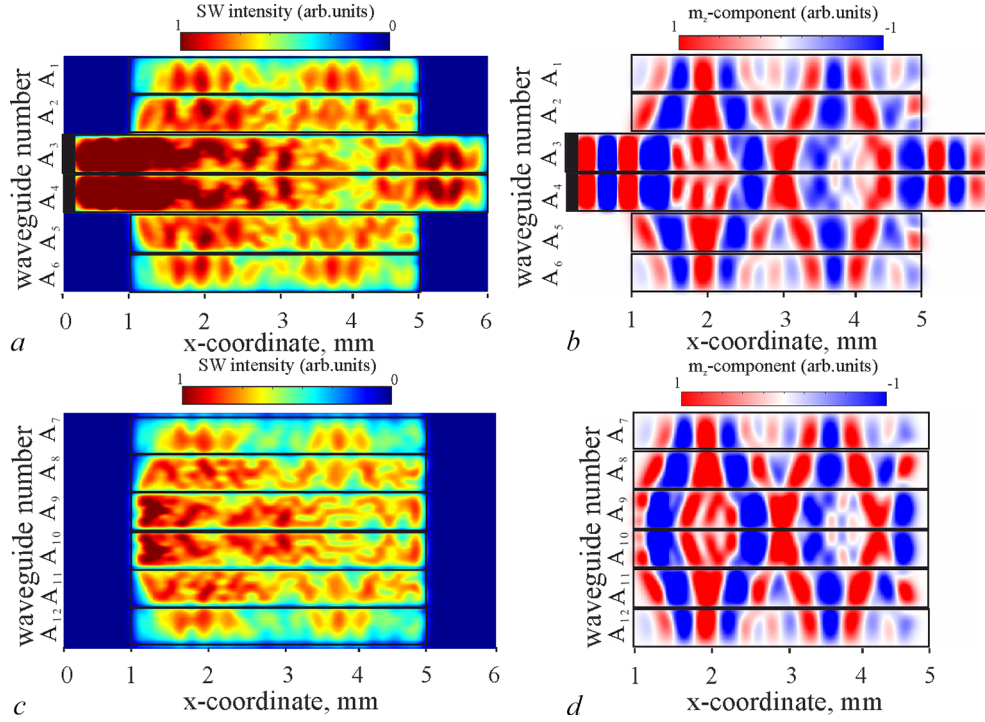


Fig 2. *a, c* – Spatial distribution of SW intensity, where the color gradations encode the SW intensity  $I(x, y)$  at the frequency 5.21 GHz; *b, d* – spatial maps distribution of the  $m_z$  component of the dynamic magnetisation for the SW propagating in the microwaveguides lattice, where the lateral and vertical gaps are 10  $\mu\text{m}$  (color online)

magnetization  $m_z(x, y)$  and the spin wave intensity. The spin wave intensity is understood as  $I(x, y) = \sqrt{m_x^2(x, y) + m_z^2(x, y)}$ .

When the lateral and vertical gaps are 10  $\mu\text{m}$ , the spin wave intensity has a maximum in two central channels. In this case, at  $x = 4$  mm, intensity redistribution is observed. The spin wave power is transferred from two central channels  $A_{3,4}$  to the lateral channels  $A_{1,2,5,6}$ . Then, in the first layer of the structure, in which the excited waveguides are located, localization of the spin wave power is observed in the region of the output sections of the central channels  $A_{3,4}$  (Fig. 2). It is seen that in the input and output regions of the central channels  $A_{3,4}$ , the spin wave length is 800  $\mu\text{m}$ . This corresponds to the calculation of the dispersion of the symmetric mode of the lateral structure with these geometric dimensions [10]. Fig. 2 shows a steady-state picture of the establishment of a wave process, in which propagation occurs both into neighboring microwave guides within one layer and in the vertical direction. In the upper layer, the spatial distribution of the  $m_z$  component of the dynamic magnetization and the intensity of the spin wave differ from those in the lower layer, namely, the minimum localization in the  $A_{3,4}$  channels of the lower layer is observed at  $x = 4.05$  mm, while in the upper layer – at  $x = 3.9$  mm.

When comparing the results of numerical simulation for two YIG layers separated by an air gap (Fig. 3) and for a 2x6 microwave array (Fig. 2), it is evident that the effect of the non-uniform distribution of the internal magnetic field in the microwave array leads to a decrease in the wavelength by 1.4 times compared to the two-layer structure. When a spin wave propagates in the central part of the two-layer film, the structure of the wave field is due to the interference of transverse spin-wave modes. In the cross section for  $1 < x < 5$  mm, a magnetization profile is observed, formed by an integer number of half-wave lengths  $3\Lambda/2$ , where  $\Lambda = 2\pi/k_y$ . This is caused by the interference of the first and third width modes with transverse wave numbers  $k_y^I = \pi/(6c)$  and  $k_y^{III} = \pi/(2c)$  and longitudinal wave numbers

$k_x^I = k_x^I(f)$  and  $k_x^{III} = k_x^{III}(f)$  [22, 23]. The field amplitude can be described by the relation  $A(x, y) = a_I \Phi_I(y) \exp(-ik_x^I x) + a_{III} \Phi_{III}(y) \exp(-ik_x^{III} x)$ , where the functions  $\Phi_{I,III}(y)$  describe the transverse profile of the first and third width modes. In the case of an array of coupled microwave guides separated by a lateral gap, it is possible to observe the formation of a periodic signal pumping in each individual waveguide along the direction of wave propagation. In this case, the maximum intensity in the lower layer is located on the line  $x = 3.1$  mm, and in the upper layer the maximum spin wave intensity occurs at  $x = 2.75$  mm.

In the case of a coupled microwave array, the signal propagation modes can be investigated at different magnetization angles using micromagnetic modeling. Fig. 4 compares two configurations: two YIG layers separated by an air gap and a 2x6 microwave array with a magnetization angle of the structure that deviates by  $15^\circ$  relative to the y-axis direction. For comparison, in the steady-state mode of spin wave propagation at  $0^\circ$ , the spin wave length changes and is localized in each of the microwaves. As a result, the transverse structure of the beam is transformed and a wave process (spin wave phase transfer) is observed inside each of the films in the array. This method of changing the spatial profile of the spin wave beam in a coupled microwave array makes it possible to implement the signal control mode at the output of channels  $A_{3,4}$ . The maximum signal amplitude can be divided between the output sections of channels  $A_3$  or  $A_4$  (Fig. 4, e). In this case, in the upper layer, the signal amplitudes in sections  $A_7$  and  $A_{12}$  are the same at  $x = 5$  mm. To explain the observed behavior of the spin wave with a change in the magnetization angle, it can be noted that the dispersion law  $k_x^I = k_x^I(f)$ ,  $k_x^{III} = k_x^{III}(f)$  changes with a change in the orientation of the magnetic field vector [2, 23]. This leads to a change in the result of the interference of transverse modes of spin waves in the region of  $1 < x < 5$  mm, which manifests itself in the zigzag shape of the dynamic magnetization profile shown in Fig. 4, a-d. In the case of a microwave guide array (Fig. 4, e-h), not only the dispersion of the spin wave in a single

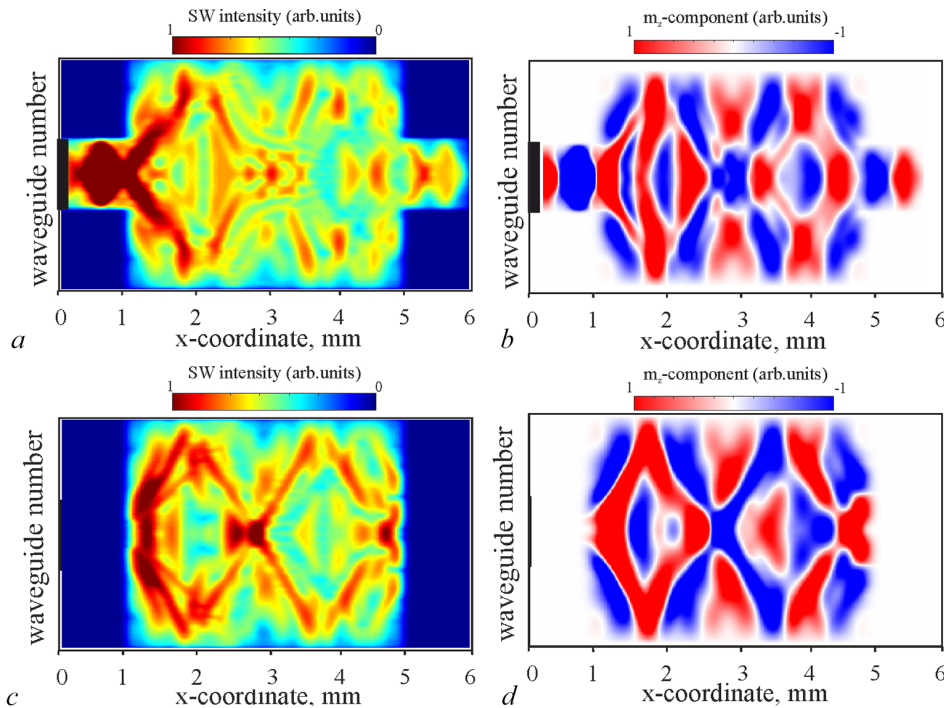


Fig 3. a, c — Spatial distribution of the SW intensity; b, d — spatial maps distribution of the  $m_z$  component of the dynamic magnetisation for the SW propagating in a bilayer structure where the vertical gap is 10  $\mu\text{m}$  (color online)

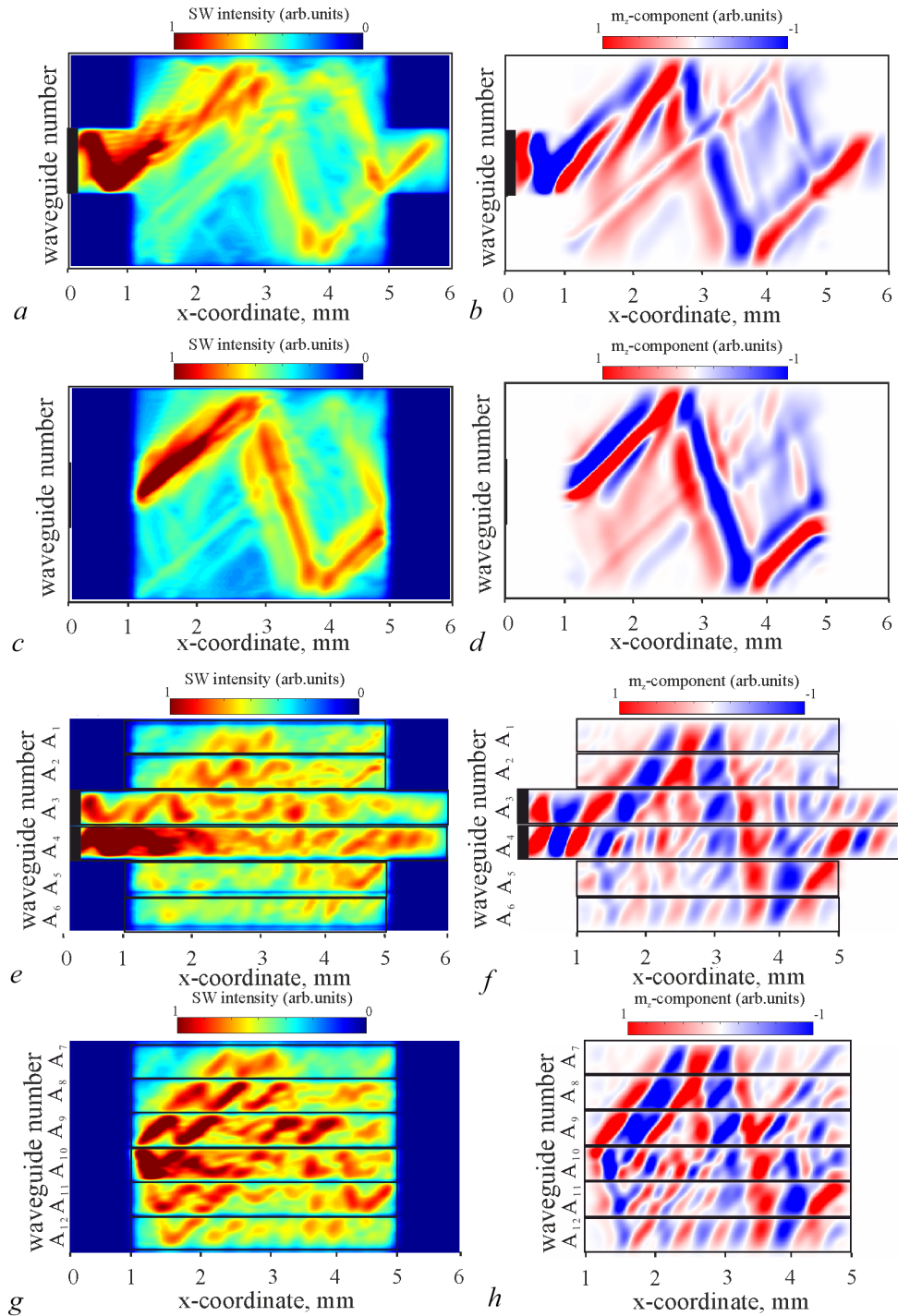


Fig 4. *a, c, e, g* – spatial distribution of the SW intensity; *b, d, f, h* – spatial maps distribution of the  $m_z$  component of the dynamic magnetisation for the SW (color online)

waveguide changes, but also the magnitude of the coupling of spin waves, determined by the overlap integral of the eigenwaves of microwave guides located next to each other.

### 3. Numerical model based on coupled discrete nonlinear Schrödinger equations

To describe the processes of nonlinear propagation of a spin wave in a microwave array, a numerical model is constructed based on coupled discrete nonlinear Schrödinger equations [25,26]:

$$i \frac{dA_{mn}}{dz} + \beta(\varphi)A_{mn} - C_L(\varphi) (A_{m(n+1)} + A_{m(n-1)}) + C_V(\varphi) (A_{(m+1)n} + A_{(m-1)n}) + \Gamma(\varphi)|A_{mn}|^2 A_{mn} = 0, \quad (1)$$

where  $A_{mn}$  is the signal amplitude in the  $n$ -th microwave guide of the  $m$ -th layer,  $\beta(\varphi)$  is the dispersion coefficient of a single microwave guide,  $C_L(\varphi)$  is the coupling coefficient between microwave guides in the lateral (horizontal) direction,  $C_V(\varphi)$  is the coupling coefficient between microwave guides in the vertical direction,  $\Gamma(\varphi)$  is the nonlinearity coefficient. In this case, one equation is written for each waveguide, i.e. a total of 12 coupled equations.

To obtain the dispersion coefficient  $\beta(\varphi)$ , the spectrum of eigenmodes of a single microwave guide was calculated and the dispersion of its first eigenmode was constructed. The calculation of this characteristic was performed in the article [10] using the finite element method. In this case,  $\beta(\varphi)$  has an explicit dependence on the magnetization angle  $\varphi$  in the range  $0 \dots 15^\circ$  and leads to an increase in the internal magnetic field in the YIG microwave guide [27].

Calculation of the coupling coefficients  $C_L(\varphi)$  and  $C_V(\varphi)$  also comes down to searching for eigenmodes, but for a system of two microwave guides having a dipole coupling in the lateral direction ( $C_L(\varphi)$ ) [10] and for a system consisting of two vertically coupled microwave guides [11, 28]. These coefficients have the following form:

$$C_{L,V}(\varphi) = \frac{\pi}{2L_h(\varphi)}, \quad (2)$$

$$L_h(\varphi) = \frac{\pi}{\Delta k} = \frac{\pi}{|k_s - k_{as}|}, \quad (3)$$

where  $k_s$  and  $k_{as}$  are the wave numbers for the symmetric and antisymmetric modes in coupled microwaves. In this case, the rotation of the external magnetic field by  $\varphi$  in the range of  $0 \dots 15^\circ$ , just like in the case of a single waveguide, will lead to an increase in the internal magnetic field in YIG microwaves and change the values of  $k_s$  and  $k_{as}$  [22, 23]. The nonlinearity coefficient  $\Gamma(\varphi)$  is expressed as [29]

$$\Gamma(\varphi) = \frac{d\beta(\varphi)}{d|A_{m,n}|^2}. \quad (4)$$

The calculation of the derivative is carried out analytically, by searching for the roots of the dispersion equation for an infinite tangentially magnetized ferrite film [30]

$$f^2 - \left( f_H(\varphi) + \frac{f_M}{2} \right)^2 = - \left( \frac{f_M}{2} \right)^2 e^{-2|\beta|t}, \quad (5)$$

where  $t$  is the film thickness,  $f_M = \gamma 4\pi M$ ,  $f_H = \gamma H_0$ ,  $\gamma = 2.8$  MHz/Oe is the gyromagnetic ratio. Under the condition  $f_M = f_{M_0} \{ |(A_{m,n})^2| \}$ , the following form of the nonlinearity coefficient for the MSSW is obtained:

$$\Gamma(\varphi) = \frac{2 (2f^2 - 2f_H^2(\varphi) - f_H(\varphi)f_M(|A_{m,n}|^2))}{\left( 4f^2 - 4f_H^2(\varphi) - 4f_H(\varphi)f_M|A_{m,n}|^2 - f_M^2|A_{m,n}|^2 \right) t}. \quad (6)$$

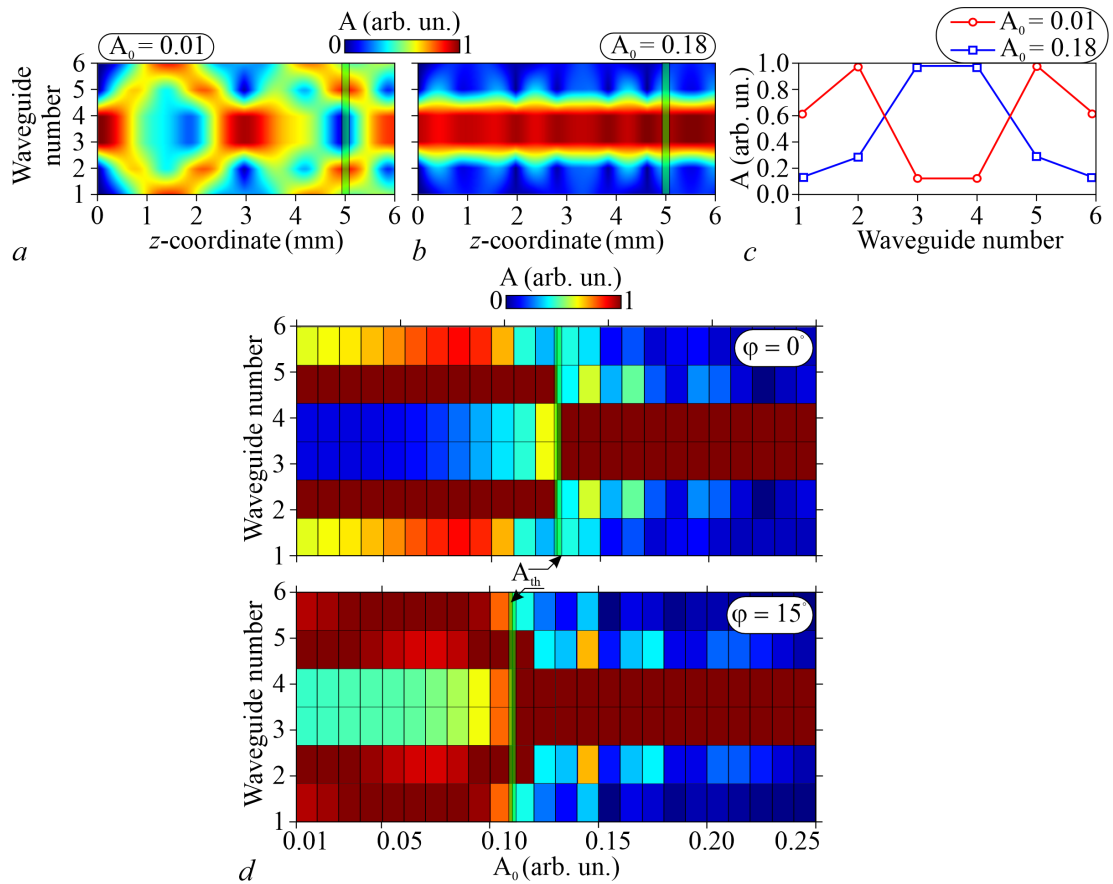


Fig 5. *a* – Spatial distribution of the SW amplitude propagating in the microwaveguides grating at the initial amplitude of the SW excitation  $A_0 = 0.01$  (*a*);  $0.18$  (*b*). *c* – Distribution of the SW amplitude along the  $y$ -axis of the magnetic microwaveguides grating in the section  $z = 5$  mm for  $A_0 = 0.01$  (red circles) and  $A_0 = 0.18$  (blue squares). *d* – Spatial maps of the SW amplitude propagating in the microwaveguides grating in the  $z = 5$  mm section as a function of the initial SW excitation amplitude for different values of the bias angle  $\varphi$  (the values of  $\varphi$  are given in the figure) (color online)

Figures 5, *a*, *b* show the spin wave intensity distribution in the YIG microwave guide array under consideration for different initial excitation amplitudes. In the case of linear spin wave excitation ( $A_0 = 0.01$ ) (Figure 5, *a*), as in the case of micromagnetic modeling, spin wave transfer between microwave guides is observed. Spin wave focusing is observed in the central region at a distance of 2.5 to 3.5 mm. With an increase in the initial excitation amplitude  $A_0 = 0.18$  (Figure 5, *b*), an increase in the spin wave intensity is observed in the region of the central microwave guides as a result of an increase in the spin wave coupling length. In this case, the spin wave self-focusing mode is observed in the array. This regime can be observed in Fig. 5, *c* when deriving the amplitude of the spin wave along the width of the microwave guides in the cross-section  $z = 5$  mm (green line in Fig. 5, *a*, *b*). In the case of  $A_0 = 0.18$  (blue squares), the maximum intensity is observed in the region of the central microwave guides, in contrast to the case of  $A_0 = 0.01$  (red circles).

The model based on coupled nonlinear Schrödinger equations allows us to study the effect of rotation of the magnetization angle  $\varphi$ , since the coefficients  $\beta$ ,  $C_{L,V}$  and  $\Gamma$  depend on the magnitude of the internal magnetic field and on  $\varphi$ . Figure 5, *d* shows the transverse distributions of the spin wave intensity in the cross section  $z = 5$  mm with an increase in the initial excitation



amplitude  $A_0$  (abscissa axis) for two values of the magnetization angle  $\varphi$ . In the case of  $\varphi = 0^\circ$  (upper map in Fig. 5, *d*) at  $A_0 = 0.14$  the nonlinear switching regime is observed in the microwave array. This value can be called the threshold value of the amplitude  $A_{th}$ , at which narrowing of the beam is observed. When the angle is rotated relative to the  $y$  axis direction ( $\varphi = 15^\circ$ ), the nonlinear switching threshold decreases to  $A_{th} = 0.11$  (lower map in Fig. 5, *d*). Thus, it becomes possible to control the spatial distribution of the spin wave intensity in the magnetic microwave array by simultaneously changing the input signal power and the orientation of the external magnetic field.

## Conclusion

In this paper, we study the dual control of spin wave characteristics in a YIG microwave guide array with simultaneous lateral and vertical coupling. We study the features of spin wave beam formation processes for a two-layer structure and an array of coupled magnetic waveguides. We reveal the features of propagation and the mechanisms of change in the spatial distribution of the spin-wave beam profile with the combined effects of anisotropic signal propagation, dipole coupling, and nonlinear dependence of the medium parameters on the power. Based on the obtained spatial distributions of the components of the dynamic magnetization of the spin wave, we demonstrate a change in the nature of the spin wave power localization in the linear and nonlinear modes in the output sections of the microwave guides. It is shown that the deviation of the magnetization angle from the direction at which the surface magnetostatic wave is excited in the central channels leads to a transformation of the transverse structure of the spin-wave beam, allowing one to change the position of the spatial localization of the spin wave power. The possibility of changing the transverse profile of the spin-wave beam with a change in the level of the initial signal amplitude is shown. In this case, when the magnetization angle of the array is varied, a shift in the threshold power value is observed, at which a characteristic narrowing of the spin-wave beam appears. Based on the results of micromagnetic modeling and calculations using model systems of discrete coupled equations, it can be concluded that the selected geometric dimensions of the structure based on magnonic microwave guides correspond to the filtering modes and the possibility of spatial-frequency demultiplexing of the signal encoded as the amplitude and phase of the spin wave. The array of laterally and vertically coupled microwave guides is an interconnection element for three-dimensional magnonic network topologies [31], demonstrating the functionality of signal processing in various applications related to magnetic microelectronics and spintronics.

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