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Influence of nonlinearity on the Bragg resonances in coupled magnon crystals

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Abstract. *Purpose.* The purpose of this paper is to investigate the effect of nonlinearity on formation mechanism and characteristics of Bragg resonances in vertically coupled magnon crystals with periodic groove system on the surface. In this paper a wave model is constructed, a nonlinear dispersion relation for surface magnetostatic waves in such a structure is obtained and the characteristics of each of the Bragg resonances are numerically studied with increasing input signal power. *Methods.* Theoretical methods of investigation of spin-wave excitations in a wide class of structures with ferromagnetic layers have been used. In particular, the following theoretical models have been used: coupled wave method, long-wave approximation. *Results.* This paper presents the results of a theoretical investigation of the effect of magnetic nonlinearity on Bragg resonances in a sandwich structure based on magnon crystals with periodic grooves on the surface separated by a dielectric layer. A mechanism for the formation of band gaps at the Bragg resonance frequencies in the presence of media nonlinearity has been revealed. It is shown that with increasing input power the frequency interval between the band gaps decreases. With increasing magnetization difference of magnon crystals, the effect of nonlinear convergence is more pronounced. *Conclusion.* The identified features extend the capabilities of sandwich structures based on magnon crystals for frequency selective signal processing by controlling the frequency selectivity, both via static coupling parameters, periodicity and layer magnetisation, and dynamically via the input signal power.

Keywords: ferromagnetic film, magnetostatic wave, magnon crystal, Bragg resonance, band gap.

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Introduction

In recent years, magnonics has attracted widespread attention from researchers due to the potential for use in a new generation of information processing devices in which information will be transmitted by magnons or spin waves [1–3]. One of the main structures proposed for the creation of a magnon component base are magnon crystals (MC) — periodic structures formed on the basis of magnetic materials [4, 5]. The use of magnonic crystals creates conditions for the formation of Bragg resonances in the spectrum of magnetostatic waves for wave numbers $k_B = \pi/L$ (L— period of the structure) satisfying the Bragg condition [6]. At the frequencies of Bragg resonances, forbidden zones are formed - non-transmission bands in the spectrum of magnetostatic waves. In this case, the task of controlling the characteristics of resonances (resonant frequency, Q-factor, number of resonances in each Brillouin zone) is important.

A ferromagnetic medium is a medium with a Kerr type of cubic nonlinearity [7–9]. Nonlinear processes in ferromagnetic structures lead to a number of new effects in the propagation of magnetostatic waves, which open up wide possibilities for using such structures for functional signal processing depending on the input power [10,11]. It is shown that one of the main nonlinear effects in the MC in the field of four-magnon decay processes is the nonlinear shift of the forbidden zones [12]. In particular, a nonlinear shift of the forbidden zones by 5-15 MHz to the low-frequency region was experimentally observed in the MC based on films of yttium-iron garnet (YIG) with an increase in the input signal power to 80 MW. This feature makes it possible to consider a waveguide structure based on MC as a nonlinear phase shifter and a signal-to-noise ratio amplifier.

The need to combine individual magnetic signal processing elements into magnon networks draws attention to the creation of interconnected structures based on ferromagnetic films. For this purpose, structures based on MC located in planar geometry (laterally connected MC) were proposed [13–16] and in vertical geometry (vertically connected MC) [11, 17–19]. In particular, the possibility of switching between the output ports of the coupled structure was demonstrated with an increase in the input power for a signal frequency that coincides with the frequency of the Bragg resonance [11]. This feature allows us to consider such structures for the implementation of the functions of a nonlinear power divider with four output ports.

However, it was shown that in the linear case, the formation of several Bragg resonances in the first Brillouin zone [20, 21] takes place in the structure of bound MC. This feature is due to the interaction of symmetric and antisymmetric direct and reflected normal waves of a coupled periodic structure at phase synchronism frequencies. In this case, the characteristics and the number of resonances depend on the type of asymmetry of the associated structure [20, 22]. It should be expected that the influence of the nonlinearity of the ferromagnetic medium on each of the resonances will be different. The purpose of this paper is to investigate the effect of nonlinearity on formation mechanism and characteristics of Bragg resonances in vertically coupled magnon crystals with periodic groove system on the surface. In this paper a wave model is constructed, a nonlinear dispersion relation for surface magnetostatic waves in such a structure is obtained and the characteristics of each of the Bragg resonances are numerically studied with increasing input signal power.

1. The model and the main relations

Let us consider a periodic structure, the diagram of which is shown in Fig. 1, *a* in the form of two coupled one-dimensional crystals MC-2 and MC-1, separated by a dielectric layer of thickness D. MC-1 and MC-2 are ferromagnetic films of thickness a with saturation magnetization $M_{01,02}$. A periodic structure in the form of grooves of depth Δ and width c, with period L is applied to one of the surfaces of MC-1 and MC-2. It is assumed that the structure is infinite in the direction of the x and y axes. The connection between MC-1 and MC-2 is realized through high-frequency magnetic fields. The system is placed in an external magnetic field H_0, directed along the x axis. In this case, in MC-1 and MC-2, magnetostatic surface waves (MSSW) propagate along the y

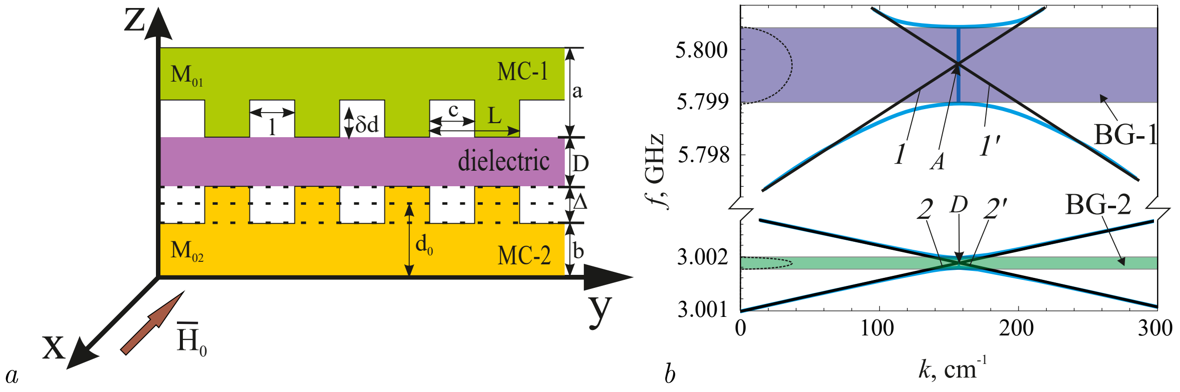

Fig 1. *a* — Layered structure scheme as MC-1 and MC-2 separated by a dielectric layer. *b* — The dispersion characteristics of the MSSW in the linear case ($u = 10^{-5}$) at $\delta d_{1,2} \neq 0$ (solid blue lines) and $\delta d = 0$ for direct waves (lines *1* and *2*) and reflected waves (lines *1′* and *2′*). Areas of band gaps BG are highlighted by a filler. Calculated parameter: $L = 200$ μm, $a = 10$ μm, $c = L/2$, $\Delta = 1$ μm, $D = 25$ μm, $M_{01} = 260$ G, $M_{02} = 60$ G, $H_0 = 800$ Oe (color online)

axis.

The dispersion relation for the MSSW propagating in the structure under study has the form [19, 22]:

$$\begin{vmatrix} D_1^+ & \beta_1 k_0 K & \theta_1^- & \theta_1^- K \\ \beta_2 k_0 K & D_2^+ & \theta_2^- K & \theta_2^- \\ \theta_1^+ & \theta_1^+ K & D_1^- & \beta_1 k_{-1} K \\ \theta_2^+ K & \theta_2^+ & \beta_2 k_{-1} K & D_2^- \end{vmatrix} = 0, \tag{1}$$

Where $D^+_{1,2} = -f^2 + f^2_{\perp 1,2} + \beta_{1,2}k_0$, $D^-_{1,2} = -f^2 + f^2_{\perp 1,2} + \beta_{1,2}k^-_{1,2}$, $\beta_{1,2} = \frac{f^2_{M_{1,2}} d_0}{2}$, $\theta^+_{1,2} = \beta_{1,2}\frac{\delta d}{2}k_0$, $\theta^-_{1,2} = \beta_{1,2}\frac{\delta d}{2}k_{-1}$, $\delta d = \frac{2\Delta}{\pi d_0}\sin\left(\frac{\pi(L-c)}{L}\right)$, $d_0 = a + \frac{\Delta(L-c)}{L}$, $f_{M_{1,2}} = 4\pi\gamma M_{01,02}$, $f_H = \gamma H_0$, $f_{\perp 1,2} = \sqrt{f_H\left(f_H + f_{M_{1,2}}\right)}$, γ — gyromagnetic ratio, $K = \exp[-kD]$, k_0 — propagation constant “0” harmonic, k_{-1} refers to the “-1” harmonic, k_0 and k_{-1} are related by the Bragg condition [6]: $k_{-1} = -k_0 + 2k_B$, where $k_B = \pi/L$ is the Bragg wave number.

At high input power levels, the ferromagnetic film is a nonlinear medium [7, 8]. The main role in nonlinear processes in ferromagnets is played by an increase in the precession angle of the magnetic moments of atoms with an increase in the signal power. This leads to a change in the longitudinal (co-directed with the external magnetic field) component of the magnetic moment. We believe that the nonlinearity of each film depends only on the saturation magnetization of this film, and the magnitude of the saturation magnetization is determined by the fields of both the first and second films. To obtain a nonlinear dispersion law in a single-frequency mode, in the absence of parametric processes, the nonlinearity of the ferromagnetic medium was introduced into the linear dispersion law (1) in the form [7, 8]:

$$f_{M_{1,2}} = 4\pi\gamma M_{0_{1,2}}\left(1 - q_{1,2}|u|^2\right), \tag{2}$$

where u — input signal amplitude, $q_{1,2} = 1/2\left(1 + f_H^2/f^2_{\perp 1,2}\right)$.

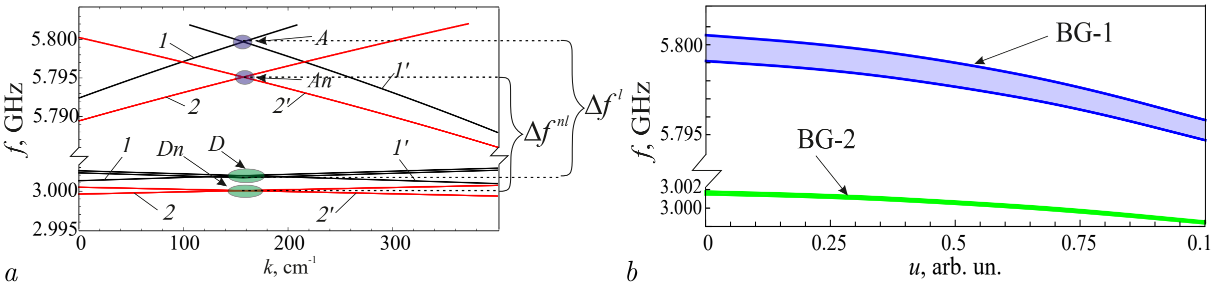

Fig 2. a — Dispersion characteristics of MSSW at $\delta d = 0$ for direct waves (lines *1* and *2*) and reflected waves (lines *1′* and *2′*) in the linear case ($u = 10^{-5}$, black lines) and nonlinear case ($u = 0.05$, red lines). b — Dependence of the width and position of the gaps BG-1 (blue shaded area) and BG-2 (green shaded area) on the input signal amplitude u. The calculated parameters are the same as in Fig. 1 (color online)

2. Results of numerical study

To study the influence of nonlinearity on Bragg resonances, the dispersion relation (1) was used, taking into account the relation (2).

In the linear case (for $u = 10^{-5}$) in coupled homogeneous films ($\delta d = 0$ and $K \neq 0$) the dispersion curve for the MSSW splits into two normal modes — symmetric and antisymmetric [17, 23, 24]. The dispersion characteristics for these waves, in the absence of coupling between them, are shown by black solid lines in Fig. 1, b: *1* — for a symmetric direct wave; *2* — for an antisymmetric direct wave; *1′* — for a symmetric wave reflected from periodic grooves; *2′* — for an antisymmetric reflected wave. For $\delta d \neq 0$ at the phase-matching frequencies with a wave number corresponding to the first Bragg resonance $k_B = \pi/L$, the interaction of the four described types of waves leads to the formation of forbidden bands (shaded areas in Fig. 1, b). The forbidden zone BG-1 (blue shaded area) is formed due to the interaction of the direct and reflected symmetric waves. In this case, the central frequency coincides with the intersection point of lines *1* and *1′* (point A). The forbidden zone BG-2 (green shaded area) is formed due to the interaction of the direct and reflected antisymmetric waves. In this case, the central frequency coincides with the intersection point of lines *2* and *2′* (point D). At the frequencies corresponding to the forbidden zones, $\mathrm{Im}\,(k)$ is different from 0 (dashed curves in Fig. 1, b), which indicates wave attenuation.

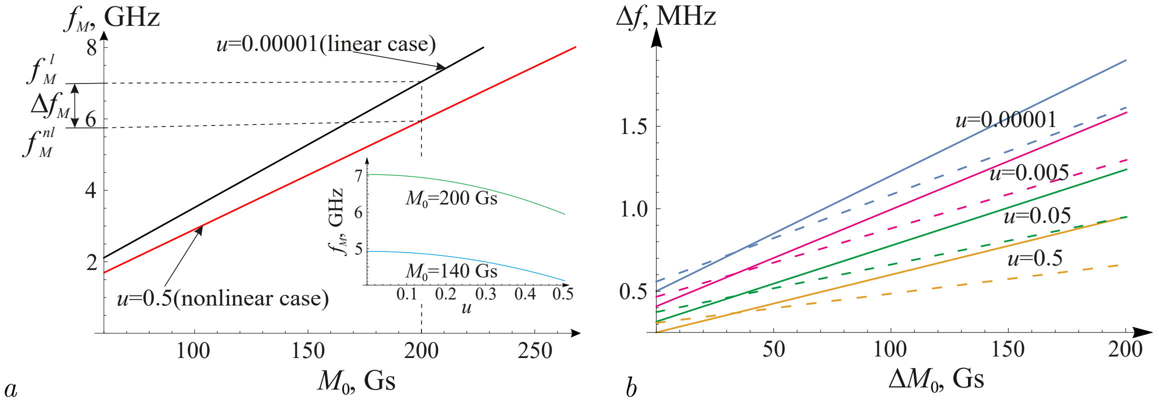

Fig 3. a — Dependences of the magnetization frequency on the saturation magnetization M_0 in linear (black line) and nonlinear (red line) cases.The inset shows the dependence of the frequency f_M on the amplitude u for different saturation magnetizations M_0. b — Dependence of the change in the frequency interval between the BG-1 and BG-2 gaps with increasing input amplitude on the difference between the saturation magnetizations of MC-1 and MC-2. At different values of u and at H_0 =800 Oe (solid curves) and $H_0 = 1000$ Oe (dashed curves)

In Fig. 2, *a* the dispersion characteristics of the MSSW in the studied structure are shown at $\delta d = 0$ for direct symmetric (line *1*) and antisymmetric (line *2*) waves and reflected symmetric (line *1′*) and antisymmetric (line *2′*) waves in the linear regime at $u = 10^{-5}$ (black lines) and in the nonlinear regime at $u = 0.05$ (red lines). The central frequency of zone BG-1 coincides with point A marked with a purple oval. The central frequency of zone BG-2 coincides with point D marked with a green oval. With an increase in the input amplitude $u = 0.05$ the central frequency of zone BG-1 coincides with point An marked with a purple oval. The central frequency of zone BG-2 coincides with the point Dn marked with a green oval. With increasing input amplitude, the central frequencies of zones BG-1 and BG-2 shift to the low-frequency region. The frequency shift is also visible in Fig. 2, *b*, which shows the dependence of the width and position of zone BG-1 (purple shaded area) and BG-2 (green shaded area) on the amplitude of the input signal u. The frequency shift is explained by a decrease in the longitudinal component of the magnetic moment with an increase in the amplitude of the input signal u in accordance with the relation (2). The frequency interval between zones BG-1 and BG-2 at $u = 10^{-5}$ ($\Delta f^l = f(A) - f(D)$) is greater than the frequency interval at $u = 0.05$ ($\Delta f^{nl} = f(A_n) - f(D_n)$), that is, there is an effect of nonlinear convergence of zones BG-1 and BG-2.

To explain this feature, the inset to Fig. 3, *a* shows the dependences of the frequency f_M on the amplitude u, constructed using the relation (2) for different saturation magnetizations M_0. It is evident that with increasing u the frequency f_M decreases. In Fig. 3, *a* the dependences of the frequency f_M on the saturation magnetization M_0 in the linear $u = 10^{-5}$ (black line) and nonlinear case $u = 0.5$ (red line) are shown[1], constructed using the relationship (2) at $M_{0_{1,2}} = M_0$ and $f_{M_{1,2}} = f_M$. It is evident that for a fixed value of M_0 the magnetization frequency in the linear case is f^l_M, and in the nonlinear case it is f^{nl}_M. That is, with an increase in the input amplitude u the frequency f_M decreases by $\Delta f_M = f^l_M - f^{nl}_M$. It is evident that Δf_M increases with an increase in M_0. That is, the greater the saturation magnetization in the linear case (M_0), the greater the nonlinear change in Δf_M.

For the studied structure consisting of MC with different saturation magnetizations, the behavior of the dispersion characteristics of symmetric waves (lines *1* and *1′*) is determined by the magnetization value of the MC with a higher magnetization (M_{0_1}). The behavior of the dispersion characteristics of antisymmetric waves (lines *2* and *2′*) is determined by the magnetization value of the MC with a lower magnetization (M_{0_2}). The nonlinear change in the magnetization frequency will be greater for symmetric waves (due to the interaction of which the BG-1 zone is formed) than for antisymmetric waves (due to the interaction of which the BG-2 zone is formed). Accordingly, the nonlinear shift of zone BG-1 ($f(A) - f(A_n)$) is greater than the nonlinear shift of BG-2 ($f(D) - f(D_n)$).

Fig. 3, *b* shows the dependence of the change in the frequency interval between the BG-1 and BG-2 zones $\Delta f = \Delta f^l - \Delta f^{nl}$ with an increase in the input amplitude from $u = 10^{-5}$ to $u = 0.05$ on the difference in the saturation magnetizations of MC-1 and MC-2 $\Delta M_0 = M_{0_1} - M_{0_2}$. The greater the difference between the saturation magnetizations of MC-1 and MC-2, the greater Δf, i.e. the effect of nonlinear convergence of the forbidden zones is more pronounced. With an increase in the external magnetic field, this effect becomes less pronounced.

Conclusion

The paper reveals the features of Bragg resonances in a periodic layered ferromagnetic structure based on magnonic crystals. It is shown that in a structure consisting of two coupled

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[1] The choice of a large value of u in the calculations is related to the clarity of the data presented in Fig. 3, *a*.

MC separated by a dielectric layer, two forbidden zones can form in the first Brillouin zone – magnetostatic wave non-transmission bands. The high-frequency forbidden zone BG-1 is formed at the phase synchronism frequency of the direct and reflected symmetric normal waves of the coupled structure. The low-frequency zone BG-2 is formed at the synchronism frequency of the direct and reflected antisymmetric waves. The zones are formed at the Bragg wave number and at frequencies different from the Bragg frequencies for each of the MC separately.

The effect of the input signal amplitude on the frequency position of the forbidden bands is investigated. It is shown that with an increase in the input signal amplitude, both forbidden bands BG-1 and BG-2 shift downward in frequency. However, nonlinearity has a different effect on the frequency position of each of the bands. The nonlinear shift of the high-frequency forbidden band BG-1 with an increase in amplitude is greater than the nonlinear shift of the low-frequency forbidden band BG-2. In this case, in the nonlinear case, the frequency interval between the bands BG-1 and BG-2 becomes smaller than in the linear case. The greater the difference between the saturation magnetizations of MC-1 and MC-2, the greater the nonlinear convergence of the forbidden bands. With an increase in the external magnetic field, this effect also becomes more pronounced.

Thus, the established features expand the capabilities of the studied structure for frequency-selective signal processing by controlling frequency selectivity both with the help of static coupling parameters, periodicity and magnetization of layers, and dynamically, with the help of the input signal power.

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