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NONLINEAR DYNAMICS OF MIXED EVOLUTIONARY STRATEGIES FOR SOLVING OPTIMIZATION PROBLEMS

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Several elementary strategies of evolution are investigated and described by simple mathematical models, leading to a highdimensional system of coupled differential equations. The stationary states of the system correspond to relative optima and the stable attractor corresponds to the final solution of the optimization problem. Special attention is devoted here to mixed Boltzmann - Darwin strategies modelling basic elements of thermodynamic and biological evolution respectively. A continous model leading to one p.d.e., the corresponding eigenvalue problem and several applications are discussed.

1. Introduction

Our world including biological species, the ecological communities and human society appears to be the result of a special search process: natural evolution. There is no external program which controls this search for well-adapted solutions. Our world is fundamentally based on the selforganization of matter which is a highly complex nonlinear process [1-7].

Evolution we understand here as in earlier work as unlimited sequences (spirals in Hegel's picture) of selforganization steps [1,2]. Selforganization is defined as the spontaneous formation of order in open entropy-exporting systems. One of the most evident features of evolution is the tendency to form ever more complex structural and behaviour modes during the course of time. The understanding and explanation of the trend to complexification and optimization forms the heart of any research program dealing with problems of evolution. Modern research has shown that the strategies developed in the process of natural evolution might also be of interest for the design and construction of technical systems. Pioneering work in this direction was done by Holland, Bremermann, Rechenberg and Schwefel [8-10].

2. Models of the Basic Strategies of Evolution

Analyzing the mechanisms of natural evolution we find several basic strategies [8-14]; the main of them are:

1. ***Boltzmann strategy***. The fundamental goal nature tried to reach in its course of evolution up to the appearance of life is the optimization of certain thermodynamic functions. In our present understanding the metagalaxis was the result of a giant vacuum fluctuation which extended through an inflationary process and led to an hot and quickly expanding plasma. The expansion of this very hot plasma started about 10-20 billions of years ago. The expanding plasma tried to maximize its entropy and this was the basic reason for the formation of the chemical elements and for the clustering of matter which finally led to galaxies, stars and planets.

The Boltzmann strategy has three important elements:

a. Motion along gradients to reach steepest ascent of entropy.
b. Various stochastic processes including thermal and hydrodynamic fluctuations. Thanks to these fluctuations locking in local maxima was avoided.
c. Decrease in temperature during the adiabatic expansion of our world.

Let us consider for example a numbered set of states $i = 1,2,\ldots,n$ each characterized by a potential energy U_i and a population $x_i(t)$ at time t. Then the simplest model of a Boltzmann process which tends to find minima of U_i is

$$\partial_t x_i(t) = \Sigma_j (A_{ij} x_i(t) - A_{ji} x_i(t)), \tag{1}$$

$$A_{ij} = A_{ij}^0 \begin{cases} 1, & \text{if } U_j > U_i, \\ \exp\{(U_j - U_i)/T\}, & \text{otherwise.} \end{cases} \tag{2}$$

$$A_{ij}^0 = A_{ji}^0.$$

In other words a hill-down transition is always carried out and a hill-up transition occurs only with a small rate which decreases exponentially with the height of the threshold.

2. ***Darwin strategy.*** This second important natural strategy appears in the universe only in the process of biogenesis i.e. about 3-4 billions of years ago. The basic elements of a Darwinian strategy are:

a. Self-reproduction of good species that show maximal fitness.
b. Mutation processes due to error reproductions that change the phenotypic properties of the species.
c. Increase of the precision of selfreproduction in the course of the evolution of life.

Considering again as an example the population of n states a simple model reads

$$\partial_t x_i = (E_i - \langle E \rangle)\, x_i + \Sigma_j (A_{ij} x_i - A_{ji} x_i). \tag{3}$$

Here E_i denotes the fitness (expressed by the rate of self-reproduction) of species i in the population and $\langle E \rangle$ is the population average.

The two strategies discussed so far will be denoted in the following as elementary strategies. For completeness let us mention only other more difficult strategies which were developed by nature in the course of evolution, as e.g. Haeckel strategies, Volterra strategies etc.[1,2,10-12].

3. ***Mixed Boltzmann - Darwin Strategy.*** Boltzmann and Darwin strategies show several parallels but also essential differences [10-11]. Both strategies are well suited to find the extrema in landscapes of potential functions. In general it will depend on the structure of this landscape, what search strategy is the better one. The qualitative analysis carried out in earlier work [11] suggests that in the case that no knowledge about the structure of the landscape is available, it will be advantageous to apply the Boltzmann strategy combined with annealing. This strategy seems to be more universal; it will always work. However, thermodynamic processes have the tendency to be locked in relative extrema surrounded by high threshold. In the other hand, Darwinian processes are able to cross high barriers by tunneling if the next minimum is close. In any case we have seen, that both strategies are quite different and we might expect that there exists a class of problems, where the Boltzmann strategy is better and another class of problems where the Darwin strategy is more appropriate. In such a situation it seems to be a good idea to develope a strategy which possesses components from both elementary strategies.

In order to model mixed strategies let us consider again a numbered set of states $i = 1, 2, \ldots, n$ each characterized by a potential energy U_i and a population $x_i(t)$ at time t. Then a simple model of a mixed strategy with the property to find minima of U_i is

$$\partial_t x_i = \gamma\,(<U> - U_i)x_i + \Sigma_j\,(A_{ij}x_i - A_{ji}x_i), \tag{4}$$

$$A_{ij} = A_{ij}^0 \begin{cases} 1, & \text{if } U_j > U_i, \\ \exp\{\beta(U_j - U_i)\} & \text{otherwise,} \end{cases} \tag{5}$$

with

$$A_{ij}^0 = A_{ji}^0, \qquad \beta = 1/\mathrm{T}. \tag{6}$$

The new models described by eqs. (4)-(6) contains as a special case the Boltzmann strategy for $\gamma = 0$. The Darwin strategy is obtained for $\gamma = 1$ and $T \to \infty$.

3. The Continuous Model of Mixed Strategies and the Associated Eigenvalue Problem

Since the mathematical problem connected with the solution of the coupled non-linear differential equations (4)-(6) is extremely difficult, let us simplify it. We introduce a corresponding continuous model leading to one partial d.e. by replacing the vector x_i by a continuous function $x(q, t)$ and restricting mutational changes to small steps [1,11]. With these assumptions we get a mixture of Fisher - Eigen and Fokker - Planck equations

$$\partial_t x(q, t) = \gamma\,[<U> - U(q)]x(q, t) + D[\Delta x(q, t) + \beta x(q, t)\nabla U(q)\,]. \tag{7}$$

For $\beta = 0$ ($T \to \infty$) this is the standard Fisher - Eigen equation which is solved for $t \to \infty$ by a Gaussian-like distribution centered around the absolute minimum of $U(q)$. For $\gamma = 0$ results the standard Fokker - Planck equation which is solved for $t \to \infty$ by the Boltzmann distribution

$$x_0(q) \sim \exp\,[-\beta U(q)] \tag{8}$$

which possesses a maximum around the lowest minimum of $U(q)$. In the general case $\gamma > 0$, $\beta > 0$ the following ansatz is useful

$$x\,(q, t) = \exp\,[\gamma \int_0^t <U>dt' - \beta U(q)/\,2]\; y(q, t). \tag{9}$$

This substitution transforms the nonlinear p.d.e. (7) into a linear equation for $y(q, t)$ which is similar to the Schrädinger equation with imaginary time

$$\partial_t y(q, t) = D[\Delta y(q, t) - V(q)]y(q, t) \tag{10}$$

with the effective potential

$$V(q) = \gamma\, U(q) - (D/2)\beta\Delta U(q) + (D/4)\,(\nabla U(q))^2. \tag{11}$$

The new linear equation which is solved by

$$y\,(q, t) = \Sigma\, a_n \psi_n(q)\exp(-\varepsilon_n t\,). \tag{12}$$

Here the eigenvalues ε_n and the eigenfunctions ψ_n are determined by the stationary Schrädinger problem

$$D\Delta\psi_n(q) + [\,\varepsilon_n - V(q)\,]\psi_n(q) = 0 \qquad (13)$$

for the effective potential. In this way we get the complete explicite solution of eq. (7) in the form

$$x(q,t) = \exp[-\beta U(q)/2\,]\cdot \Sigma\, a_n\psi_n(q)\exp(-\varepsilon_n t)/\Sigma c_n \exp(-\varepsilon_n t). \qquad (14)$$

The coefficients a_n and c_n are defined by

$$a_n = \int dq\; x(q,0)\;\psi_n(q),$$
$$c_n = a_n \int dq\;\psi_n(q). \qquad (15)$$

In the limit $t \rightarrow \infty$ our solution converges to

$$x_0(q) = \exp\,[-\beta U(q)/2\,]\cdot(a_0/c_0)\;\psi_0(q) \qquad (16)$$

where $\psi_0(q)$ is the ground state wave function which is centered around the deepest minimum of the effective potential $V(q)$. A deeper mathematical analysis of the solution given above is in preparation [15].

4. Application to Optimization Problems

Let us first show that the dynamics of the mixed process is problem-solving that means it will find the absolute minimum of $U(q)$ which is the target of the search. Since the Boltzmann factor has a maximum at the deepest minimum of the potential $U(q)$ we see that $x_0(q)$ is centered near to the latter. In other words, the dynamics of the mixed strategy converges to a point attractor near to the minimum of $U(q)$ which is searched, i.e. the strategy is indeed problem solving. In order to guarantee the exact convergence to the minimum of $U(q)$ we may use a kind of annealing $\gamma \rightarrow 0$ and $\beta \rightarrow \infty$. Since for $\gamma = 0$ the ground state solution of eq. (13) is

$$\psi_0(q) = \exp[-\beta U(q)/2\,] \qquad (17)$$

we see that $x_0(q)$ degenerates in the limit of the annealing process to a δ-function centered around the deepest minimum of $U(q)$. This proves that the dynamic process converges in the limit $\gamma \rightarrow 0$, $\beta \rightarrow \infty$ and $t \rightarrow \infty$ indeed to the absolute minimum of $U(q)$.

In earlier work we considered the application of mixed Boltzmann - Darwin strategies based on the discrete form of the mixed strategies given by eqs. (4)-(6) to the travelling salesman problem (TSP) and the related cost problem (CP) [13]. For example we considered routes between 100 towns stochastically distributed on a square as well as routes connecting 16 «real» towns. The potential corresponding to the TSP is the total length of a closed tour connecting all towns [13]

$$U = \Sigma\, L(i,j) \qquad (18)$$

where $L(i, j)$ is the distance between the towns i and j. The «real» towns considered in our earlier work were: Schwerin (B), Rostock (A), Stralsund (P), Neubrandenburg (C), Stendal (H), Potsdam (D), Berlin (I), Frankfurt (E), Cottbus (Z), Dresden (R), Chemnitz (T), Gera (N), Halle (K), Leipzig (S), Suhl (O), Erfurt (L). The distances then were taken from the data given in a common calendar for automobile kilometers. The problem was of course not, to solve a realistic touring problem but, to find good strategies which yield short tours in a modest time. It could be shown by simulations that in this respect mixed Boltzmann - Darwin strategies have very good search properties. We mention also another variant of those strategies which includes aging of the searchers, a strategy which was called a Haeckel-type strategy [12].

In our simulations the number of representatives (salesmen) in the ensemble was varied between 2^0 and 2^7. We calculated the mean value of the dispersion and the best results obtained by a search in a given fixed total computer time. It was shown that ensembles with 4-32 representatives give the best results with respect to the mean value, the best value and the dispersion of the results. The interpretation of these findings is the following: certain amount of parallelism is a useful element of good search strategies. The computer time which is lost for simulating a number of parallel searchers is gained by certain advantages of parallel search as e.g. the possibility of exchange of experience. In our mixed strategy this is modelled by the Darwinian elements: competition between searchers, survival of the fittest. However when the number of searchers working in parallel is too high, in our case exceeding the number 32, the parallelism costs more than one can gain from it. According to our experience, a successful strategy requires a fine balance between parallelism and individualism on one side and between Darwinian and Boltzmann elements on the other side.

Similar results were obtained by the analysis of a cost problem with the potential

$$U = \Sigma\, C(i,j) \tag{19}$$

where the cost for travelling from town i to town j was derived from the distance by multiplication with a random number in the range $(0.2 \div 1.2)$. The cost problem does not satisfy a triangle inequality and must be considered therefore as a different class of optimization problems. The results show again certain advantage of including Darwinian elements into the search strategy. Let us underline that $N = 1$ corresponds to a pure Boltzmann strategy and $N = 2,4,...,128$ to a mixed Boltzmann - Darwin strategy. In this case we are simulating a population of N salesman which are searching simultaneously and which are coupled by a competition for the best results. The simultaneous search was simulated always on a single sequential computer. Of course it could in principle be carried out also on a net of parallel processors. Since the coupling (acts of selection) between the elements of a Darwinian ensemble is a rather seldom event, the speed up by using such an N-processor net might be near to N. Possibly the real power of mixed strategies including Darwinian elements will show up only on parallel computers with 4 - 32 parallel processors [14]. This question has to be left to future work, as well as a deeper mathematical analysis of the mixed strategies [15], which were described here in a more qualitative way.

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НЕЛИНЕЙНАЯ ДИНАМИКА СМЕШАННЫХ ЭВОЛЮЦИОННЫХ СТРАТЕГИЙ ДЛЯ РЕШЕНИЯ ПРОБЛЕМ ОПТИМИЗАЦИИ

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Исследуются несколько простых стратегий эволюции. Они описываются простыми математическими моделями, сводящимися к многомерной системе связанных дифференциальных уравнений. Стационарные состояния системы соответствуют относительным оптимальным условиям, а устойчивый аттрактор соответствует окончательному решению проблемы оптимизации. Особое внимание уделяется смешанной стратегии Больцмана - Дарвина, моделирующей основные элементы термодинамической и биологической эволюции. Обсуждается распределенная модель, сводящаяся к одному дифференциальному уравнению в частных производных, проблема собственных значений и ряд применений.

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