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Mathematical model for controlling brain neuroplasticity during neurofeedback[1]

I. V. Nuidel[1] ✉, *A. V. Kolosov*[1], *S. A. Permyakov*[2], *I. S. Egorov*[2], *S. A. Polevaya*[2], *V. G. Yakhno*[1,2]

[1]FRC A. V. Gaponov-Grekhov Institute of Applied Physics of the Russian Academy of Sciences, Russia
[2]National Research Lobachevsky State University of Nizhny Novgorod, Russia
E-mail: ✉nuidel@ipfran, 7418880@mail.ru, permyakov@fsn.unn.ru, igor.egorov.unn@yandex.com, s453383@mail.ru, yakhno@ipfran.ru
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Abstract. The *purpose* of this work is to apply a model of interaction between thalamocortical system modules to control brain neuroplasticity. *Methods.* Psychophysiological experiments on neurofeedback are being carried out, which consist of light stimulation of the eyes with monofrequency light pulses in the range of 4...20 Hz and recording the bioelectrical activity of the brain. As a characteristic of maturity, brain rhythms use the combination of the presence or absence in the bioelectrical activity of the brain of a dominant peak frequency in the alpha range of the EEG, the effect of assimilation of the rhythms imposed by stimulation, and the presence of a multiplying effect from the rhythms imposed by stimulation. Solutions to the model of an elementary thalamocortical cell, which is described by a system of differential equations, corresponding to a psychophysiological experiment are considered. The model is implemented using the Python. *Results.* The model parameters are selected in such a way as to achieve a qualitative correspondence of the spectral characteristics of the obtained solutions with the bioelectrical activity of the subject's brain. Rhythmic maturity is assessed based on the parameters of the thalamocortical cell model. The brightness and frequency characteristics of light stimuli are selected based on the prediction of the model, the input of which is supplied with various variants of pulse sequences. *Conclusion.* A method has been developed for digital diagnostics of the level of brain rhythm maturity based on a comparison of modeling results and data from a psychophysiological experiment on neurofeedback. The evolution of model solutions depending on its parameters simulates the process of biocontrol of brain neuroplasticity, taking into account the initial level of rhythmic maturity and stress-induced distortions of neurodynamics. Experiments on the model with different parameters of the model and external signal can be used in the development of new neurofeedback protocols.

Keywords: thalamocortical cell model, neurofeedback, neuroplasticity, bioelectrical activity of the brain, assessment of brain rhythm maturity.

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Introduction

How to assess a person's ability and readiness to perceive information for further work with it, to solve problems, to learn, the possibility of restoring cognitive abilities after stress and traumatic effects? This is the task of functional diagnostics of one of the main human systems — his brain. After functional diagnostics in each specific case, if necessary, steps follow for further special testing and training of cognitive functions. The paper proposes a version of digital diagnostics of the level of maturity of brain rhythms based on a comparison of experimental data on brain rhythms obtained during psychophysiological experiments with the results of mathematical modeling.

To determine the level of functional efficiency of the cortical structures, the assessment characteristics of the levels of maturity of the brain rhythms (the reference level and 3 levels of regression of the brain rhythms: weak, moderate, deep) [1] were introduced. Different levels of maturity of the rhythms correspond to different stages of the ontogenesis of the neural networks of the brain, providing recurrent cycles of signal processing for the implementation of cognitive functions. Neuroplasticity of the brain (the ability of the human brain to change under the influence of experience or in response to external influences) allows you to change the level of maturity of the brain rhythms [2–4]. You can move to a more mature rhythm by controlling the activation parameters of the neural networks of the brain during a psychophysiological experiment.

Currently, adaptive neurostimulation, organized according to the principle of a closed loop with feedback from the EEG, is successfully used to control activation parameters [5–7]. It has been established that presenting subjects in a state of anxiety and stress with rhythmic light stimuli, automatically generated in real time based on their own EEG, leads to a reliable increase in EEG power, as well as a decrease in the level of emotional maladaptation and stress in subjects [8–10]. Functional disorders caused by stress are corrected.

To select a protocol for a psychophysiological experiment on neurobiocontrol, it is necessary to assess the degree of maturity of brain rhythms. The paper proposes to use a thalamocortical cell model [11] for this purpose, calculations on which qualitatively reproduce the effects of spectral dynamics in humans in response to an external signal similar to the experimental one. It is possible to "construct" an external signal and watch the model's response with different parameters, simulating a sequence of protocols corresponding to the transition to the next stage of rhythmic maturity. Such calculations can be useful in developing new neurobiocontrol protocols.

In this sense, the model can be considered a simulator of personal thalamocortical reactions. A promising goal of this work is to use the model parameters and calculations when selecting optimal stimulation modes (for example, photostimulation [12, 13]) to form useful EEG patterns in order to influence the characteristics of the personal thalamocortical system of the subject (patient) and improve his emotional state.

1. Terms

Brain adaptability is the brain's ability to adjust to new conditions, changes in the environment, or demands. It includes the processes of neuroplasticity. Let's list several key aspects of brain adaptability.

1. *Neuroplasticity.* The brain is able to change its structure and function in response to experience. This may involve forming new connections between neurons, changing the strength of existing synapses, and even creating new neurons [2–4].
2. *Learning and Memory.* The brain adapts to new information through learning and memory formation. By strengthening synaptic connections and changing the structure of neural networks, the brain can improve its cognitive abilities.
3. *Compensating for Damage.* When the brain is damaged, such as by injury or disease, the brain may attempt to compensate for the loss of function by activating other areas or changing its structure to adapt to new conditions.
4. *Development Across the Lifespan.* The brain's adaptability occurs throughout the lifespan. Even in adulthood, the brain can continue to change and adapt to new challenges, although these processes may slow down with age.
5. *Capacity to reorganize.* The brain has the capacity to reorganize its structural and functional components in response to changes in external conditions, demands, or tasks.

The adaptability of the brain is an important aspect of our ability to learn, adapt to our environment, and cope with changes in life. Brain plasticity (neuroplasticity) is the ability of the nervous system to change its structure and function in response to experience, learning, adaptation, or injury. This means that the brain can form new connections between neurons (nerve cells), change the strength of existing connections, and even create new neurons in a process called neurogenesis. Brain plasticity involves changes at the synapse level. The brain's capacity for plasticity is most pronounced in early childhood, when neural networks are being intensively formed, but it persists throughout life. This process of plasticity plays an important role in learning, adapting to change, recovering from injury, and it affects various aspects of cognitive function, including memory, attention, thinking, and learning. Understanding brain plasticity has implications for education, training, rehabilitation, and treatment of neurological conditions. [14].

2. Dynamics of EEG signal. Experiment

The article uses experimental data from earlier studies. A detailed description of the protocols of psychophysiological experiments on dynamic photostimulation is given in the works [1, 5–10, 13–18].

In a dynamic photostimulation experiment, a flickering infrared lamp is directed at the subject's closed eyes. [1, 17]. A sequence of monofrequency light pulses in the range of 4...20 Hz is used as light stimulation. The subject's EEG is recorded before exposure to pulsed infrared radiation, during exposure, and after exposure. Based on the recorded frequencies of the subject's alpha rhythm (at rest with eyes closed), an external signal with the same frequency characteristics (alpha peak and alpha range frequencies) is generated.

Based on the results of the "dynamic photostimulation" functional test, it is possible to determine the nature of the subject's rhythm: the reference rhythm, or the reference level of rhythm maturity, and several levels of brain rhythm regression (weak, moderate, deep). It has been established that the same types of disturbances in the dynamic structures of brain rhythms are manifested in the case of structural and functional destruction of neural networks of the brain. The same distortions of the dynamic structures of brain rhythms are manifested in children with organic delayed brain development or attention deficit hyperactivity disorder, and in adults with depression, emotional burnout, and stress of various origins. The degree of disturbances is determined by the depth of regression of the level of maturity of rhythms. Restoration of rhythms during neurobiofeedback corresponds to the stages of maturation. Each transition along

the maturity scale corresponds to the protocol of the resonance biofeedback procedure. Thus, the use of neurobiofeedback (resonance biofeedback) for rehabilitation is carried out through a sequence of protocols (procedures) that ensure the transition to the next stage of maturity.

The reference level of maturity of brain rhythms, providing a personal optimum of cognitive functions, is formed by the age of 7. The effectiveness of cognitive functions and stress resistance are associated with the stability of the reference rhythm. The reference level of maturity is characterized by the following features: in the background — a clearly expressed alpha rhythm in the range of 8...14 Hz; with dynamic photostimulation, the intrinsic alpha rhythm is preserved; rhythms with stimulation frequencies appear (rhythm acquisition), that is, the adaptability and plasticity of the brain are realized through the reproduction of light signal oscillations in the EEG; rhythms with frequencies multiple of the stimulation frequency appear (multiplication), that is, the adaptability and plasticity of the brain are realized through the generation of a new rhythm; after stimulation, the alpha rhythm power is no higher than before stimulation. Fig. 1, *a* shows the amplitude spectrum and the dynamic spectrum of the EEG of the subject with a clearly expressed peak frequency in the alpha range (an example of reference rhythmics).

Based on the results of the "dynamic photostimulation" functional test, three levels of brain rhythm regression can be distinguished: deep, moderate, and weak. Note that at this stage of the research, the detection of brain rhythm features, including the spectral peak in the alpha range, is performed by an expert. Weak regression of brain rhythm is characterized by the following features: in the background — a weak frequency peak in the alpha rhythm range (8...14 Hz); with dynamic photostimulation, at least one feature of the reference brain rhythm is absent: the alpha peak is reduced or absent, the effects of rhythm assimilation and multiplication are absent or weakly expressed. Moderate regression of brain rhythm is characterized by the following features: in the background — there is no frequency peak in the alpha rhythm range (8...14 Hz); with dynamic photostimulation, the effects of rhythm assimilation and multiplication are absent or weakly expressed. Deep regression of brain rhythms is characterized by the following signs: in the background — there are no frequency peaks in the alpha (8...14 Hz) and theta (3...8 Hz) ranges; with dynamic photostimulation, there are no rhythm assimilation and multiplication effects; after stimulation, the alpha rhythm power is higher than before stimulation. Fig. 1, *b* shows the dynamic spectrum (spectrogram) of the EEG of the subject, which shows the absence of a peak frequency in the alpha range, i.e. the EEG spectrum picture is considered "blurred" (regression of rhythms).

The effectiveness of resonant neurofeedback for improving cognitive functions is associated with the formation of a peak in the alpha range, an increase in the alpha peak frequency, with the effects of rhythm assimilation and the effects of rhythm multiplication, respectively [14, 19]. Dynamic EEG spectra during the experiments on resonant biofeedback are shown in Fig. 1, *c, d*. In psychophysiological tests on dynamic photostimulation, the following effects are observed in different subjects: an alpha peak in the EEG spectrum is formed / not formed; stable / unstable alpha range during the experiment; resonant spectral peaks are observed / not observed at the stimulation frequency; resonant spectral peaks are observed / not observed at frequencies multiple of stimulation (rhythm multiplication).

For example, in the spectral dynamics of the subject (see Fig. 1, *c*), the alpha range of the EEG is formed, the alpha peak is registered; when an external signal is applied (see Fig. 1, *c*, on the right, time 200...450 s), the rhythm assimilation is registered, the peak frequency in the alpha rhythm is preserved. It is evident that already at the first light effects and as their frequency increases, resonant spectral peaks are observed in the EEG spectra, exactly coinciding in frequency with the stimulation frequency acting at the moment. These spectral peaks form an inclined straight line on the dynamic spectrum of the EEG (see Fig. 1, *c*, on the left), reflecting

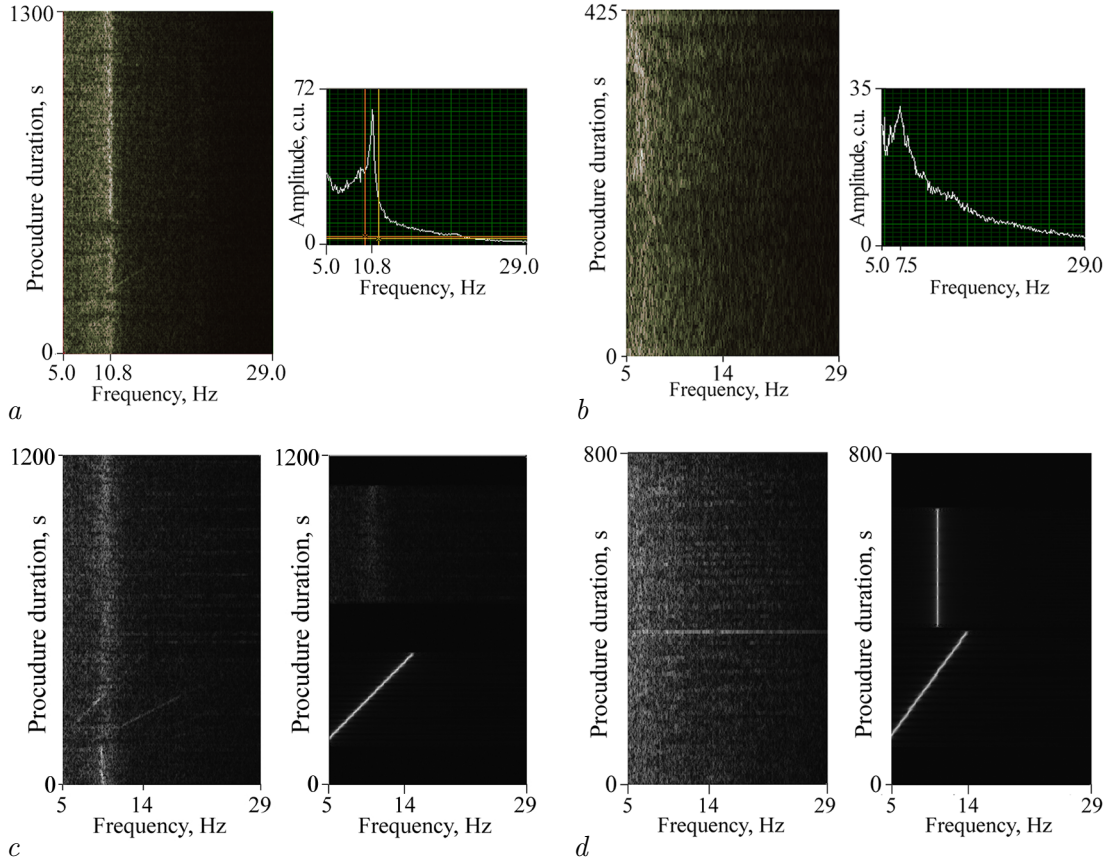


Fig 1. Dynamic EEG spectra during neurofeedback experiments. On the abscissa axis — spectrum frequency, Hz. Along the ordinate axis — experiment time, s. Along the Z axis — spectral density reflected in brightness: a — example of reference rhythmics; b — regression of rhythmics; c, d — dynamic EEG spectra of the subject during the experiment (left) and external signal (right)

the resonant activation of brain activity at the stimulation frequency (an example of mature rhythmicity). In the spectral dynamics of the subject (Fig. 1, *d*), the alpha range of the EEG is not formed. In response to an external signal (Fig. 1, *d*, on the right), no resonance effects are observed in the EEG spectrum, which indicates a complete lack of rhythm assimilation, multiplication and stability of the alpha range (rhythmic regression).

The diversity of neurobiocontrol effects depending on the type of stimulation, maturity and functional state of the brain poses the task for researchers to systematize the effects and targeted stimulation for their appearance.

As a characteristic of the maturity of brain rhythms, neurophysiologists use the combination of the presence or absence of a dominant peak frequency in the alpha range of the EEG in the bioelectrical activity of the brain, the effect of assimilation of rhythms imposed by stimulation, and the presence of a multiplier effect from rhythms imposed by stimulation. Neuroplasticity of the brain — the ability of the human brain to change under the influence of experience or in response to external influences, allows you to change the level of maturity of brain rhythms [15]. You can go from regressive rhythms to more mature ones by controlling the activation parameters of the neural networks of the brain during a neurophysiological experiment. The authors propose the following assessment of rhythm maturity using ranking on a scale from 0 to 3 points of the effects of the dominant peak frequency in the EEG alpha range (presence — 1 point, absence — 0 points), the effect of assimilation of rhythms imposed by stimulation (presence — 1 point,

Table 1. Assessment of the degree of rhythmic maturity

Peak frequency in α-range (0;1)	Adaptability: rhythm acquisition (0;1)	Plasticity: multiplication (0;1)	Rhythmic maturity assessment	Rhythmic maturity	N neurofeedback protocols
0	0	0	0	Deep regression	3
1	0	0	1	Moderate regression	2
1	1	0	2	Weak regression	1
1	1	1	3	Reference rhythm	0

absence — 0 points) and the presence of a multiplier effect from rhythms imposed by stimulation (presence — 1 point, absence — 0 points). The effectiveness of the proposed algorithm for assessing rhythm maturity on a scale from 0 to 3 points using the assessment of three effects was demonstrated in the analysis of EEG correlates of learning problems in primary school children [1]. The test results are recorded in Table 1. Using the neurofeedback procedure, with an increase in the number of sessions and in combination with correctly selected experimental scenarios, it is possible to achieve a transition to a more mature rhythm. The problem of maintaining the effects of the neurofeedback procedure for a certain period of time or permanently is a separate task.

The sequence of neurofeedback procedure protocols leads to the transition of the brain rhythm to the next level of maturity (Fig. 2). From the regression of the brain rhythm (Fig. 2, *a*) after several sessions of photostimulation, there is a transition to mature rhythm: the appearance of a peak alpha rhythm (Fig. 2, *b*), then the assimilation of the rhythm (Fig. 2, *c*) and the appearance of a multiplier effect from the rhythm imposed by stimulation (Fig. 2, *d*).

Thus, the use of neurobiofeedback for rehabilitation is carried out through a sequence of protocols that ensure the transition to the next stage of maturity (see Fig. 2). The transition to the reference level of maturity is accompanied by subjective sensations of improvement in the condition of the subjects and an improvement in cognitive abilities. [1, 5–10, 13–16, 18].

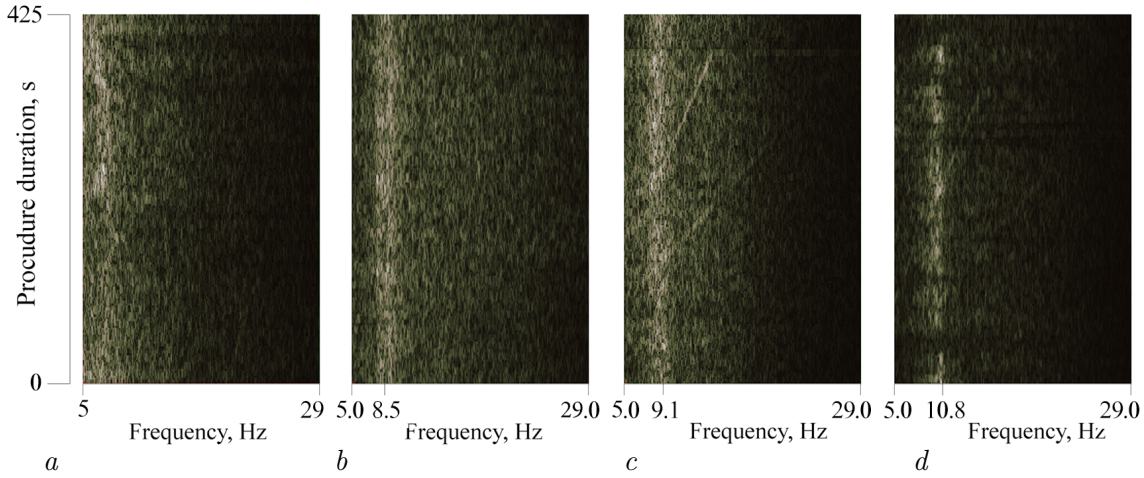


Fig 2. Dynamic EEG spectra of a subject during the neurofeedback procedure. Transition from immature to mature rhythmicity

3. Modeling neuroplasticity control in biofeedback

3.1. Model. To model the human EEG oscillatory system, we propose a phenomenological model of neuroplasticity control based on the interaction of ensembles within one thalamocortical module or several interacting modules. This model was developed and studied in detail earlier [20–22].

In choosing this approach to modeling this complex phenomenon using a simple model, we rely on the data of experimental neurophysiological studies that interconnected neuronal modules — specific thalamus, cortex, reticular nuclei of the thalamus — play an important role in information processing. Brain activity is recorded in EEG signals.

One neural module consists of ensembles of pyramidal neurons and inhibitory interneurons of the cortex (Cortex), neurons of specific thalamic nuclei (Thalamus) and inhibitory neurons of the reticular nucleus of the thalamus (NRT), interconnected [11].

The scheme of intermodular interaction is shown in Fig. 3, *a* [22]. Triangles in the scheme show excitatory connections, and circles show inhibitory connections between modules. An external sensory signal enters the system through the thalamus. The arrow at the bottom of Fig. 3, *a* is the sensory input to the thalamus.

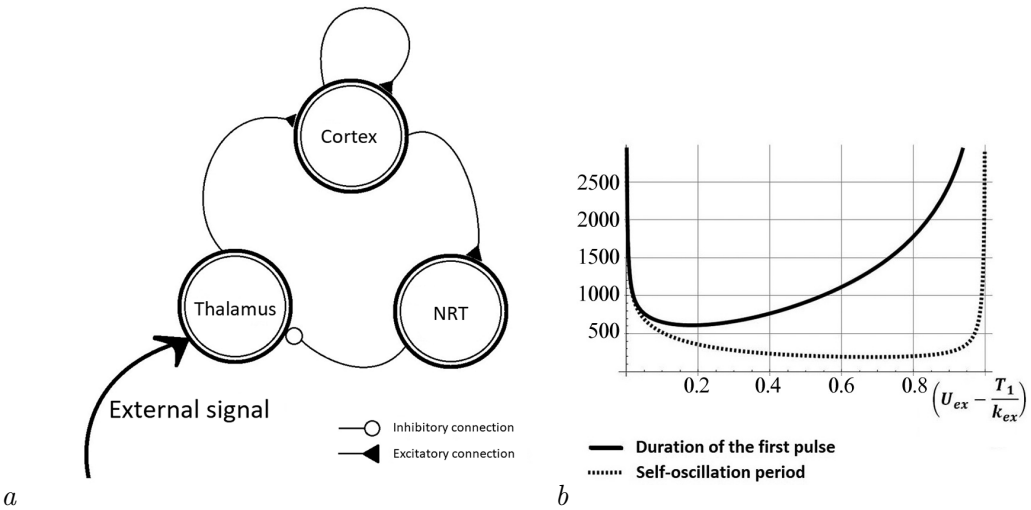
The model of the elementary thalamocortical cell, corresponding to the diagram in Fig. 3, *a*, is described by a system of differential equations(1)–(3):

$$\frac{dU_1}{dt} = -\frac{U_1}{\tau_1} + k_1 \cdot F_1[-T_1 + k_{ex}U_{ex} - k_{13}U_3], \tag{1}$$

$$\frac{dU_2}{dt} = -\frac{U_2}{\tau_2} + k_2 \cdot F_2[-T_2 + k_{21}U_1 + k_{22}U_2], \tag{2}$$

$$\frac{dU_3}{dt} = -\frac{U_3}{\tau_3} + k_3 \cdot F_3[-T_3 + k_{32}U_2], \tag{3}$$

where U_1, U_2, U_3 are the average activity of neurons in the selected areas of the thalamus, cortex, and reticular nuclei of the thalamus, respectively; τ_i is the characteristic decay time of activity



a *b*

Fig 3. *a* — diagram of functional connections between subsystems in one thalamocortical cell; *b* — dependence of the duration of the first pulse (solid line) and the period of self-oscillations following it on the constant value of the external signal (dotted line)

in the corresponding neural ensembles; k_i is the amplitude of generation of impulse activity by the corresponding neural ensembles; T_i are the average values for the excitation thresholds of the corresponding neural ensembles; U_{ex} is the input signal to the thalamus; k_{ij} are the coefficients of mutual coupling between the subsystems in the thalamocortical cell; $F_i[f]$ are step-shaped functions whose steepness characterizes the spread of threshold values around the average values in the ensemble under consideration (another name is the energy supply function); in square brackets is an analogue of the postsynaptic potential on the membranes of the corresponding neuronal ensembles.

We will use this model to interpret experiments on biocontrol, i.e. control of the parameters of the human EEG oscillatory system by an external signal with the parameters of the EEG of this person.

On the one hand, in a psychophysiological experiment, the maturity of the rhythm, as mentioned above, is assessed by the presence / absence of three effects in the EEG signal of the subject: the effects of the dominant peak frequency in the alpha range of the EEG; the effect of assimilation of rhythms imposed by stimulation and the presence of a multiplier effect from rhythms imposed by stimulation.

On the other hand, the dynamic modes of the model are determined by its parameters and the parameters of the external signal.

It is proposed to assess the maturity of the rhythm by certain parameters of the thalamocortical cell model, which are adjusted in such a way as to correspond to the dynamics of the spectral components of the bioelectrical activity of the brain examined during background recording and during stimulation with a sequence of light pulses.

Since the model can be used to obtain dynamics corresponding to the "mature cortex" by changing its parameters, it can be considered a model for controlling neuroplasticity [17].

To select the model parameters for assessing the maturity of the cortex, an important dependence on the external signal of the duration of the first pulse and the period of the following self-oscillations was studied; its appearance is shown in Fig. 2, *b* [22]. An important parameter is also the slope of the step-shaped functions $F_i[f]$ and the response thresholds for the model variables.

Let us consider the dynamic modes in the mathematical model of an elementary thalamocortical cell with a constant external signal. The equations given below represent a reduced model with the same relaxation times and certain values of the coefficients of the mutual coupling of the subsystems:

$$\frac{dU_1}{dt} = -U_1 + F[U_{ex}(t) - U_3], \tag{4}$$

$$\frac{dU_2}{dt} = -U_2 + F[-0.5 + U_1], \tag{5}$$

$$\frac{dU_3}{dt} = -U_3 + F[-0.5 + U_2]. \tag{6}$$

$U_{ex}(t)$ —the external signal is set (7) on a limited time interval (Fig.5, *a, g*).

$$U_{ex} = A(t)|\sin(\omega_{ex}(t)t)|, \quad \omega_{ex}(t) = \omega t + b. \tag{7}$$

The structure of the three-dimensional phase space (U_1, U_2, U_3) shows how, under the selected initial conditions $0 < U_{ex} < 1$, the point moves from a given initial state. When $U_{ex} = 1$ or $U_{ex} = 0$, the cycle merges with the equilibrium state to form a stable equilibrium state. In a non-autonomous system, the equilibrium state moves along the phase space together with

the cycle when U_{ex} changes. The speed of the equilibrium state over time is determined by the speed (frequency) of change of the external signal. The natural frequency of the system determines the speed of movement of the representative point along the trajectory of a stable limit cycle. Then here we can distinguish several modes that were revealed exclusively in the numerical analysis of the system of differential equations describing the model. We will consider a harmonic external signal in the form of a sinusoid with a linearly changing frequency(7).

3.2. Calculation results. The model was used to calculate signals and their spectra in the case of frequency modulation of the thalamic signal by an external signal with a linearly increasing frequency. It is known that the external signal enters the cortex through the thalamus.

The input sensory signal is a frequency-modulated signal (in the psychophysical experiment it is similar to an infrared high-frequency modulated signal in relation to the natural frequency of the EEG signal). The signal from the variable cortex is similar to the integral bioelectric signal of the EEG.

In this work, of all the parameters, only the value of the external signal U_{ex} was changed. Numerical values of the parameters: $k_i = 1, i = 1, 2, 3$; $\tau_i = 1, i = 1, 2, 3$; $T_1 = 0$; $T_2 = 0.5$; $T_3 = 0.5$; $k_{ex} = 1$; $k_{13} = 1$; $k_{21} = 1$; $k_{22} = 0$; $k_{32} = 1$. The energy supply function depends on the parameter d: the larger d, the flatter the function. For different parameters d, the dynamics obtained in numerical experiments correspond to the reference rhythm and different degrees of rhythm regression in psychophysiological experiments on neurobiocontrol (Fig. 4).

$$F_i = 0.5 \cdot \left(1 + \operatorname{erf}\left(\frac{t}{d}\right)\right) = 0.5 \cdot \left(1 + \frac{2}{\sqrt{\pi}} \int_0^{\frac{t}{d}} e^{-z^2} dz\right). \tag{8}$$

For U_{ex}, a signal (7) was taken, which is similar to the experimental one: the frequency of the oscillating part increases linearly in a certain frequency range (Fig.5, *a, g*).

The calculations form a library of modes corresponding to the data of psychophysiological experiments. Some of these modes with different parameters of the system and the external signal are shown in the figures Fig. 5, 6, 7.

In all Fig. 5, 6, 7 of the calculations on the model for signals along the ordinate axis — the amplitude of the signals in the interval from 0 to 1, along the abscissa axis — the time interval from 0 to 100. For dynamic spectra along the abscissa axis — the frequency in relative units and

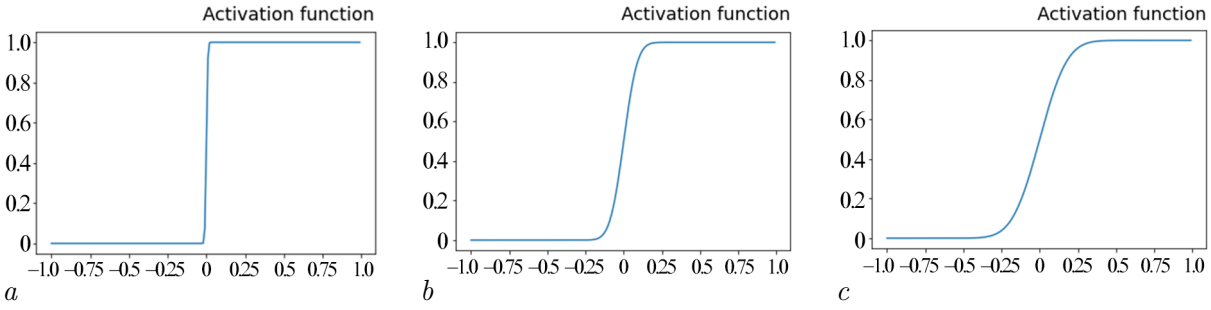


Fig 4. Функции энергообеспечения при различных значениях параметра d: *a* — соответствует системе с эталонной зрелостью ритмики, $d = 0.01$; *b* — соответствует системе с умеренной регрессией ритмики, $d = 0.1$; *c* — соответствует системе с глубокой регрессией ритмики, $d = 0.2$

Fig. 4. Energy supply functions for different values of the parameter d: *a* — corresponds to a system with standard rhythmic maturity, $d = 0.01$; *b* — corresponds to a system with moderate rhythmic regression, $d = 0.1$; *c* — corresponds to a system with deep rhythmic regression, $d = 0.2$

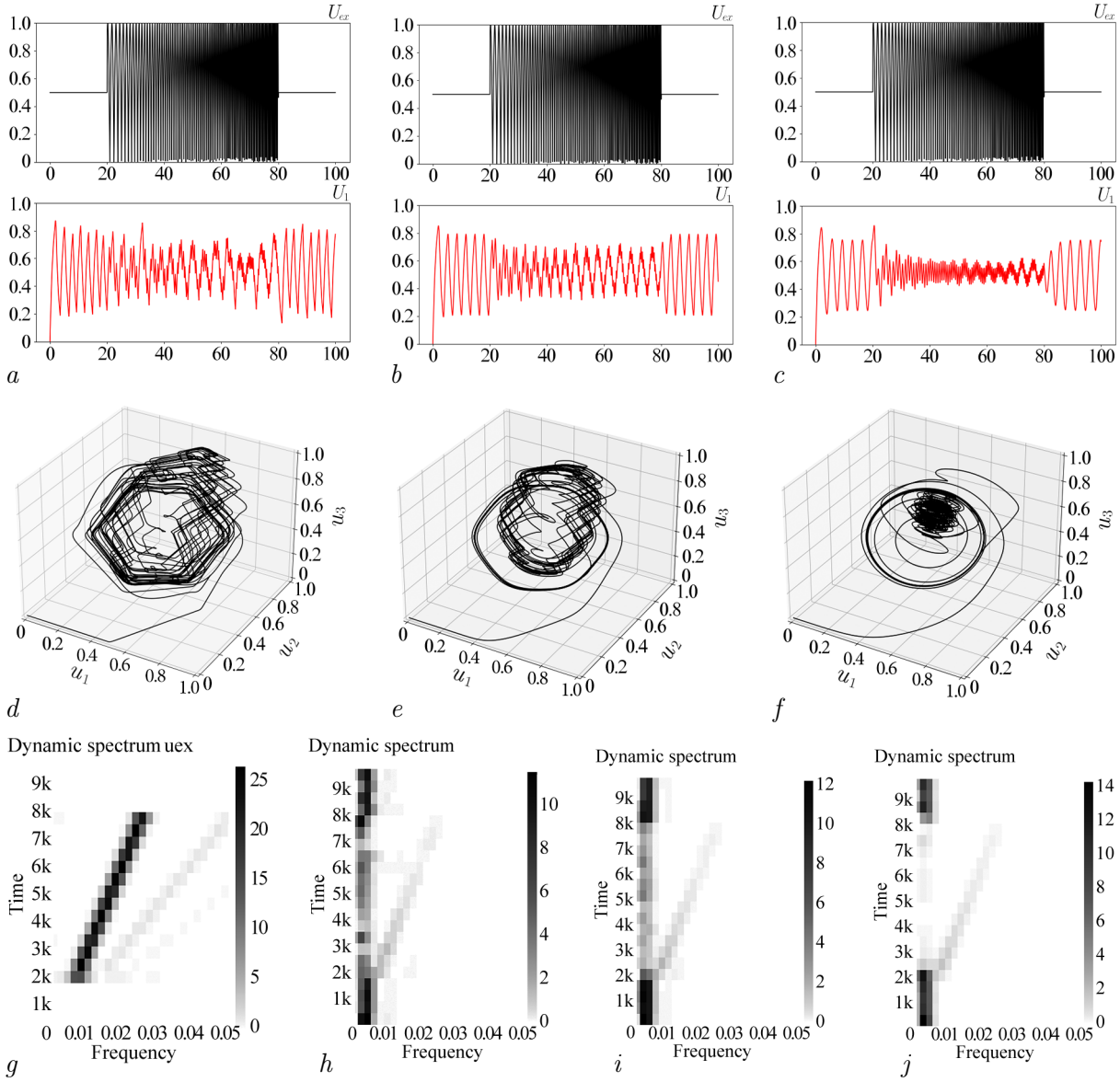


Fig 5. Model responses to an external signal with a smoothly varying frequency for different values of the parameter d: a — corresponds to a system with standard rhythmic maturity, $d = 0.01$; b — corresponds to a system with moderate rhythmic regression, $d = 0.1$; c — corresponds to a system with deep rhythmic regression, $d = 0.2$. d — Phase space of the system for $d = 0.01$, e — phase space of the system for $d = 0.1$, f — phase space of the system for $d = 0.2$. g — Dynamic spectrum of the external signal; h — dynamic spectrum corresponding to mature rhythm, $d = 0.01$; i — dynamic spectrum corresponding to rhythm with moderate regression, $d = 0.1$; j — dynamic spectrum corresponding to rhythm with deep regression, $d = 0.2$

the interval from 0.01 to 0.05, along the ordinate axis — the time in counting steps from 0 to 10000 taking into account the step interval of 0.01.

In Fig. 5 The vertical columns show the model's responses to an external signal with a smoothly changing frequency for different values of the parameter d. The first column of Fig. 5 $d = 0.01$ corresponds to a system with standard rhythmic maturity; the second column $d = 0.1$ corresponds to a system with moderate rhythmic regression; the third column $d = 0.2$ corresponds to a system with deep rhythmic regression. The external signal and the system's response are

shown from top to bottom. The ordinate axis shows the signal amplitude in the interval from 0 to 1, and the abscissa axis shows the time interval from 0 to 100. In Fig. 5, *d, e, f* are the phase portraits of the system for different parameters d. In Fig. 5, *g, h, i, j* — dynamic spectrum of the external signal; dynamic spectrum corresponding to mature rhythm $d = 0.01$; dynamic spectrum corresponding to rhythm with moderate regression $d = 0.1$; dynamic spectrum corresponding to rhythm with deep regression $d = 0.2$. The abscissa axis shows the frequency in relative units and the interval from 0.01 to 0.05, the ordinate axis shows the time from 0 to 1, and the abscissa axis shows the time in counting steps from 0 to 10000 taking into account the time step 0.01.

Fig. 6 shows the responses of the model with the parameter $d = 0.01$ to an external signal with incorrectly specified parameters of the external signal. Fig. 6, *a* a low-amplitude external signal (in Fig. 6, *b* is the dynamic spectrum of the external signal) does not evoke a response in the cortex variable (Fig. 6, *c* is the dynamic spectrum of the cortex variable signal). In Fig. 6, *d* a high-frequency external signal (in Fig. 6, *e* is the dynamic spectrum of the external signal)

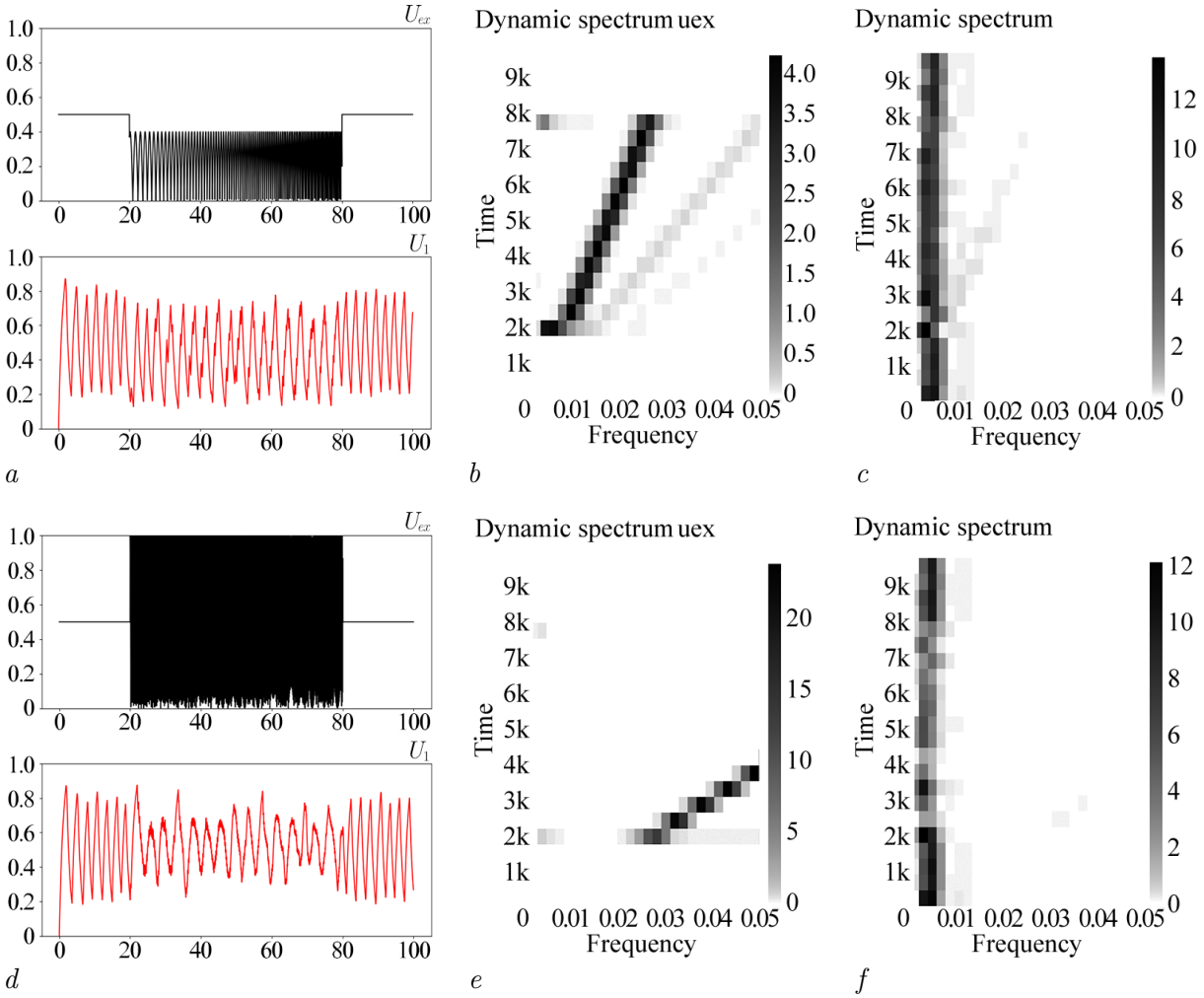


Fig 6. Responses of a model with parameter $d = 0.01$ to an external signal with incorrectly specified parameters of the external signal: *a* — external signal of small amplitude, call of variable cortex; *b* — dynamic spectrum of the external signal; *c* — dynamic spectrum of the variable cortex signal; *d* — external high frequency signal, variable bark call; *e* — dynamic spectrum of the external signal; *f* — dynamic spectrum of the variable cortex signal

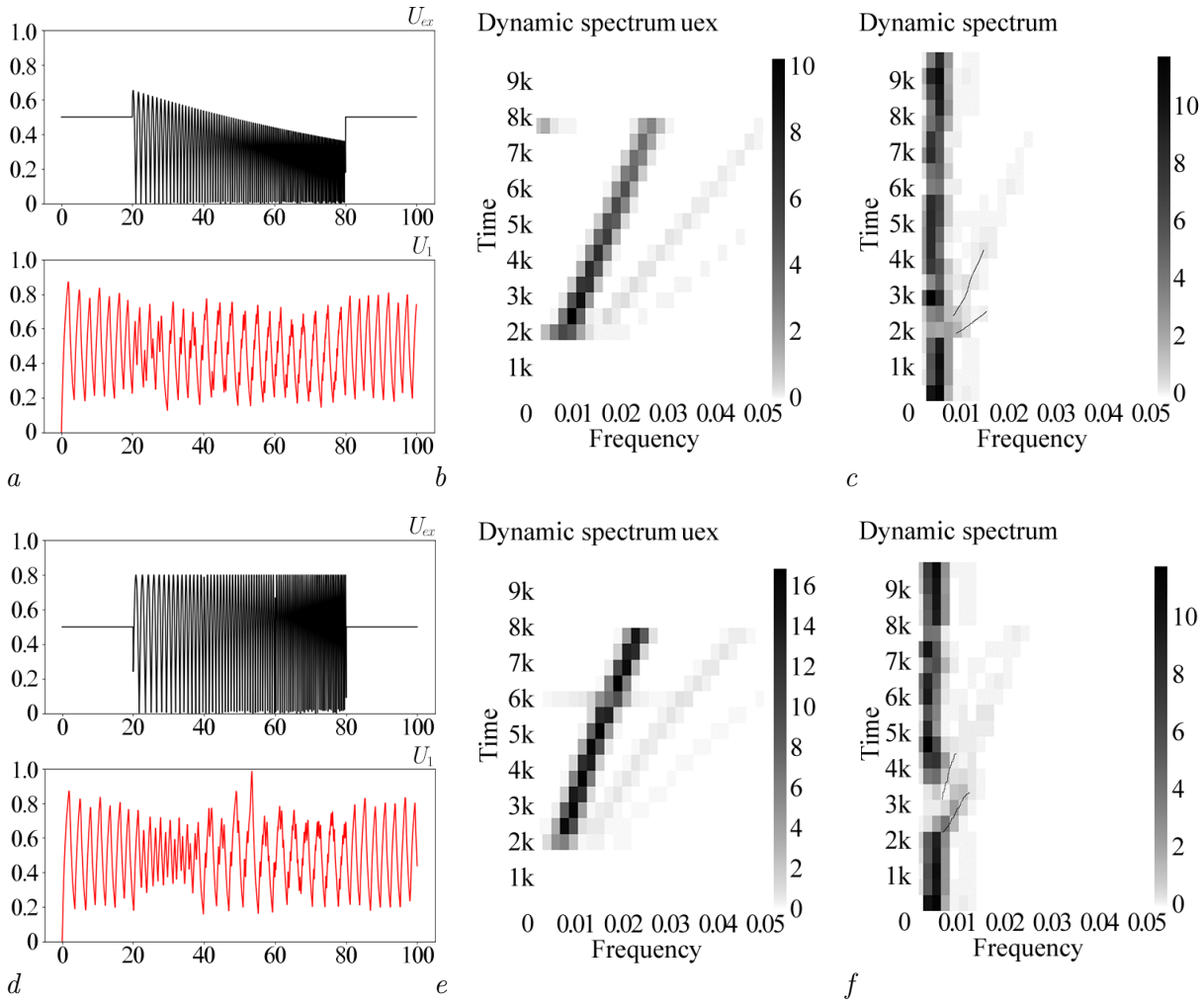


Fig 7. Examples of rhythm animation by various external signals: *a* — external signal with varying frequency and amplitude, variable cortex signal; *b* — dynamic spectrum of the external signal; *c* — dynamic spectrum of the variable cortex signal; *d* — external signal with variable frequency (the frequency of the signal changes from different initial frequencies in each third of the time interval of the external signal), variable bark signal; *e* — dynamic spectrum of the external signal; *f* — dynamic spectrum of the variable cortex signal

does not evoke a response in the cortex variable (Fig. 6, *f* is the dynamic spectrum of the cortex variable signal).

Fig. 7 shows examples of rhythm multiplication by various external signals: an external signal with changing frequency and amplitude, an external signal with interval frequency changes in each third of the time interval of the external signal. Responses to the external signal of the cortex variable are shown; dynamic spectra of the external signal and the signal — response of the variable cortex.

Conclusion

The phenomenological model of neuroplasticity yielded regimes similar to experimental ones, which allow us to qualitatively explain some mechanisms of the emergence of various resonance modes of the thalamocortical system (Table 2). The study of the model showed that the amplitude of the external signal is one of the important control parameters of various resonance

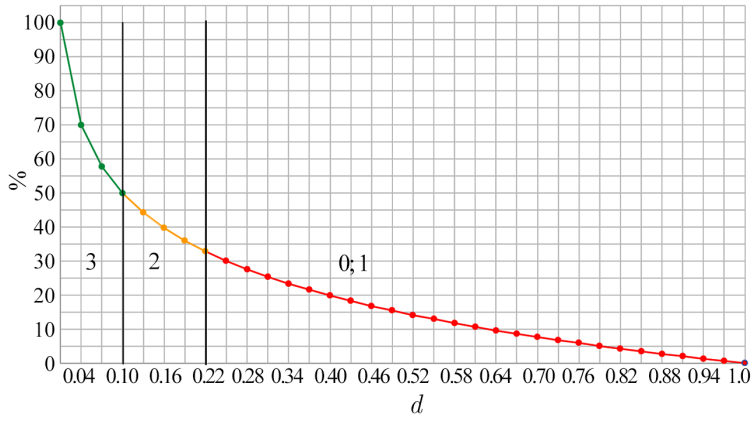
Table 2. Experiment and simulation results

№	Data	Calculation results
1	Alpha activity is recorded with eyes closed. Mature reference rhythm.	In response to a constant signal of a given amplitude $U_{ex} = 0.5$ or to the sum of several harmonic signals of different frequencies in a given interval of subthreshold amplitude, impulse activity with a natural frequency of self-oscillations in the thalamocortical system occurs.
2	An alpha peak is formed in the EEG spectrum.	The appearance of the main rhythm depends, in addition to the amplitude of the external signal, on the slope parameter d of the energy supply function $F[f]$. The smaller d (the steep $F[f]$), the slower the oscillations decay and the main rhythm is maintained, comparable in the experiment with the alpha rhythm of Fig. 4, *a, b*, fig. 5, *a, d, g, h* areas in the absence of an external signal.
3	The alpha peak in the EEG spectrum is not formed.	At large d (flat $F[f]$) the basic rhythm is not formed.
4	Resonant spectral peaks are observed at the stimulation frequency while maintaining the alpha rhythm	The basic rhythm is maintained, and an external frequency is imposed. At a low frequency of the external signal, oscillations are divided into two frequency scales: the frequency of the response envelope is equal to the frequency of the external signal; the frequency of filling with self-oscillations is determined by the system parameters fig. 4, *a, b*, fig. 5, *a, d, g, h*
5	No resonant spectral peaks are observed at the stimulation frequency while maintaining the alpha rhythm.	The frequency of the experimental signal is higher than the natural frequency of the thalamocortical system. The basic rhythm is maintained. The rhythm is not imposed, and the main rhythm is modulated by a high-frequency signal. The amplitude of the experimental signal is below the threshold for excitation of oscillations in the thalamocortical system Fig. 4, *a*, Fig. 6.
6	Resonant spectral peaks are observed at the stimulation frequency, the alpha rhythm is not maintained.	The frequency of the main rhythm is reduced due to the flat function $F[f]$ (d corresponds to insufficient maturity of the cortex rhythms), which is comparable with the transition to other low-frequency delta and theta ranges. The system operates in the forced oscillation mode when an external high-frequency signal is applied Fig. 4, *a*, Fig. 5, *b, c, e, f, g, i, j*.
7	Rhythm multiplication while maintaining the alpha rhythm.	The mode is obtained on the model with an external signal with a smoothly decreasing amplitude Fig. 4, *a*, Fig. 7, *a, b, c*. In the dynamic spectrum of the signal, weak lines are observed at multiple frequencies (drawn). The basic rhythm is preserved, rhythm imposition and multiplication occur. The mode is obtained on the model with an external signal with a linearly changing frequency in a certain interval (3 frequency change intervals during the presentation of the external signal) Fig. 4, *a*, Fig. 7, *d, e, f*. In the dynamic spectrum of the signal, weak lines are observed at multiple frequencies (drawn). The basic rhythm is preserved, rhythm imposition and multiplication occur.

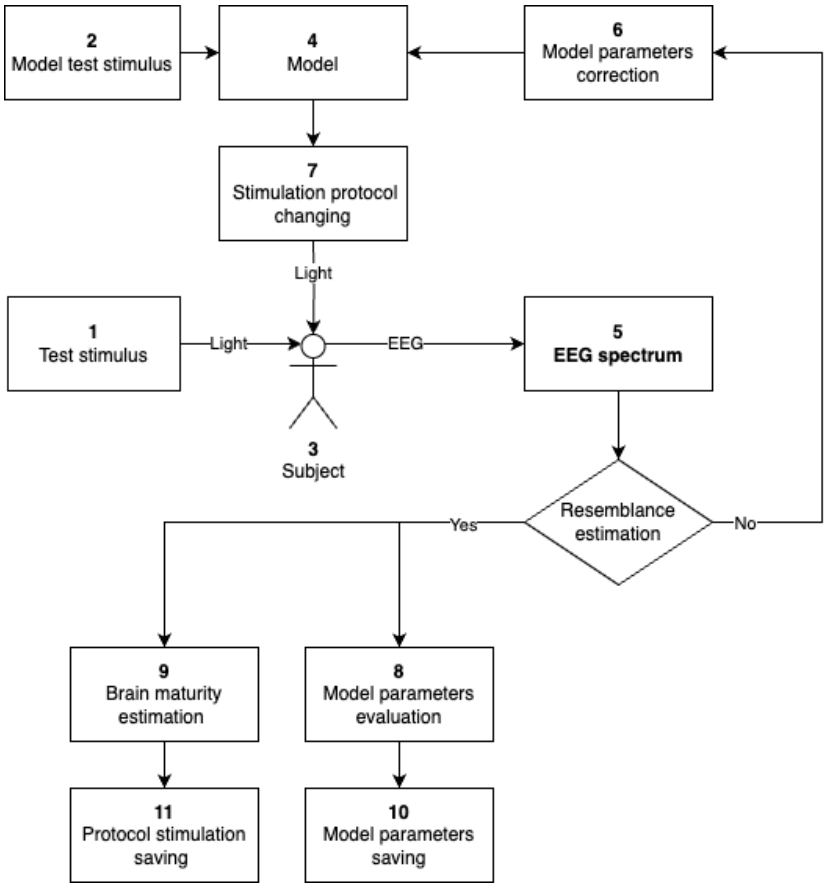
modes in such a system. Therefore, in biocontrol scenarios, it is important to consider not only the frequency, but also the amplitude characteristics of the external signal (see Fig. 3, *b*). The model can be complicated by considering, for example, several interacting cells.

Based on the model prediction, in a neurophysiological experiment, in order to increase the stimulation efficiency, it is necessary to control the signal amplitude (brightness of the light stimuli) (see Fig. 3, *b*), and to select the frequency range of the external signal according to the bioelectrical activity of the subject's brain (see Fig. 3, *b*).

According to the model calculations, the maturity of rhythmicity can be assessed by the calculated parameter d of the thalamocortical cell model, the parameters of which are adjusted in such a way as to correspond to the dynamics of the spectral components of the bioelectrical activity of the subject's brain during background recording and during stimulation with a sequence of light pulses. Fig. 8, *a* shows the inversely proportional dependence of the level of maturity of rhythmicity in percent (ordinate axis) depending on the model parameter d (abscissa axis).



a



b

Fig 8. *a* — Dependence of the degree of rhythmic maturity as a percentage (ordinate axis) on the value of the model parameter d (abscissa axis), 3 — mature rhythmicity, 2 — weak rhythmic regression, 1; 0 — moderate and deep rhythmic regression; *b* — scheme for digital assessment of the degree of maturity of brain rhythms

In the Table 2 a generalization of the model calculations and the data of the psychophysiological experiment is clearly presented.

Scheme of digital assessment of the degree of maturity of the cortex. Based on the data of psychophysiological experiments on neurobiocontrol and model calculations, the following scheme of digital assessment of the degree of maturity of the brain rhythm can be proposed (Fig. 8, *b*).

1. In the absence of an external signal, with eyes closed, the alpha rhythm boundaries and the presence of a peak frequency in the alpha rhythm are determined. A preliminary assessment of the degree of maturity of the cortex rhythm is performed (table 2).
2. A test stimulus of the model with parameters corresponding to the preliminary assessed degree of maturity of the rhythm according to the psychophysiological experiment is presented.
3. A stimulation protocol is formed according to item 1.
4. A computational experiment is carried out with model parameters corresponding to the conducted psychophysiological experiment.
5. The calculated spectra and spectra of the real EEG signal recorded during the psychophysiological experiment are evaluated.
6. The model parameters are changed to obtain similar resonant dynamics of the signal in the form of assimilation of the rhythm of the external signal.
7. Correction of the amplitude and frequency characteristics of the external signal in the psychophysiological experiment. Repeating items 3, 4, 5 until satisfactory similarity of the obtained spectra is achieved.
8. The degree of maturity of the subject's cortical rhythm is assessed.
9. The model parameter is estimated, which is compared with the degree of maturity of the cortical rhythm (the model parameter d is recalculated into percentages, Fig. 8, *a*).
10. The model parameters are saved and a new calculation of item 2 is prepared according to the scenario «Stimulation of the plasticity effect by an external signal».
11. The result of the psychophysiological experiment is recorded and a new experiment is prepared according to the scenario «Stimulation of the neuroplasticity of the cortex».

Conclusions. A mathematical model of interaction of thalamocortical system modules has been successfully applied to control brain neuroplasticity. In the course of comparing the results of psychophysiological and computational experiments on neurobiofeedback, a library of model modes (dynamic spectra, test signals) is formed to describe a «prototype model» corresponding to different levels of cortical rhythmic maturity.

A method for digital diagnostics of the level of brain rhythmic maturity has been developed based on a comparison of modeling results and data from a psychophysiological experiment on neurobiofeedback.

The evolution of model solutions depending on its parameters simulates the process of brain neuroplasticity biocontrol taking into account the initial level of rhythmic maturity and stress-induced distortions of neurodynamics.

Experiments on the model with different parameters of the model and external signal can be used in the development of new neurobiofeedback protocols.

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