Modeling of Global Processes. Nonlinear Dynamics and Humanities

Izvestiya Vysshikh Uchebnykh Zavedeniy. Applied Nonlinear Dynamics. 2024;32(5)

Article DOI: 10.18500/0869-6632-003123

Modeling language competition in a bilingual community*

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Received 29.11.2023, accepted 10.05.2024, available online 11.09.2024, published 30.09.2024

Abstract. The *purpose* of this study — construction and research of a new mathematical model of a bilingual community, which takes into account: the effect of mutual assistance within a group of speakers of the same language, the effect of language acquisition by children of bilingual parents at an early age, different prestige of languages for adults. *Methods.* A new model is being built that takes into account new effects. The model is studied using classical methods with an unlimited increase in dynamics time. The effect of mutual assistance is compared with the effect of language volatility introduced by Abrams and Strogatti. Based on the observed statistical data, using the regression method, the parameters of some languages of England and Canada are determined: Welsh, Scottish, English, French. A forecast is being made for the further development of dynamics. *Results.* The effects taken into account in the model are confirmed by the correspondence of the development of language dynamics to the characteristics of the language: large values of the parameters of mutual assistance correspond to such a development of language dynamics in which one language displaces the second; at low values of mutual assistance, languages coexist. To model language dynamics using the new model, real statistical data on language pairs is used: Welsh-English, Scots-English, French-English. A forecast is being made for the further development of dynamics by language. *Conclusion.* General concepts in language dynamics have been supplemented with new ones — the power of mutual assistance within a group of speakers of the same language. The similarity between the effect of language volatility and the effect of mutual assistance is noted.

Keywords: extinction of languages, mutual aid effect, language volatility, bilingualism, language competition, language dynamics, language preservation, mathematical model, ordinary differential equations.

For citation: Medvedev AV, Kuzenkov OA. Modeling language competition in a bilingual community. Izvestiya VUZ. Applied Nonlinear Dynamics. 2024;32(5):691–708. DOI: 10.18500/0869-6632-003123

Introduction

The problem of language extinction is currently very acute. It is studied by various methods, including mathematical modeling [1–7]. Among the first in mathematical modeling of language dynamics were Abrams and Strogatti [1]. Their model allows us to explain historical data on the

*The paper presents materials of a talk given at the conference "Nonlinear dynamics in cognitive research — 2023".

decline of Welsh, Scots and other disappearing languages. The Abrams–Strogatti (AS) model assumes that any member of the community in question, regardless of what languages s/he speaks, currently prefers only one of the two. It also assumes that newborn children learn and use the language their parents prefer, so the change of generations does not affect the proportion of language speakers in society. Abrams and Strogatti introduced the concepts of language prestige and language volatility as the readiness of language speakers to change it. The number of community members was assumed to be constant. The AS model showed that one language, given the assumptions made, is always displaced by another over time. The latter was called language death in the AS model of language dynamics [1]. In 2005, the Mira and Peredes model [2] appeared. This model introduced a new characteristic – language similarity. This model showed that if languages are very similar, they can coexist for a long time.

In 2006, the Castello model [3] appeared. Its author introduced a new group of bilingual individuals — bilinguals, and showed the possibilities of stable coexistence of two languages in a community. The model of interaction between monolingual and bilingual populations by Baggs and Friedman [4, 5] demonstrated the possibilities of developing dynamics, in which language groups can coexist or displace each other. Whyburn and Hayward in their model demonstrated the importance of a stable bilingual group for the coexistence of two languages [6]. Diaz and Schwidke introduced a new concept — language status [7].

Abrams and Strogatti reasonably believed that a child will speak the language of his parents, and this hypothesis was subsequently retained in new models, including models with bilinguals. But the idea that bilingual children, like their parents, acquire two languages is not entirely accurate. A child growing up in a bilingual environment can acquire two languages to some extent, but one of them may be lost due to lack of use [8, 9]. An effect has been noted in which children of bilingual parents first acquire the first or second language with varying degrees of probability and only later, presumably in adulthood, with some probability learn the second language, while children whose parents speak only one language first acquire the language of their parents and acquire the second language with some probability. That is, the acquisition of two languages by children occurs sequentially [8, 10].

It is worth noting that the initial AS model did not include the language volatility indicator. It was added by the authors to better match the dynamics of the model with the dynamics observed in reality. We assume that the dynamics are also influenced by other effects, such as the mutual aid effect in the corresponding language group. Mutual aid within a group of native speakers of one language directly affects the attractiveness of the second language for them. Thus, if mutual aid is high, then nothing stimulates the native speaker of this language to change it, and vice versa. In this regard, we supplement the general concepts of language dynamics with new characteristics: the probability of early language acquisition by children, the probability of second language acquisition by adults, and the strength of mutual aid within a single-language group. The studies made the first attempts to take into account the effect of early language acquisition by children [11, 12], but this effect was considered without taking into account the different probabilities of second language acquisition by adults. The resulting models describe situations in which there is stable bilingualism or only one of the two languages is preserved. In the present study, we study a model that takes into account all the described effects. This may lead to the emergence of new qualitative features in the behavior of the model.

The purpose of this study is to construct and investigate a new model of a bilingual community that takes into account: the effect of mutual assistance within groups of one language; the effect of language acquisition by children of bilingual parents at an early age, taking into account the different probabilities for acquiring a second language; and the different prestige of a language for speakers.

1. Methodology

1.1. Model of a bilingual community. We will accept the following hypotheses for constructing the model:

- community members can speak one of two languages, conventionally called «first» and «second», or both at once; z_1 — the proportion of community members who speak only the first language, z_2 — the proportion of community members who speak only the second language, z_{12} – the proportion of community members who speak two languages (bilinguals);
- the proportion of individuals who do not speak either language is negligibly small;
- the size of any language group is non-negative: $0 \leqslant z_1, z_2, z_{12} \leqslant 1$;
- the size of the community is constant over time (the number of births is equal to the number of deaths), $z_1 + z_2 + z_{12} = 1$;
- the coefficient r simultaneously characterizes both the birth rate and the death rate;
- the probability of simultaneous (spontaneous) acquisition of two languages by a child is negligibly small;
- bilingual children initially acquire the first or second language with probabilities c_1 and c_2, respectively; $c_1 + c_2 = 1$, it is assumed that $c_1 > c_2$ [8, 9];
- within language groups there is a mutual aid effect, which is determined by the coefficients: α-for native speakers of the first language and β-for native speakers of the second language;
- the strength of the mutual aid effect has a linear dependence with the opposite sign on the number of native speakers: when the proportion of native speakers is close to zero, the strength of the effect is close to its maximum; when the proportion is close to one, the strength of the effect is close to zero;
- prestige, introduced by Abrams and Strogatti, is determined by the coefficients b_1 and b_2 for the first and second languages, respectively;
- when native speakers of different languages meet (the frequency of which is directly proportional to the product of their proportions), a change of language is possible with coefficients $b_{1,2}$ for the first and second languages, respectively;
- it is assumed that monolingual members of a community can be taught a second language by bilinguals [3];
- the principle of interaction between native speakers of languages in a community generalizes the well-known hypothesis of effective meetings [13].

Taking into account the accepted hypotheses, we find that the dynamics of language speakers in society is characterized by the following system:

$$\begin{cases} \dot{z_1} = c_1 r z_{12} - b_1 z_1 (z_2 + z_{12}) + \alpha z_1^2 (1 - z_1), \\ \dot{z_2} = c_2 r z_{12} - b_2 z_2 (z_1 + z_{12}) + \beta z_2^2 (1 - z_2), \\ \dot{z_{12}} = z_1 (b_1 - \alpha z_1)(z_2 + z_{12}) + z_2 (b_2 - \beta z_2)(z_1 + z_{12}) - r z_{12}. \end{cases} \tag{1}$$

The initial values of the proportions of native speakers cannot be negative, based on the understanding of the meaning of these values. The validity of the statement that for non-negativity of solutions for all phase coordinates, under non-negative initial conditions, it is necessary and sufficient to satisfy the following requirement: when any phase coordinate is zero, the right-hand side of the corresponding equation of the model (1) must be non-negative (these conditions are called quasi-positivity conditions [14]). For this system, these conditions are satisfied. Preservation of a constant sum of phase coordinates is ensured by the fact that the sum of all equations of the system is identically equal to zero: $\dot{z_1} + \dot{z_2} + \dot{z_{12}} = 0$. For the system (1) this condition is also satisfied. Since at the initial moment $z_1 + z_2 + z_{12} = 1$, this equality will be preserved at all subsequent moments of time. In the work [1], by studying statistical data, approximate values of the parameters were established: $\alpha, \beta = 1.31 \pm 0.25$. Based on this, the present study is carried out for the following realistic limits of change of the specified parameters: $\alpha, \beta = 1.5 \pm 0.5$.

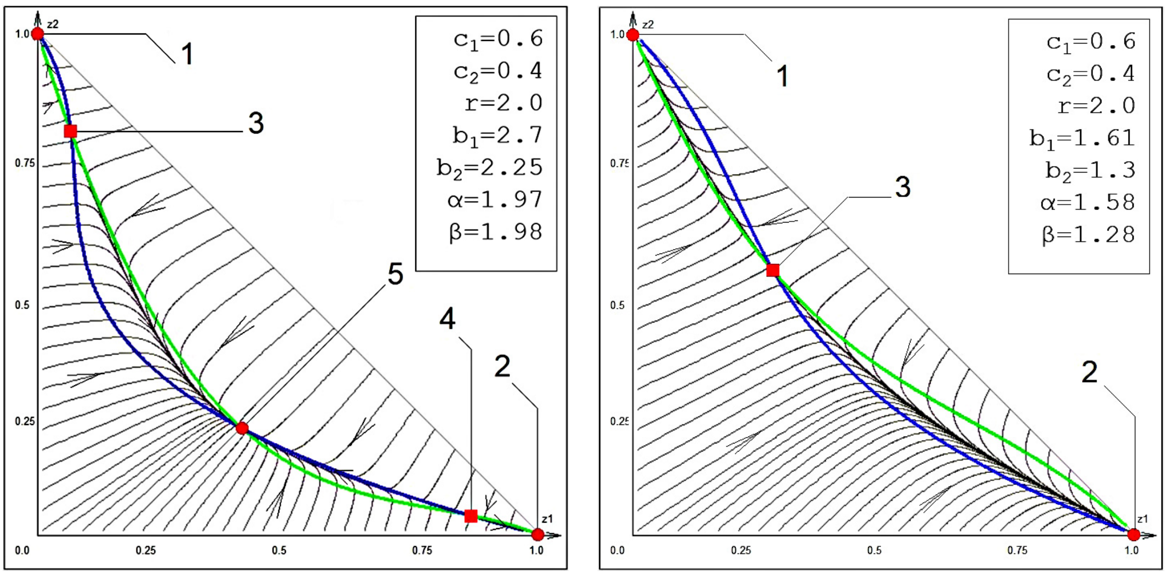
2. Results

2.1. Model study. The phase space of the system (1) is the three-dimensional standard simplex [14]. By projection, expressing z_{12} in terms of z_1 and z_2, this model can be reduced to a system on the plane

$$\begin{cases} \dot{z_1} = c_1 r(1 - z_1 - z_2) - b_1 z_1 + (\alpha + b_1) z_1^2 - \alpha z_1^3, \\ \dot{z_2} = c_2 r(1 - z_1 - z_2) - b_2 z_2 + (\beta + b_2) z_2^2 - \beta z_2^3, \end{cases} \tag{2}$$

Since $z_{12} \geqslant 0$, then the inequality $z_1 + z_2 \leqslant 1$ holds for the model (2). Let us check the quasi-positivity conditions for the model (2). Let $z_1 = 0$, then $\dot{z_1} = c_1 r(1 - z_2)$, and since $0 \leqslant z_2 \leqslant 1$, then $\dot{z_1} \geqslant 0$. Let $z_2 = 0$, then $\dot{z_2} = c_2 r(1 - z_1)$, and since $0 \leqslant z_1 \leqslant 1$, then $\dot{z_2} \geqslant 0$.

The system (2) was investigated by standard qualitative methods. Phase portraits of the system (2) for different parameters of the tongues are shown in (Fig. 1). Red dots mark stable equilibrium states, and red squares mark unstable ones. Blue and green curves show isoclines of horizontal and vertical slopes, respectively. The number of intersections of isoclines with each other determines the number and nature of equilibrium points, as well as possible bifurcations. A



a *b*

Fig. 1. Phase plane for model (2): *a* — coexistance of two languages and bilinguals, $r - 0.873 - (\min[\frac{b_1}{\alpha}, \frac{b_2}{\beta}])^3 < 0$; *b* — displacement of one language by another, $r - 0.873 - (\min[\frac{b_1}{\alpha}, \frac{b_2}{\beta}])^3 > 0$; *blue curve* — isocline of horizontal slope; *green curve* — is the isocline of the vertical slope (color online)

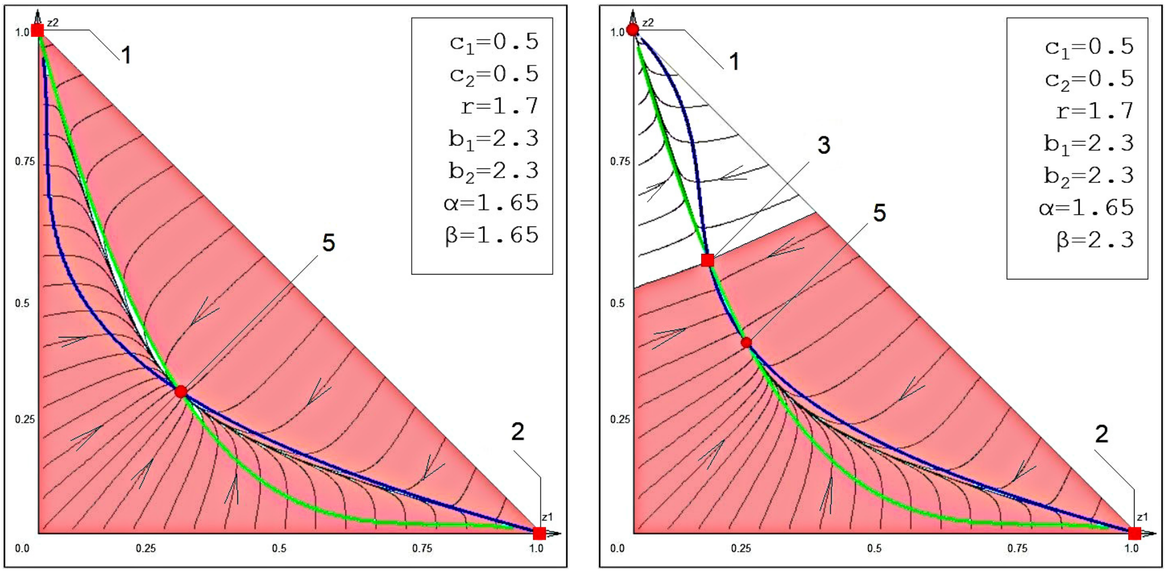
detailed consideration of bifurcations is given in Section 2.2. States No.1 and No.2, depending on the parameters, are either stable nodes or unstable of the saddle type, and are always located on the boundary of the phase space at the points $(0, 1)$ and $(1, 0)$ (see Fig. 1). The stability of these states was determined by the Lyapunov method by determining the eigenvalues. The inequality $\alpha > b_1$ was obtained, under which the point $(1, 0)$ is a stable node, otherwise it is unstable of the saddle type. The inequality $\beta > b_2$ was obtained, under which the point $(0, 1)$ is a stable node, otherwise it is unstable of the saddle type. Taking this into account, we can reliably assert the coexistence of languages under the simultaneous fulfillment of the conditions $\alpha < b_1, \beta < b_2$. The remaining equilibrium states are located inside the region defined by the following constraints:

$$\begin{cases} 0 \leqslant z_1, z_2 \leqslant 1, \\ z_1 + z_2 \leqslant 1. \end{cases} \tag{3}$$

Their coordinates were determined by searching for the intersection points of the vertical and horizontal slope isoclines. This problem was solved using the WolframAlpha software package. Calculations showed that the system (2) can have 2–5 equilibrium states. The equilibrium character of the remaining states was determined by numerically constructing a phase portrait: states 3 and 4 are unstable of the saddle type, and states 1, 2 and 5 are stable of the node type (see Fig. 1). The following range of parameters was used to construct the phase portrait: $0.0 \leqslant c_1, c_2 \leqslant 1.0$; $1.0 \leqslant r, b_1, b_2 \leqslant 9.0$; $0.5 \leqslant \alpha, \beta \leqslant 2.5$; grid step $step = 0.005$. A qualitative study of the system (2) shows two possible variants of dynamics: the survival of only one language or their coexistence. Numerical estimates of the parameters were obtained that guarantee the implementation for the first case $r - 0.873 - (\min[\frac{b_1}{\alpha}, \frac{b_2}{\beta}])^3 > 0$, for the second case $r - 0.873 - (\min[\frac{b_1}{\alpha}, \frac{b_2}{\beta}])^3 < 0$.

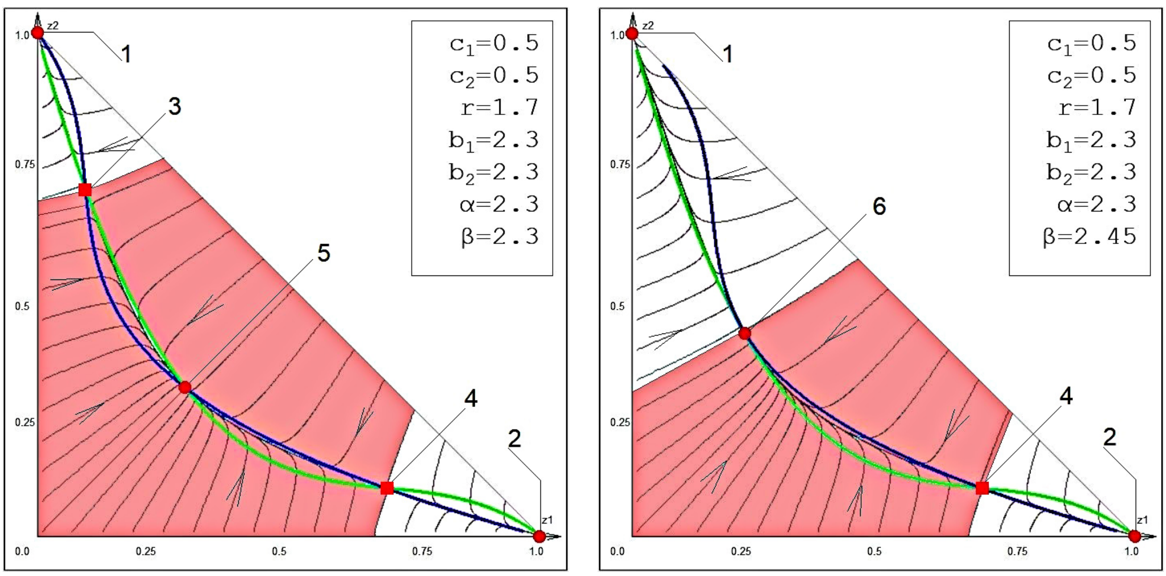
2.2. The influence of different values of mutual assistance coefficients on language dynamics. Let us consider the variant of the model parameters (2), when bilingualism is guaranteed to be preserved in the community $\alpha < b_1, \beta < b_2$, and the estimated parameters $r - 0.873 - (\min[\frac{b_1}{\alpha}, \frac{b_2}{\beta}])^3 < 0$ are also true. At the first stage of consideration, we will choose the parameters of two competing languages equal to: $c_1 = c_2, b_1 = b_2, \alpha = \beta$. To consider possible bifurcations in the system, we will increase the parameters of mutual assistance, starting with β. If the parameter of mutual assistance is small enough, then the language dynamics will develop in such a way that stable coexistence of languages will be possible for any initial values of the shares of the number of languages. This is clearly seen in the basin of attraction of stable state No.5 on the phase plane (Fig. 2, *a*), which is located in the middle of the region (3). The boundaries of the basin of attraction of state No.5 were determined numerically. States No.1 and No.2 are unstable. Increasing the mutual assistance parameter for the second language β will lead to the inequality $\beta \geqslant b_2$, and as a result of the saddle-node bifurcation, the unstable state No.1 will split into two new states: No.1 — a stable node and No.3 — an unstable saddle type. The basin of attraction of stable state No.5, the dynamics of whose trajectories correspond to the coexistence of languages, will shrink (see Fig. 2, *b*). Increasing the mutual assistance parameter for the first language α will lead to similar changes in the phase portrait for the opposite corner of the region (3) — the unstable node at point $(1, 0)$ as a result of saddle-node bifurcation will turn into two new states: No.2 — a stable node, No.4 — an unstable saddle-type node (Fig. 3, *a*).

Further increase of the mutual assistance parameter β will result in the basin of attraction of the stable state No.5, in which the languages coexist, being reduced on the side of the z_2 axis, and the state of equilibrium No.3 — saddle — will approach the state of equilibrium No.5 — stable node. Continuing to increase the parameter β, we will obtain such changes in the phase



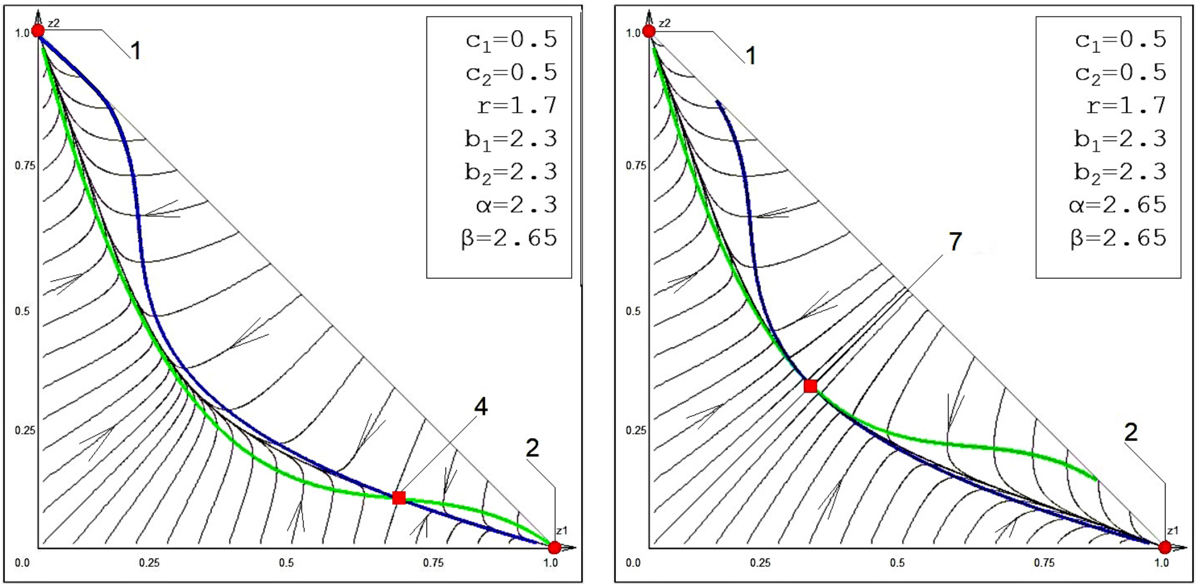
a *b*

Fig. 2. Steady State attraction pool No.5 and No.6 models (2): the are a in which two languages and bilinguals coexist is marked in *red*; *blue curve* — isocline of horizontal slope; *green curve* — is the isocline of the vertical slope; *a* — language mutual aid parameters: $\alpha < b_1, \beta < b_2$; *b* — language mutual aid parameters: $\alpha < b_1, \beta \geqslant b_2$ (color online)



a *b*

Fig. 3. Pools of attraction for equilibrium states of the model (2): the are a in which two languages and bilinguals coexist is marked in *red*; *blue curve* — isocline of horizontal slope; *green curve* — is the isocline of the vertical slope; *a* — language mutual aid parameters: $\alpha \geqslant b_1, \beta \geqslant b_2$; *b* — language mutual aid parameters: $\alpha \geqslant b_1, \beta > b_2$ (color online)



a *b*

Fig. 4. Phase trajectories of the model (2): case of one language being replaced by another; *blue curve* — isocline of horizontal slope; *green curve* — is the isocline of the vertical slope; *a* — language mutual aid parameters: $\alpha \geqslant b_1, \beta > b_2$; *b* — language mutual aid parameters: $\alpha > b_1, \beta > b_2$ (color online)

portrait of the system, in which states No.3 and No.5 as a result of a saddle-node bifurcation will merge into one state No.6 according to the saddle-node type (see Fig. 3, *b*). The isoclines of the vertical and horizontal slope will touch each other. This situation is not rough and with a further increase of the parameter β the state of equilibrium No.6 will disappear (Fig. 4, *a*). The coexistence of languages will become impossible, and one language will always displace the other. The dynamics of the model in the vicinity of equilibrium state No.6 behaves in such a way that the number of speakers of the first language decreases. If the ratio of the first and second languages is above a certain level, then over time a fixed proportion is established at this level between bilinguals and speakers of two languages. But if their ratio is below this level, then the first language is lost over time. If in the system shown in (see fig. 3, *b*), the parameter β is fixed and the parameter α begins to increase, then equilibrium state No.4 (saddle) will begin to shift towards equilibrium state No.6 (saddle-node) and as a result of saddle-node bifurcation will merge with it into state No.7 (saddle). This will also lead the system to a case in which the coexistence of languages is impossible. An increase in the coefficients of mutual assistance leads to the fact that in the middle of the phase plane there remains one unstable equilibrium state of the saddle type, and the dynamics of the system corresponds to the second case — displacement of one language by another (see Fig. 4, *b*). It turns out that with an increase in the coefficients α and β, the effect of mutual assistance within groups of one language gains strength and begins to act in such a way that bilingualism becomes impossible. Based on this analysis of the dynamics of the system, it can be argued that mutual assistance within a group prevents the formation of bilingualism and promotes the dominance of the language in the community under consideration.

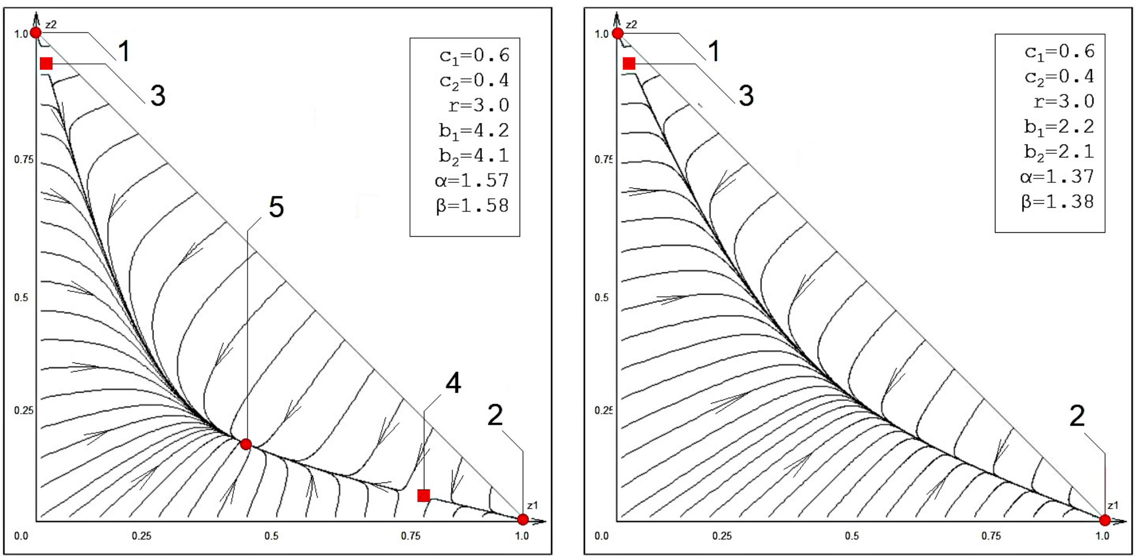
2.3. Model with the effect of language volatility of Abrams and Strogatti. To compare two models with different effects (mutual aid and language volatility), we consider a model based on the classical equations of language volatility. Given the hypotheses adopted for constructing the model (1) and replacing the hypothesis of mutual aid with the hypothesis of language volatility used in the AS model, we obtain that the dynamics of language speakers in society is determined by the following system:

$$\begin{cases} \dot{z_1} = c_1 r z_{12} - b_1 z_1 (z_2 + z_{12})^{\alpha}, \\ \dot{z_2} = c_2 r z_{12} - b_2 z_2 (z_1 + z_{12})^{\beta}, \\ \dot{z_{12}} = b_1 z_1 (z_2 + z_{12})^{\alpha} + b_2 z_2 (z_1 + z_{12})^{\beta} - r z_{12}. \end{cases} \tag{4}$$

The phase space for (4) is a three-dimensional unit simplex. Phase portraits for various parameters of the languages of the system (4) are shown in Fig. 5. Equilibrium states No.1 and No.2, depending on the parameters, are either stable nodes or unstable saddle-type ones, and are always located on the boundary of the simplex at points $(0, 1)$ and $(1, 0)$. Their stability was investigated by the Lyapunov method. By searching for eigenvalues it was established that for $\frac{\alpha}{2} \geqslant b_1$ the point $(1, 0)$ is a stable node, otherwise it is unstable of the saddle type, and for $\frac{\beta}{2} \geqslant b_2$ the point $(0, 1)$ is a stable node, otherwise it is unstable of the saddle type. Taking this into account, we can confidently assert the coexistence of languages under the simultaneous fulfillment of the conditions $\frac{\alpha}{2} \geqslant b_1, \frac{\beta}{2} \geqslant b_2$. The remaining equilibria are located inside the simplex, their first coordinates are determined by an equation of the following form:

$$1 - \big(1 - z_1(1 + \frac{b_1}{c_1 r}(1 - z_1)^{\alpha}\big)\left(1 + \frac{b_2}{c_2 r}\left(1 - \left(1 - z_1\big(1 + \frac{b_1}{c_1 r}(1 - z_1)^{\alpha}\big)\right)^{\beta}\right)\right) = 0. \tag{5}$$

The equation (5) was solved numerically for all possible parameter values. The constraints on the parameters $b_{1,2}, c_{1,2}$ are determined by the data observed in reality. The constraints on the volatility parameters $\alpha, \beta = 1.31 \pm 0.25$ were introduced by Abrams and Strogatti [1]. As a result,



a *b*

Fig. 5. Фазовые портреты модели (4): a — сосуществование двух языков и билингвов, $r - 1.1125 - (\min[\frac{b_1}{\alpha}, \frac{b_2}{\beta}]) > 0$; b — вытеснение одного языка другим, $r - 1.1125 - (\min[\frac{b_1}{\alpha}, \frac{b_2}{\beta}]) < 0$ (цвет онлайн)

Fig. 5. Phase plane for model (4): a — coexistance of two languages and bilinguals, $r - 1.1125 - (\min[\frac{b_1}{\alpha}, \frac{b_2}{\beta}]) > 0$; b — displacement of one language by another, $r - 1.1125 - (\min[\frac{b_1}{\alpha}, \frac{b_2}{\beta}]) < 0$ (color online)

it was found that the number of roots of the equation on the interval $[0,1]$ can be from 2 to 5. There are always two roots 1 and 0.

The equations of isoclines of vertical and horizontal slope are determined:

$$\text{Ver}: z_2 = 1 - z_1 - \frac{b_1}{c_1 r} z_1 (1 - z_1)^{\alpha}, \text{Hor}: z_1 = 1 - z_2 - \frac{b_2}{c_2 r} z_2 (1 - z_2)^{\beta}. \tag{6}$$

States No.3, No.4, and No.5 were determined numerically as the intersection points of the vertical and horizontal tilt isoclines, and the nature of their equilibrium was determined by numerically constructing a phase portrait. For (Fig. 5), the equilibrium states have the following type: No.1, No.2, and No.5 are stable nodes, while No.3 and No.4 are unstable saddles. The following range of parameters was used to construct the phase portrait: $0.0 \leqslant c_1, c_2 \leqslant 1.0$; $1.0 \leqslant r, b_1, b_2 \leqslant 9.0$; $0.5 \leqslant \alpha, \beta \leqslant 2.5$; grid step step $= 0.005$. A qualitative study of the model (4) showed that if at least one of the inequalities $\frac{\alpha}{2} \geqslant b_1$ and $\frac{\beta}{2} \geqslant b_2$ is satisfied, the coexistence of languages is determined by the presence of stable states inside the simplex. Numerical estimates of the parameters were obtained that guarantee the implementation for the first case $r - 1.1125 - (\min[\frac{b_1}{\alpha}, \frac{b_2}{\beta}]) \geqslant 0$, for the second case $r - 1.1125 - (\min[\frac{b_1}{\alpha}, \frac{b_2}{\beta}]) < 0$. The dynamics of the two models are almost identical, and a comparison of the phase portraits for (4) and (1) revealed their close similarity. Such a correspondence gives grounds to speak about the similarity of the effects of language volatility and mutual assistance. Expressing z_{12} through z_1 and z_2, we reduce the model (4) to a model on a plane, and approximate the power terms of the right-hand sides of the equations of the model (4) using Taylor series.

$$\begin{cases} \dot{z_1} = c_1 r (1 - z_1 - z_2) - b_1 z_1 + \alpha b_1 z_1^2 - \dfrac{\alpha(\alpha - 1)}{2} z_1^3, \\ \dot{z_2} = c_2 r (1 - z_1 - z_2) - b_2 z_2 + \beta b_2 z_2^2 - \dfrac{\beta(\beta - 1)}{2} z_2^3. \end{cases} \tag{7}$$

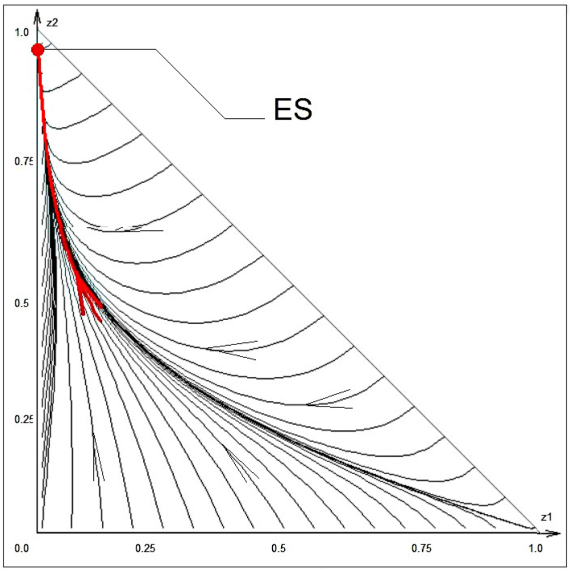
The resulting approximation model (7) turned out to be almost identical to the model (2). They differ only in the coefficients of the second- and third-degree variables: for (2) they are equal to $\alpha + b_1, \beta + b_2, \alpha, \beta$, and for (7) $\alpha b_1, \beta b_2, \frac{\alpha(\alpha-1)}{2}, \frac{\beta(\beta-1)}{2}$. Taking into account the restrictions on the volatility parameters $\alpha, \beta = 1.31 \pm 0.25$ introduced by Abrams and Strogatti [1], the numerical expressions for the parameters of the models (2), (7) become close. This approximation is very useful from a mathematical point of view, since it simplifies the equations of the dynamic system and its study. This approximation also allows us to explain mutual assistance between speakers of individual languages through the already known effect of language volatility, and vice versa. In the original model (7), the parameters of language volatility did not have an obvious explanation, but after comparing its approximation with the model (2), the effect of language volatility can be interpreted as a manifestation of the mutual assistance effect. In this regard, the terms of the right-hand sides of the equations with the coefficients α and β in the models (7 and 4) can be interpreted as the influence of the mutual assistance effect within a group of speakers of the same language, and the values of α and β as the strength of this mutual assistance. A second-order approximation is sufficient to preserve the effect. In the model (7) the dependence of the marginal development of language dynamics on the parameters of language volatility was investigated. The results were similar to those obtained in Section 2.2 for the model (1): with an increase in the coefficients α and β the effect of language volatility gains strength and begins to act in such a way that bilingualism becomes impossible.

2.4. Application of the model to statistical data. The parameters of the models (1) and (4 were identified using the regression method; the values of the coefficients are given in Table 1.

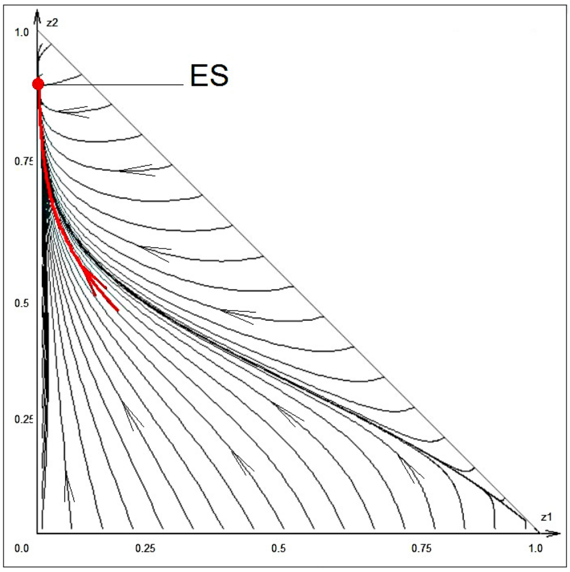
Table 1. Coefficients of models (1) and (4) for language pairs

Language pair	c_1	c_2	b_1	b_2	r	α	β	diff
Welsh and English (1)	0.1	0.9	8.5	5.5	6.1	2.4	0.2	0.012
Welsh and English (4)	0.1	0.9	8.0	0.5	2.7	2.3	0.1	0.007
Scottish and English (1)	0.1	0.9	8.5	1.0	8.8	2.4	0.5	0.111
Scottish and English (4)	0.1	0.9	0.5	0.5	4.2	2.4	0.3	0.724
French and English (Canada) (1)	0.3	0.7	1.0	3.0	1.5	1.5	0.5	0.000
French and English (Canada) (4)	0.7	0.3	0.5	2.5	3.2	0.1	0.5	0.000
French and English (Montreal) (1)	0.5	0.5	6.5	3.5	1.6	0.1	2.1	0.000
French and English (Montreal) (4)	0.6	0.4	7.1	3.1	3.1	0.1	2.3	0.000

The statistical data on the shares of Welsh and English languages in England for 1901–2001 are considered [15, 16]. The models (1) and (4) showed that although English is displacing Welsh, it will not displace it completely. The dynamics will come to a stable coexistence of languages, while the Welsh language group will be very small (Fig. 6, *a*, *b*). The result was obtained based on statistical data on the shares of the Scots and English languages in England for 1891–1971 [15, 16]. The modeling results for the two models showed different results for the outcome of the dynamics. Model (1) showed that the Scots language will be displaced over time (Fig. 7, *a*). Model (4) showed that although the English language prevails over Scots, it will not displace it completely. The dynamics will come to a stable coexistence of languages, while the Scots language group will be very small (see Fig. 7, *b*). Since the models showed different results for the marginal behavior of the dynamics, the values between the actual data and the data obtained as a result of modeling were taken for comparison. For the model (1) the difference in values was diff = 0.111, and for the model (4) diff = 0.724 (table 1). Thus, the new

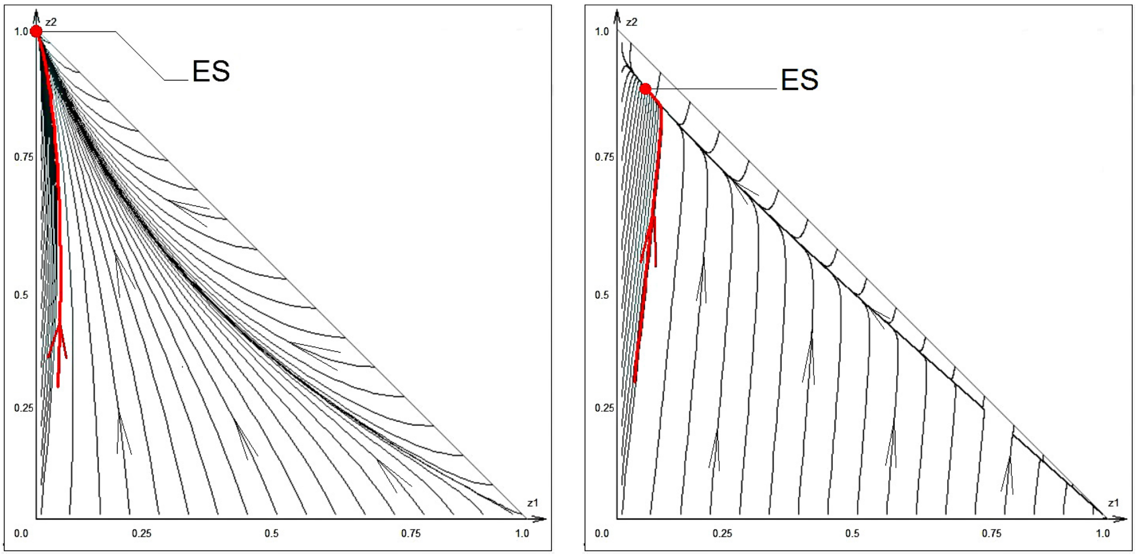


a



b

Fig. 6. Welsh-English language pair. *a* — phase plane of the model (1), first analytical case. *b* — phase plane of the model (4), first analytical case. The bold trajectory corresponds to the current development of dynamics (color online)

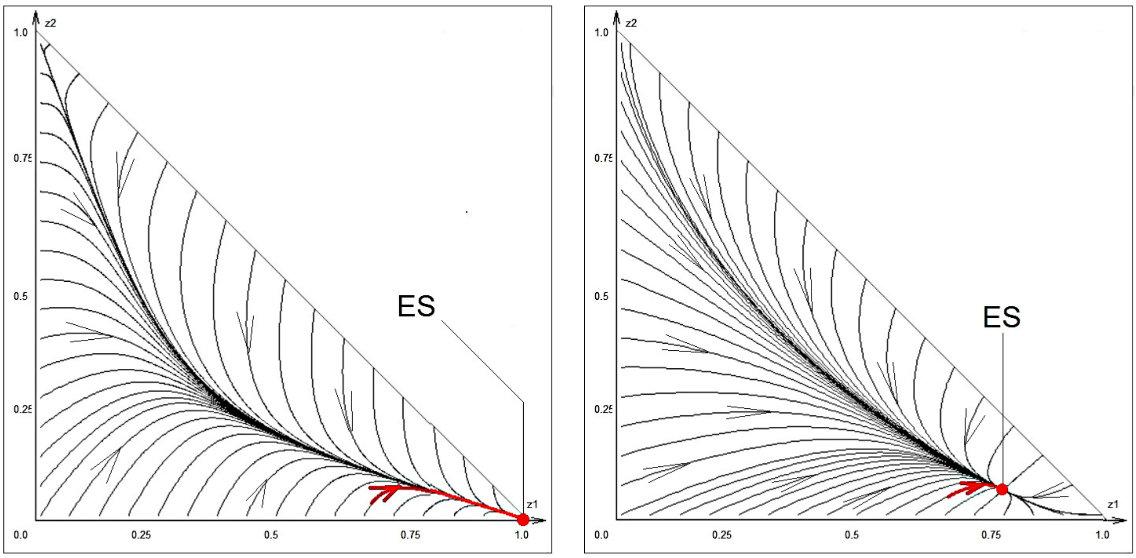


a *b*

Fig. 7. Gaelic-English language pair. *a* — phase plane of the model (1), English is replacing Scots. *b* — phase plane of the model (4), coexistence of languages. The bold trajectory corresponds to the current development of dynamics (color online)

model showed different results in the limiting development of dynamics, but at the same time its modeling accuracy was higher than that of the previous model.

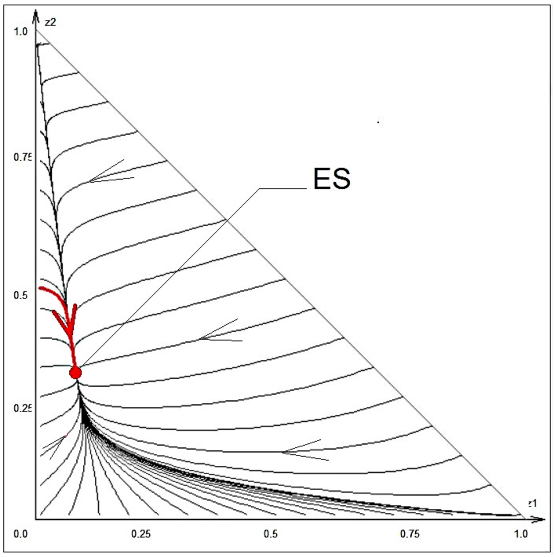
The statistical data on the shares of French and English languages for all of Canada and separately for Montreal for 1996–2016 are also considered [15, 17]. The model (1) showed that French is displacing English, but the rate of displacement is extremely low (Fig. 8, *a*). The model (4) showed that the languages coexist, the dynamics are close to a stable state (Fig. 8, *b*).



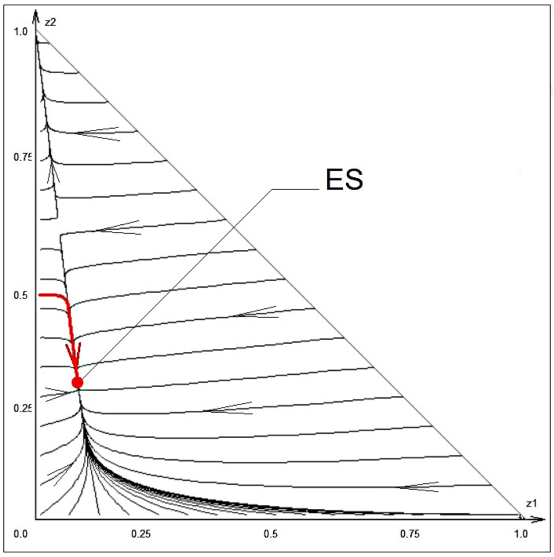
a *b*

Fig. 8. French-English language pair (Canada). *a* — phase plane of the model (1), French is replacing English. *b* — phase plane of the model (4), coexistence of languages. The bold trajectory corresponds to the current development of dynamics (color online)

Both models showed different results of the limiting development of dynamics, while the accuracy of modeling for the two models is approximately the same (Table 1). According to Montreal, both models showed compliance with the first case — languages coexist together with bilinguals (Fig. 9). The accuracy of modeling the language dynamics for the (1) and (4) models turned out to be approximately the same. A comparison of the data obtained as a result of modeling and the data taken from statistics for the Welsh and English languages is shown in (Fig. 10, *a*),

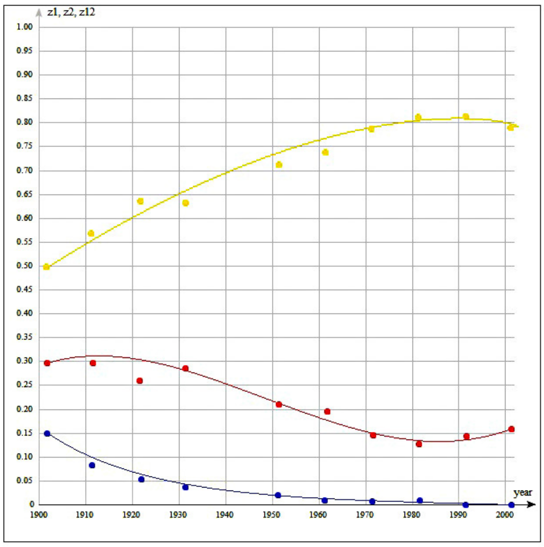


a

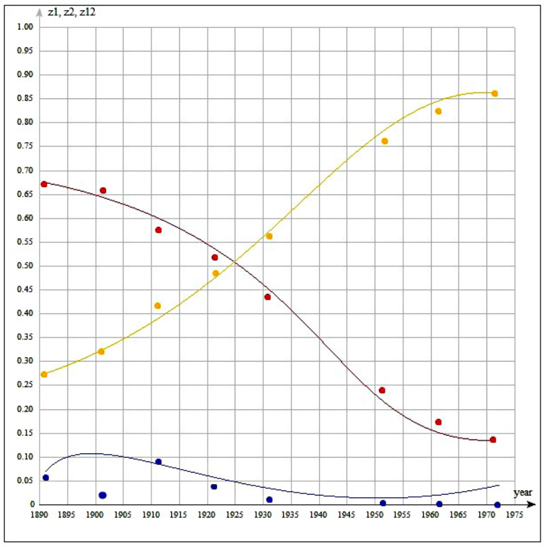


b

Fig. 9. French-English language pair (Montreal). *a* — phase plane of the model (1), coexistence of languages. *b* — phase plane of the model (4), coexistence of languages. The bold trajectory corresponds to the current development of dynamics (color online)

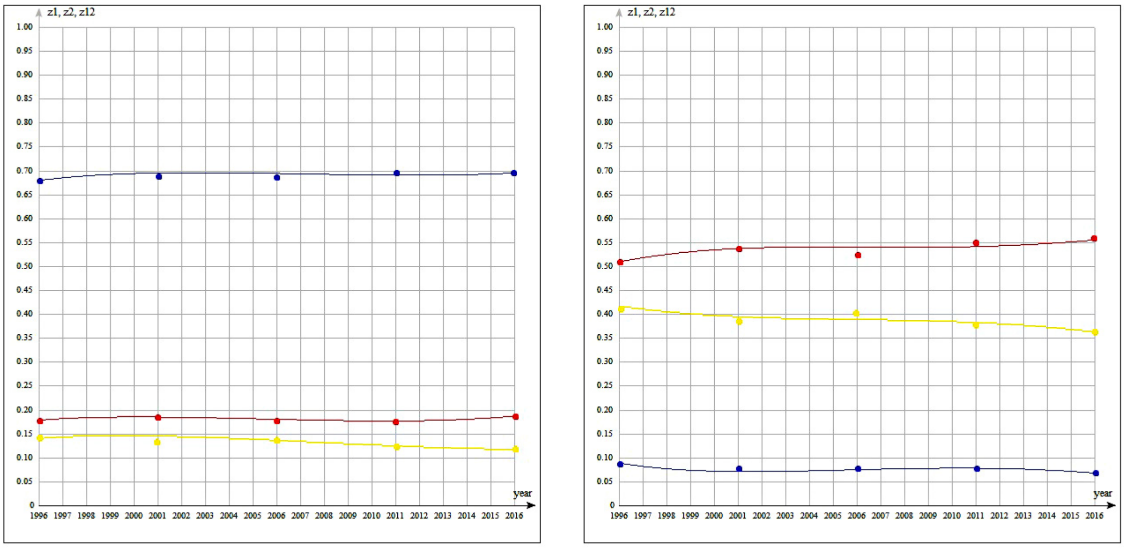


a



b

Fig. 10. The dots mark the values of the shares of languages taken from statistical data, the curves show the shares of languages obtained as a result of modeling: *red* — bilinguals; *blue* — Gaelic or Welish; *yellow* — English. *a* — Gaelic-English language pair. *b* — Welish-English language pair (color online)



a *b*

Fig. 11. The dots mark the values of the shares of languages taken from statistical data, the curves show the shares of languages obtained as a result of modeling: *red* — bilinguals; *blue* — Franch; *yellow* — English. *a* — in Kanada. *b* — in Montreal (color online)

for Scots and English in (Fig. 10, *b*), English and French for the whole of Canada in (Fig. 11, *a*), and only for Montreal in (Fig. 11, *b*). The dots mark the values of the shares of languages taken from the statistical data, the curves show the shares of languages obtained as a result of modeling. To check the quality of the model (1) three parameter identifications were made by the regression method: the first and second parameter identifications were made on the initial fragments of natural data of different periods (4 and 5 years), the third parameter identification was made on the full fragment of data lasting 10 years. The comparison of parameters is presented in Table 2.

Analysis of the data from Table 2 shows that the values of the coefficients obtained on the initial fragments of data are close to the values obtained on the full fragment of natural data. Based on this, it can be argued that the model (1) is capable of adequately predicting the outcome of language competition.

Table 2. Model parameters (1) for different time periods

Data chunk size		c_1	c_2	b_1	b_2	r	α	β
4 values	1901–1921	0.2	0.8	7.5	8.5	6.7	1.1	1.2
5 values	1901–1931	0.1	0.9	7.0	8.5	6.7	2.4	1.2
10 values	1901–2001	0.1	0.9	6.5	8.5	6.4	2.2	1.2

3. Discussions

The parameter of language volatility, which appeared as a result of Abrams and Strogatti's observation of nonlinearity in language dynamics, is an important discovery made in their work. They interpreted it as the readiness of community members to switch languages, but such an explanation is not obvious. In the present work, other possible interpretations of this parameter are considered. The volatility introduced by Abrams and Strogatti was reflected in the equations of the new dynamic system by terms of the equations that can be interpreted as mutual assistance within a group of one language. In fact, mutual assistance within a language group and the readiness to switch languages are very close in meaning. The higher the mutual assistance, the more actively members of a language group learn their language and are less inclined to switch languages, and the lower it is, the higher the readiness to switch languages, because there is no motivation to use only the language of their language group. Based on such reasoning, the parameter of language volatility can also be explained as an effect of mutual assistance within a group of one language, since, firstly, it is close to it in semantic interpretation, and secondly, it also finds its explanation in a mathematical expression. This is confirmed by the comparison of two models given in this paper.

Considering the statistical data and the data obtained as a result of modeling for language pairs of England, it is worth noting the very high value of the parameters of mutual assistance for the Welsh and Scottish languages. They exceed the corresponding value for the English language by 8-20 times, and this order is confirmed by the two models considered. English is displacing them due to its high prestige, but due to high mutual assistance, small groups of these two languages are still preserved. Analyzing the data for Canada, it is worth noting that the dynamics of the French and English languages are close to a stable state and do not actually change. French dominates in Canada, but in its capital, due to high mutual assistance within the English language group, which is reflected in the parameters of the model, the advantage has shifted to English.

Using the example of the dynamics of these two languages, it is also worth noting that languages with completely different parameters are capable of stably coexisting. The new and old models showed different results in modeling the marginal development of dynamics for two language pairs: Scottish-English and French-English (Canada). At the same time, for the first language pair, the accuracy of the new model (1) turned out to be higher than that of the model (4), for the second language pair, the accuracy was the same. Based on this, we can conclude that the new model is more accurate in forecasts that do not coincide with the forecasts of the old model. At the time of testing the model, its parameters were adjusted on the initial fragment of natural data, and the outcome of the competition was compared with the data at the end of the period. The forecast of the trial result turned out to be very accurate. This allows us to make a forecast for the future and say that at the moment the dynamics of the languages under consideration are close to stable states. Under unchanged conditions, English will continue to dominate in England, and French in Canada. The effects under consideration (mutual assistance and language volatility) turned out to be quite similar and showed a very close result in modeling, but when considering the limiting development of the dynamics, they can give both coinciding and different forecasts.

The symmetry of the system equations in form does not mean that the languages are the same, because the difference in languages is reflected not in the system equations, but in its parameters. This allows us to consider a language pair of the «international–national» type within the framework of the studied models. In reality, there may be cases when, due to political, social or other reasons, one of the two competing languages is subjected to targeted pressure and begins to exist in fundamentally different conditions. In the course of this study, it was noted that when considering language pairs of the «international–national» type, mutual assistance for the international language is close to the value of 0, and for the national language it is close to the maximum value. In terms of language prestige, the situation is the opposite — the prestige of the international language is close to the value of 1, and that of the national language is close to the value of 0. Taking these features into account in subsequent studies may lead to the emergence of new models with asymmetric equations.

Conclusion

The paper constructs and studies a new model of language dynamics for a bilingual community. The general concepts of language dynamics are expanded by new characteristics of languages: the forces of mutual assistance within groups of native speakers of the same language. The effect of mutual assistance is empirically confirmed. The effect of sequential acquisition of languages by children at an early age is taken into account. The paper examines language dynamics based on real statistical data for some languages of England and Canada: Scottish, Welsh, English, and French. The observed statistics were compared with the results of mathematical modeling and confirmed the adequacy of the new model. A forecast for the further development of the dynamics of these languages is constructed. The similarity of the effect of mutual assistance of native speakers of the same language with the effect of language volatility in the AS model is noted. The effect of mutual assistance is demonstrated by the example of how a change in the parameters of mutual assistance is followed by a corresponding change in the phase portraits of the model.

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