

Anisotropy and amplification of terahertz electromagnetic response enabled by direct electric current in graphene

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Abstract. The purpose of this study is to investigate the polarization conversion and amplification of electromagnetic terahertz (THz) wave incident normally upon graphene monolayer with direct electric current flowing at arbitrary angle to the electric vector of incident wave. *Methods.* The expressions for the elements of the dynamic conductivity tensor of graphene were obtained in hydrodynamic approximation. The electromagnetic response is calculated by solving the Maxwell equations with standard boundary conditions for lateral components of the electric and magnetic fields. *Results.* It is shown that the dynamic conductivity of graphene depends on value and direction of the electron drift velocity even in the absence of the spatial dispersion. This results in the polarization conversion of electromagnetic radiation at THz frequencies. The real parts of elements of graphene dynamic conductivity tensor can become negative which leads to the amplification of THz oscillations. *Conclusion.* The polarization conversion and amplification of electromagnetic THz wave incident upon graphene with direct electric current is demonstrated. Polarization conversion efficiency can be as high as 97 percent.

Keywords: hydrodynamic graphene conductivity, terahertz radiation, polarisation conversion, amplification.

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Introduction

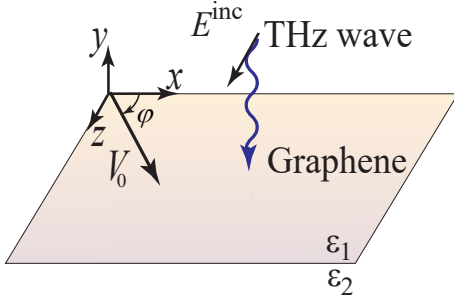


Fig 1. Schematic representation of the studied structure. The structure under study consists of a graphene monolayer located in the plane x - z at the interface of two semi-infinite dielectric media with different permittivities (Fig. 1). A plane homogeneous transverse electromagnetic (TEM) wave falls normally on graphene from medium 1.

Detection [1, 2], amplification [3, 4] and polarization conversion [5] of terahertz (THz) radiation are actively investigated in graphene-based structures. The latter is possible in structures with broken symmetry, which can be created due to the anisotropy of the geometry of the structure [6], by the action of a static magnetic field [7], as well as due to electron drift in graphene [8]. The

motion among charge carriers in graphene can be described within the framework of a hydrodynamic approach, which is valid when the frequency of interparticle collisions of charge carriers in graphene is the highest frequency in system [9]. The hydrodynamic behavior of charge carriers in graphene has been confirmed experimentally [10, 11]. In this paper, we study the polarization conversion and amplification of a homogeneous electromagnetic wave incident on hydrodynamic graphene with a direct electric current directed at an arbitrary angle relative to the direction of the electric field of the incident wave. The structure under study consists of a graphene monolayer located in the plane x - z at the interface of two semi-infinite dielectric media with different permittivities (Fig. 1). A plane homogeneous transverse electromagnetic (TEM) wave falls normally on graphene from medium 1.

1. Methods

1.1. Hydrodynamic conductivity of graphene. The dynamic (high-frequency) conductivity of graphene is obtained using a hydrodynamic approach by solving hydrodynamic equations, which in the case of a homogeneous (stationary plus oscillating) electric field are written as

$$\frac{\partial \mathbf{S}}{\partial t} + e\mathbf{E}N = -\gamma\mathbf{S}, \quad (1)$$

$$\frac{\partial W}{\partial t} + eN\mathbf{E} \cdot \mathbf{V} = 0, \quad (2)$$

where equation (1) is the momentum balance equation of charge carriers, and equation (2) is the energy balance equation of the two-dimensional motion of charge carriers in graphene. In equations (1)–(2) N is the concentration of charge carriers, \mathbf{V} is the hydrodynamic velocity, \mathbf{S} is the macroscopic momentum density, W is the macroscopic energy density, e is the elementary charge ($e > 0$), \mathbf{E} is the electric field in the graphene plane, τ is the relaxation time of the charge carrier momentum in graphene. Specifically, it is assumed that the charge carriers in doped (with a finite Fermi energy) graphene are electrons. The relations between the physical quantities entered equations (1)–(2), written as

$$\mathbf{S} = M\mathbf{V}, \quad W = MV_F^2 - P, \quad P = M(V_F^2 - V^2)/3, \quad (3)$$

where M is the hydrodynamic mass density, P is the hydrodynamic pressure, $V_F = 10^6$ m/c is the Fermi velocity in graphene. Equations (1)–(3) are linearized by representing all the required quantities as the sum of stationary and small oscillating terms $A = A_0 + A_1 \exp(-i\omega t)$, where ω is the angular frequency of the oscillation, and the indices 0 and 1 refer to stationary and oscillating quantities, respectively. It follows from the equation of continuity (the balance of the

number of particles) that in the case of homogeneous oscillations, the electron concentration in graphene does not depend on time and is equal to the stationary electron concentration N_0 .

Linearized equation (1), taking into account (3), gives expressions for the components of the oscillating hydrodynamic velocity:

$$V_{x1} = -\frac{M_1 V_{x0} (\gamma - i\omega) + eE_{x1} N_0}{M_0 (\gamma - i\omega)}, \quad (4)$$

$$V_{z1} = -\frac{M_1 V_{z0} (\gamma - i\omega) + eE_{z1} N_0}{M_0 (\gamma - i\omega)}, \quad (5)$$

where E_{x1} and E_{z1} are the components of the oscillating electric field, M_0 and M_1 are the stationary and oscillating hydrodynamic mass densities, respectively, $V_{x0,z0}$ are the components of the stationary velocity drift velocity, $V_{x1,z1}$ are the components of the oscillating hydrodynamic velocity, γ is the relaxation rate of the electron momentum. From equation (2), taking into account (3), we obtain an expression for the oscillating mass density of charge carriers in graphene:

$$M_1 = \frac{3eN_0(E_{x1}V_{x0} + E_{z1}V_{z0}) + V_{x1}(3eE_{x0}N_0 - 2iM_0V_{x0}\omega) + V_{z1}(3eE_{z0}N_0 - 2iM_0V_{z0}\omega)}{i\omega(2V_F^2 + V_{x0}^2 + V_{z0}^2)}, \quad (6)$$

where $E_{x0,z0}$ are the components of a stationary electric field causing the electron drift. The varying hydrodynamic mass is a unique physical property of graphene, which, as follows from the following description, determines the anisotropy and amplification of the electromagnetic response of graphene when direct electric current is passed through it. As can be seen from equation (6), the mass density of charge carriers oscillates only in the presence of electron drift in graphene. Solving equations (4)–(6) together, we obtain expressions of the form $V_{x1}(E_{x1}, E_{z1})$ and $V_{z1}(E_{x1}, E_{z1})$ and substituting those in the expression for oscillating current density,

$$J_{x1,z1} = -eN_0V_{x1,z1},$$

we obtain expressions for the elements of the graphene high-frequency hydrodynamic conductivity tensor:

$$\begin{aligned} \sigma_{xx} &= \Sigma \left[\omega (2V_F^2 - 2V_{x0}^2 - V_{z0}^2) - 3i\gamma (V_{x0}^2 - V_{z0}^2) \right], \\ \sigma_{zz} &= \Sigma \left[\omega (2V_F^2 - 2V_{z0}^2 - V_{x0}^2) + 3i\gamma (V_{x0}^2 - V_{z0}^2) \right], \\ \sigma_{xz} &= \sigma_{zx} = \Sigma(\omega + 6i\gamma)V_{x0}V_{z0}, \end{aligned} \quad (7)$$

where

$$\Sigma = \frac{ie^2 N_0 (V_F^2 - V_{x0}^2 - V_{z0}^2)}{(\omega + i\gamma) [\omega (2V_F^2 - V_{x0}^2 - V_{z0}^2) + 3i (V_{x0}^2 + V_{z0}^2) \gamma] \mathcal{E}_F}$$

with \mathcal{E}_F being the stationary Fermi energy, $V_{x0} = V_0 \cos \varphi$ and $V_{z0} = V_0 \sin \varphi$. Note that even in the absence of spatial dispersion, the dynamic conductivity of graphene depends on the velocity and direction of stationary drift due to breaking the Galilean invariance in the system of electrons with zero mass of an individual electron in graphene [12]. This significantly distinguishes graphene from classical two-dimensional electronic systems with massive electrons.

The anisotropy created by passing direct electric current through graphene can be used for changing the polarization of electromagnetic radiation. The physical reason for the electrically induced anisotropy of graphene is that oscillations of the hydrodynamic momentum in the transverse direction to electron drift occur as in the usual hydrodynamics of massive particles with their inertia determined by a stationary hydrodynamic mass (since the concentration of electrons in graphene does not depend on time and coordinate in the case of homogeneous oscillations, the

hydrodynamic mass can be normalized per unit area, and per electron as well). At the same time, in the direction of electron drift, oscillations of the hydrodynamic momentum occur both due to stationary hydrodynamic inertia and due to the drift of the oscillating part of the hydrodynamic mass. As a result, the oscillating hydrodynamic momentum (and, consequently, the oscillating hydrodynamic velocity) depend differently on the oscillating electric field depending on its polarization.

1.2. Polarization conversion coefficient. Let us introduce reflection and transmission coefficients

$$Y_n^m = \frac{|S_n^m|}{|S^{\text{inc}}|}, \quad (8)$$

where S_n^m is the energy flow of a reflected ($m = R$) or transmitted ($m = T$) wave with an orthogonal ($n = \perp$) or initial ($n = \parallel$) polarization of the electric field with respect to the polarization of the incident wave, S^{inc} is the energy flow of an incident wave. We also introduce the polarization conversion coefficient for the transmitted wave:

$$P^T = \frac{|S_{\perp}^T|}{|S_{\text{tot}}^T|}, \quad (9)$$

where S_{tot}^T is the total (by two orthogonal polarizations) energy flow of the transmitted wave.

2. Results

The appearance of an orthogonal (relative to the polarization of the incident wave) component of the electric field in reflected and transmitted waves is associated with the presence of nonzero off-diagonal elements of the graphene conductivity tensor for the electron drift directed at an angle relative to the electric field of the incident wave (Fig. 2, *a*). As a result, partial waves with orthogonal polarization with respect to the polarization of the incident wave appear in the reflected and transmitted waves (Fig. 2, *b*). For the drift angle $\varphi \approx 30^\circ$ at frequency $\omega/2\pi = 0.1$ THz an almost complete polarization cross-conversion occurs in the transmitted wave ($P^T \approx 0.97$, Fig. 3). For certain drift angles, the real parts of the graphene conduction tensor elements become negative, which leads to an amplification of both the wave with the

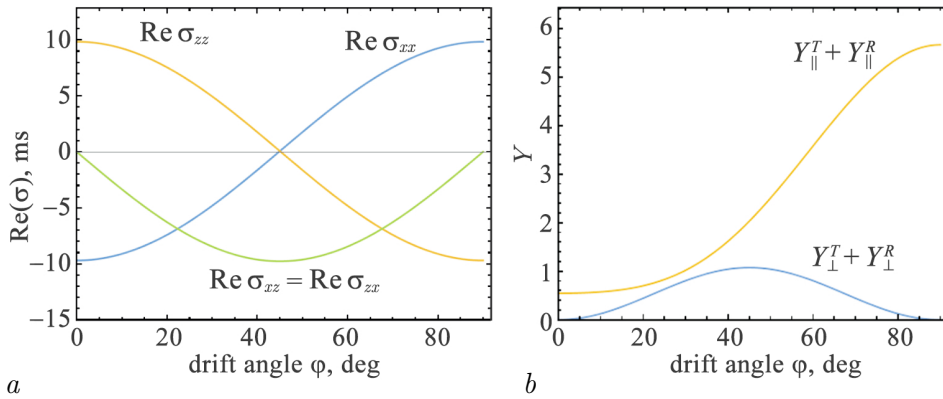


Fig 2. *a* – The real parts of the elements of the graphene dynamic conductivity tensor and *b* – the reflection and transmission coefficients for waves with initial and orthogonal polarizations in dependence on the electron drift angle for drift velocity value $V_0 = 0.7V_F$ at frequency $\omega/2\pi = 0.1$ THz. Graphene parameters are $\tau = 0.1$ ps and $\mathcal{E}_F = 500$ meV (Color online)

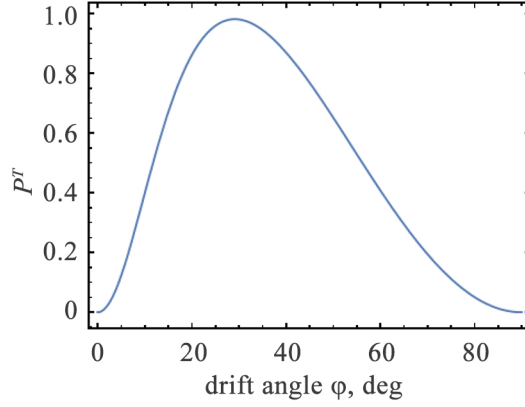


Fig 3. The coefficient of polarization conversion P^T in dependence on electrons drift angle for drift velocity $V_0 = 0.7V_F$ at frequency $\omega/2\pi = 0.1$ THz. Graphene parameters are $\tau = 0.1$ ps and $\mathcal{E}_F = 500$ meV

initial polarization and the induced wave with the orthogonal polarization (see Fig. 2, b). At the same time, the wave with orthogonal polarization is amplified insignificantly, and mainly the wave with the initial polarization is amplified. The amplification can be interpreted as the occurrence of negative dissipation in graphene when direct current is passed [8].

For drift angles of 0 and 90 degrees, the reflected and transmitted waves have no the electric field components with cross-polarization, since $\sigma_{xz} = \sigma_{zx} = 0$. At $\varphi = 90^\circ$ (the drift is co-directed with the electric field of the incident wave), the amplification of the wave with the initial polarization is maximal, which is associated with reaching the maximum negative conductivity value $\text{Re } \sigma_{zz}$. In this case, a fairly clear explanation of the physical mechanism of amplification can be given. A detailed examination shows that amplification occurs when the oscillations of the hydrodynamic mass occur in phase with oscillations in the hydrodynamic velocity. At each moment of time, the total kinetic energy of the hydrodynamic motion of charge carriers, both as a result of their oscillations and as a result of stationary drift, is equal to

$$\frac{1}{2}M(t)\{V_0 \pm V_1(t)\}^2 = \frac{1}{2}M(t) \{V_0^2 + V_1^2(t) \pm 2V_0V_1(t)\}, \quad (10)$$

where the signs «+» and «-» correspond to the co-directional and counter-directional stationary drift velocity V_0 and the hydrodynamic oscillation velocity $V_1(t)$. The first two terms in formula (10) represent, respectively, the kinetic energy values of stationary drift and oscillations. The last term should be interpreted as the amount of kinetic energy supplied by the source of stationary drift to the oscillations (with the sign «+») or transferred from the oscillations to the source of stationary drift (with the sign «-»). In a positive oscillation half-cycle, the hydrodynamic mass becomes larger than the stationary mass, and the oscillating velocity is directed along the stationary drift velocity. In this half-cycle, kinetic energy is transferred from the stationary drift source to the oscillations. In the next (negative) half-cycle, the hydrodynamic mass becomes smaller than the stationary value, and the oscillating velocity is directed against the stationary drift velocity, and therefore smaller kinetic energy is transferred from the oscillations to the stationary drift compared to the kinetic energy received by the oscillations from the source of the stationary drift in the previous (positive) half-cycle of oscillations. As a result, on average over the oscillation period, kinetic energy is supplied to the oscillating system from a source of constant drift, which swings the oscillations. Since the only physical mechanism of energy exchange between drift and oscillations in the hydrodynamic model is collisions of electrons with the loss of their hydrodynamic momentum (which is responsible for the real part of the conductivity), the amplification can be interpreted as a consequence of negative dissipation.

Conclusion

The polarization conversion and amplification of a normally incident electromagnetic wave on graphene with direct current are investigated. It is shown that the dynamic conductivity of graphene depends on the value and direction of the stationary electron drift velocity even in the case of a homogeneous oscillating electric field, which is unusual for classical two-dimensional electronic systems with massive electrons. The tensor character of the graphene dynamic conductivity leads to the conversion of the polarization of an electromagnetic wave incident on graphene with into the cross polarization an efficiency of up to 97 percent at THz frequencies. The real part of each element of the conductivity tensor of graphene with electron drift can be negative, which leads to the amplification of THz wave.

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