Applied Problems of Nonlinear Oscillation and Wave Theory

Article DOI: 10.18500/0869-6632-003195

Stability of multi-machine power grid with a common load to connecting and disconnecting of generators

V. A. Khramenkov[1], *A. S. Dmitrichev*[1]✉, *V. I. Nekorkin*[2]

[1]FRC A. V. Gaponov-Grekhov Institute of Applied Physics of the RAS, Nizhny Novgorod, Russia
[2]National Research Lobachevsky State University of Nizhny Novgorod, Russia
E-mail: hramenkov.vlad@ipfran.ru, ✉admitry@ipfran.ru, vnekorkin@ipfran.ru
Received 21.07.2025, accepted 5.09.2025, available online 12.09.2025, published 30.01.2026

Abstract. The *purpose* of this work is studying the stability of the power grid, consisting of an arbitrary number of synchronous generators supplying a common passive linear load, to disconnection and connection of generators. *Methods.* In this paper, numerical modeling of the power grid operation and the Lyapunov direct (or second) method are used. *Results.* Conditions for safe disconnection and connection of generators have been revealed, under which a synchronous mode is established in the disturbed power grid. *Conclusion.* The power grid consisting of an arbitrary number of synchronous generators supplying a common passive linear load is considered. Using the approach based on the Lyapunov direct method, conditions on parameters are found that ensure safe disconnection of generators, including, if any, a generator involved in the «inhomogeneous» load supply path, that differs from the others in current and transmitted power. These estimates are validated numerically for networks of various sizes. The evolution of the area corresponding to the safe connection of a generator to a five-generator power grid is also numerically traced.

Keywords: power grids, synchronous machines, synchronous modes, stability, multistability, disconnection and connection of generators.

Acknowledgements. This work was supported by the Russian Science Foundation, project No. 24-12-00245.

For citation: Khramenkov VA, Dmitrichev AS, Nekorkin VI. Stability of multi-machine power grid with a common load to connecting and disconnecting of generators. Izvestiya VUZ. Applied Nonlinear Dynamics. 2026; 34(1):49–67. DOI: 10.18500/0869-6632-003195

This is an open access article distributed under the terms of Creative Commons Attribution License (CC-BY 4.0).

Introduction

Currently, there is a continuous growth of power grids, which is accompanied by the creation of large power systems consisting of a large number of generators, load nodes (consumers), and intra- and inter-system connections (transmission lines). Operating such systems is a very complex task that requires various approaches and methods, depending on the complexity and completeness of the description [1–9].

During their operation, power grids are constantly exposed to various disturbances, such as fluctuations in generated and consumed power, short circuits, tripping of relay protection devices, and shutdowns of transmission lines and generators, etc. The resulting transient processes can be accompanied by significant fluctuations in currents and voltages. These fluctuations, due to further tripping of relay protection devices, can lead to both single and short-term power supply failures affecting relatively small parts of the system, as well as cascading failures [10, 11] with severe and widespread power outages [12–16].

© *Khramenkov V. A., Dmitrichev A. S., Nekorkin V. I., 2025*

Therefore, studying the stability of power grids to various types of disturbances is an important task from both a fundamental and applied perspective.

Previously, the effects arising from the switching of power transmission lines [17–19], generators and consumers [20], as well as from single-time step-like (impulsive) [21–28] and long-term noise [28–33] disturbances in the generated and consumed power, have been studied.

In our previous works [28, 34], we considered a power grid consisting of synchronous generators operating on a common passive linear load. We have shown that in the case when one of the generators is «closer» to the load (due to a shorter transmission line and/or longitudinal reactance compensation), a reduced effective network model in the form of an ensemble with a hub topology (topology «star») can be used to describe the dynamics of such a power grid. It is found that two different types of synchronous modes can be established in the grid: homogeneous and heterogeneous. The first one is characterized by equal powers and currents flowing through all but one load supply path. The second one has another additional path, which differs from the others in current and transmitted power. Moreover, the currents flowing along the same path, but in different modes, vary. The coexistence of homogeneous and heterogeneous synchronous modes, as well as quasi-synchronous and asynchronous modes, is demonstrated; the corresponding regions in the parameter space are highlighted.

This paper further investigates this power grid by analyzing its stability under generator disconnection and connection events. Using numerical modeling and the Lyapunov direct (or second) method, we derive conditions for safe generator switching that ensure the establishment of a synchronous mode in disturbed power grid. Note that generator disconnection may occur, for example, due to the operation of transmission line relay protection (e.g., overcurrent protection) triggered by short circuits, or during scheduled maintenance. Conversely, generators may be connected in response to a sharp increase in load. In such cases, automatic transfer switch (ATS) devices activate backup generators to provide the necessary power reserve [35].

Section 1 presents the architecture of the power grid and the model used for description of its dynamics. Section 2 discusses general aspects of the grid's stability in response to changes in the number of active generators. Section 3 analyzes the resistance of the power grid to generator disconnection, while Section 4 focuses on generator connection. Conclusion provides a brief discussion of the results.

1. The architecture of the power grid and its model

Let us consider a multi-machine power grid, the schematic diagram of which is shown in Fig. 1, *a*, and the equivalent circuit of its typical part is shown in Fig. 1, *b*. In the grid, a group of synchronous generators $G_i (i = \overline{1,n},\ n \geqslant 3))$ supplies one common passive linear *load (represented by the impedance Z_{load}) through transmission systems. The transmission systems inside input transformers $T_i^{\rm in}$ (represented by impedances $Z_{i,T}^{\rm in}$), output transformers $T_i^{\rm out}$ (represented by impedances $Z_{i,T}^{\rm out}$) and transmission lines (replaced by a standard T-shaped circuit with impedances $Z_i^{\rm line}$ and conductivities $Y_i^{\rm sh}$). Each generator is characterized by the amplitude $|E_i|$ and the phase angle δ_i of its electromotive force (EMF), $E_i = |E_i| \exp(i\delta_i)$; internal impedance $Z_i^{\rm int} = r_i^{\rm int} + ix_i^{\rm int}$, $x_i^{\rm int} > 0$; constant inertia C_i of its rotating part*

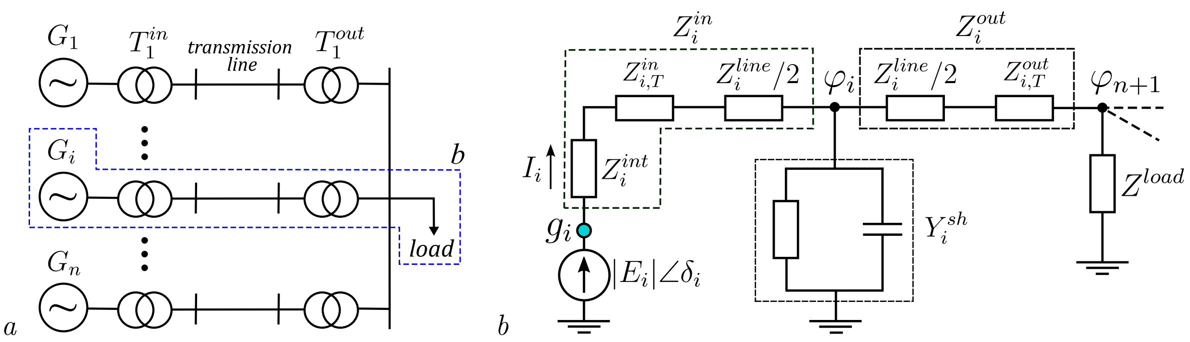

Fig. 1. Architecture of power grid: *a* — schematic diagram; *b* — equivalent circuit of the *i*-th grid's part, containing the *i*-th generator and a load

(turbine and rotor); the damping coefficient D_i, summing up the influence of both mechanical (friction) and electrical (the appearance of asynchronous power) damping factors; as well as the mechanical power of the turbine $P_{T,i}$.

Let us assume that the first generator G_1 is much closer to the load than the other generators $G_2, G_3, \ldots, G_n$, and/or a longitudinal rectance compensation is performed on the transmission line of this generator. For definiteness, we shall assume that the constants of inertia and damping coefficients of all generators are the same, i.e. $C_i \equiv C$, $D_i \equiv D\,(i = \overline{1,n})$, and all generators except the first one, as well as their transmission systems, have the same parameters, i.e

$$P_{T,k} = P_{T,2}, \quad |E_k| = |E_2|, \quad Z_k^{\text{in}} = Z_2^{\text{in}}, \quad Z_k^{\text{out}} = Z_2^{\text{out}}, \quad Y_k^{\text{sh}} = Y_2^{\text{sh}} \quad (k = \overline{2,n}).$$

This effectively means that the generators have the same design and are connected to the load in the same way, except for the transmission system of the first generator. In [34], it was shown that the dynamics of such a power grid can be described using the reduced effective network model with the following equations:

$$\begin{cases} \dot{\varphi}_i = y_i, \\ \dot{y}_i = \Delta - \mu y_i - \sin(\varphi_i + \alpha) - \sum\limits_{j=1}^{N} \sin(\varphi_j - \alpha), \\ i = \overline{1,N}, \quad N = n-1. \end{cases} \tag{1}$$

In dimensionless system (1), defined in the cylindrical phase space $G = \mathrm{S}^N \times \mathrm{R}^N$, the variables $\varphi_i = \delta_1 - \delta_{i+1}$ define the relative angles of the EMF (rotors) of the generators, the variables $y_i = \dot{\delta}_1 - \dot{\delta}_{i+1}$ are the instantaneous frequencies of change of the relative angles, and the dot denotes differentiation with respect to time $\tau = \sqrt{K/C}\,t$. The parameter Δ is proportional to the difference in the power of the turbines of the first and $(i+1)$-th generators; the parameter μ represents the normalized damping coefficient, and the parameter α characterizes the passive part of the equivalent circuit. These parameters are expressed in terms of the quantities characterizing the equivalent circuit as follows:

$$\Delta = \frac{A_1 - A_2}{K}, \quad \mu = \frac{D}{\sqrt{CK}}, \quad \alpha = \pi/2 - \arg(Y_{1,2}^{-1}),$$

where

$$K = |E_1 E_2 Y_{1,2}|, \quad A_k = P_{T,k} - |E_k|^2 |Y_{k,k}| \sin(\alpha_{k,k}), \quad \alpha_{k,k} = \pi/2 - \arg(Y_{k,k}^{-1}),$$

and

$$\begin{aligned} &Y_{1,1} = \frac{1}{Z_1^{\text{in}}}\left[1 - \frac{Z_1^{\text{out}}}{R_1}\left(1 + \frac{Z_1^{\text{in}}}{C_R R_1 Z_1^{\text{out}}}\right)\right], \\ &\begin{cases} Y_{i,i} = \frac{1}{Z_2^{\text{in}}}\left[1 - \frac{Z_2^{\text{out}}}{R_2}\left(1 + \frac{Z_2^{\text{in}}}{C_R R_2 Z_2^{\text{out}}}\right)\right], Y_{1,j} = \frac{1}{C_R R_1 R_2}, \\ Y_{i,j} = \frac{1}{C_R R_2^2}, \; j \neq i, \\ i, j = \overline{2,n}, \end{cases} \\ &C_R = \frac{1}{Z_{\text{load}}} + \frac{1}{R_1}(1 + Y_1^{\text{sh}} Z_1^{\text{in}}) + \frac{n-1}{R_2}(1 + Y_2^{\text{sh}} Z_2^{\text{in}}), \\ &R_k = Z_k^{\text{in}} + Z_k^{\text{out}} + Y_k^{\text{sh}} Z_k^{\text{in}} Z_k^{\text{out}}, \; Z_k^{\text{in}} = Z_k^{\text{line}}/2 + Z_{k,T}^{\text{in}} + Z_k^{\text{int}}, \\ &Z_k^{\text{out}} = Z_k^{\text{line}}/2 + Z_{k,T}^{\text{out}}, \; k = 1, 2. \end{aligned} \tag{2}$$

Here $Y_{i,i}$ and $Y_{i,j}$ are the so-called intrinsic and mutual complex conductivities of the generator branches. More information about the expressions that relate the parameters of system (1) to those of the equivalent circuit can be found in [34].

System (1) has an absorbing region

$$G^+ = \left\{\phi_i \in S^1, \; y_i \in \left[\, y^-, y^+ \,\right], \; i = \overline{1,N}\right\}, \tag{3}$$

where $y^{\pm} = \dfrac{1}{\mu}\big[\Delta \pm (2|\cos(\alpha)| + N - 1)\big]$, *which attracts all trajectories with initial conditions outside this region and contains all the attractors of the system.*

System (1) is symmetric under the permutation of any pair of elements. Consequently, all solutions obtained through such permutations belong to the same class; for instance, if one solution is stable, then all its permuted counterparts are also stable.

A study of system (1) showed that for parameters from the region

$$S_N^{hom,st} = \Big\{\alpha, \Delta, \mu \Big|\ 0 \leqslant \alpha \leqslant \pi/2,\ -f(N,\alpha) < \Delta < f_1(N,\alpha), \mu > 0\Big\} \tag{4}$$

there is a stable homogeneous steady state

$$O_1^{\mathrm{hom}}(\varphi_i = \varphi^{\mathrm{hom}};\ y_i = 0), \tag{5}$$

where

$$\begin{gathered} f(N,\alpha) = \sqrt{N^2 + 2N\cos(2\alpha) + 1},\ f_1(N,\alpha) = 1 + N\cos(2\alpha), \\ \varphi^{\mathrm{hom}} = \beta^{\mathrm{hom}} + \arcsin\left(\frac{\Delta}{f(N,\alpha)}\right),\ \sin\beta^{\mathrm{hom}} = \frac{(N-1)\sin\alpha}{f(N,\alpha)},\ \cos\beta^{\mathrm{hom}} = \frac{(N+1)\cos\alpha}{f(N,\alpha)}, \end{gathered} \tag{6}$$

which corresponds to a homogeneous synchronous power grid mode characterized by equal powers and currents flowing through all load paths except the first one.

Conversely, for parameters within the region

$$S_N^{inh,st} = \Big\{\alpha, \Delta, \mu |\ \alpha_1 \leqslant \alpha \leqslant \alpha_2,\ -g(N,1,\alpha) < \Delta < g_1(N,\alpha,\mu), \mu > 0\Big\} \tag{7}$$

there are N stable heterogeneous steady states

$$O_j^{inh,1}(\varphi_j = -\alpha + \pi - \Phi_1^{(1)}, \quad \varphi_{i\neq j} = -\alpha + \Phi_1^{(1)}; \quad y_i = 0), \quad j = \overline{1,N}, \tag{8}$$

where

$$\begin{gathered} g(N,1,\alpha) = \sqrt{(1 + N\cos 2\alpha)^2 + (N-2)^2\sin^2 2\alpha}, \quad g_1(N,\alpha,\mu) = \left\{\Delta : \tfrac{\sqrt{4\varkappa_c - \varkappa_b^2}}{-\varkappa_b\mu^2} = 1\right\}, \\ \varkappa_b = a' + d' + (N-2)c',\ \varkappa_c = a'd' + (N-2)c'd' - (N-1)c'b', \\ a' = -2\cos(\alpha)\cos(\Phi_1^{(1)} - \alpha),\ b' = \cos(\Phi_1^{(1)} + 2\alpha),\ c' = -\cos(\Phi_1^{(1)} - 2\alpha), \\ d' = 2\cos(\alpha)\cos(\Phi_1^{(1)} + \alpha),\ \Phi_1^{(1)} = \pi - \Omega_1^{inh} + \beta_1^{inh},\ \Omega_1^{inh} = \arcsin\left(\tfrac{\Delta}{g(N,1,\alpha)}\right), \\ \sin\beta_1^{inh} = \frac{(N-2)\sin 2\alpha}{g(N,1,\alpha)},\ \cos\beta_1^{inh} = \frac{N\cos 2\alpha + 1}{g(N,1,\alpha)}, \end{gathered} \tag{9}$$

each of which corresponds to a heterogeneous synchronous mode of the power grid, in which, in addition to the homogeneous mode, there is another «heterogeneous» $(j+1)$-th load supply path that differs from the others in terms of current and transmitted power.

In addition, the coexistence of homogeneous and heterogeneous synchronous modes, as well as quasi-synchronous and asynchronous modes, is demonstrated.

Next, we examine the behavior of a power grid initially operating in one of the synchronous modes subject to either disconnection or connection of generators.

2. About safe disconnection and connection of generators

We assume that generator disconnection (or connection) occurs instantaneously compared to the characteristic time scales of the disturbed power grid's dynamics. Consequently, its initial state is either fully (in the case of disconnection) or partially (in the case of connection) determined by the synchronous *mode of the initial power grid, i.e., by the coordinates of the corresponding steady state of system (1). We also assume that any generator, expect for the first one, may be disconnected, and that the parameters of any newly connected generators and their transmission systems are identical to those of the majority of the initial power grid units. In this case, the dynamics of the disturbed power grid is still described by system (1), taking into account its new size and the correspondingly redefined parameters* α, Δ *and* μ *(see the expression for* C_R *in (2)). Let us find the conditions under which we can neglect the change in the parameters* α, Δ, *and* μ *when we switch from the initial to the disturbed power grid. To do this, consider a simplified transmission line equivalent circuit consisting of active and reactive resistances connected in series. In this case, the transmission system of the i-th generator can be replaced by the impedance* $Z_i = r_i + ix_i$, *so the formulas for* $Y_{i,i}$ *and* $Y_{i,j}$ *(see (2)) take the following form*

$$Y_{1,1} = \left[1 - Z_2 Y^*\right]/Z_1,$$
$$\begin{cases} Y_{i,i} = \left[1 - Z_1 Y^*\right]/Z_2, \ Y_{1,j} = Y^*, \\ Y_{i,j} = \left[Y^* Z_1\right]/Z_2, \ j \neq i, \\ i, j = \overline{2, n}, \end{cases}$$

where $Y^* = Z_{\text{load}}/\left[Z_1 Z_2 + Z_{\text{load}} Z_2 + N Z_{\text{load}} Z_1\right]$. *Note that* $Y_{i,i}$ *and* $Y_{i,j}$ *depend on the number of generators only through* Y^*. *Therefore, all the differences between the parameters* α, Δ *and* μ *of the initial and disturbed power grids are determined by* Y^*. *Let us assume that the transmission line of the first generator has only active resistance, i.e.,* $Z_1 = r_1$, *which is typical, for example, for a cable line [3]. Then*

$$Z^* = (Y^*)^{-1} = r_2\left(1 + \frac{r_1}{r_{\text{load}}}\right) + N r_1 + i x_2\left(1 + \frac{r_1}{r_{\text{load}}}\right).$$

It is easy to see that only the real part of Z^* *changes when* N *changes. Thus, when the generators are disconnected* ($Z^* \to Z^*_-;\ N \to N - m;\ N - m > 1$) *or connected* ($Z^* \to Z^*_+;\ N \to N + m;\ N > 1$), *we have*

$$\text{Re}\,(Z^*_\mp) = r_2\left(1 + \frac{r_1}{r_{\text{load}}}\right) + N r_1 \mp m r_1 > (r_2 + N r_1) \mp m r_1,$$

where m is the number of disconnected (connected) generators. From this, we can see that when

$$m r_1/\left[r_2 + N r_1\right] \ll 1, \tag{10}$$

$$\text{Re}\,(Z^*_\mp) \approx \text{Re}\,(Z^*).$$

Condition (10) is satisfied when the number of generators that are being disconnected (connected) is relatively small compared to the number of initial generators ($m \ll N$), *and also when* $r_1 \ll r_2$. *The latter condition is satisfied when the conditions for the transition to a reduced effective network are met, which is the case we are considering here. In this case, when generators are being disconnected or connected, the values of* Z^* *and* Y^* *remain almost unchanged, and therefore the parameters* α, Δ, *and* μ *do not change significantly.*

It should be noted that the connection and disconnection of generators can be safe only if a stable synchronous mode exists in the disturbed power grid, as well as in the initial power grid. Therefore, first of all, it is necessary to have a non-empty intersection of the regions of existence of stable steady states of systems (1) corresponding to the initial and disturbed power grids. It is easy to show that such a region (let us denote it by a) exists regardless of the size ratio between the initial and disturbed power grids, the parameters of which satisfy condition (10). In general, the region a consists of three subregions $a_i, i = 1, 2, 3$, *corresponding to various combinations of stable steady states of the initial and disturbed systems (1). Possible combinations are given in Table, where* N_- *and* N_+ *denote, respectively, the sizes of the smallest and largest of the power grids, that is, when disconnected (connected), the size of the initial power grid is* N_+ (N_-), *and the disturbed one is* $N_- = N_+ - m$ ($N_+ = N_- + m$).

If the parameters of the initial and disturbed power grids belong to the region a_1, *then there are only homogeneous synchronous modes corresponding to the steady states* O_1^{hom} *(see (5)) of the respective systems (1). If the parameters belong to the region* a_2, *then in the initial power grid, in the case of the*

Table. Combinations of stable steady states of the initial and disturbed systems (1)

a_i N	$N_- > 1$	$N_+ > 2$
a_1	O_1^{hom}	O_1^{hom}
a_2	O_1^{hom}	O_1^{hom} and $O_j^{\text{inh},1}$, $j = 1, 2, ..., N_+$
a_3	O_1^{hom} and $O_j^{\text{inh},1}$, $j = 1, 2, ..., N_-$	O_1^{hom}

disconnection of generators ($N = N_+$), there are both homogeneous and heterogeneous synchronous modes corresponding to the steady states O_1^{hom} and $O_j^{\text{inh},1}$ (see (8)) of the respective system (1), and in the case of connection of generators ($N = N_-$), there is only a homogeneous synchronous mode corresponding to the steady state O_1^{hom}. Conversely, in the disturbed power grid, if the generators are disconnected ($N = N_-$), there is only a homogeneous synchronous mode, and if the generators are connected ($N = N_+$), there are both homogeneous and heterogeneous synchronous modes. Finally, in the region of a_3, the combination of synchronous modes of the initial and disturbed power grids is completely opposite to a_2. Thus, the safe disconnection of generators is always associated with the establishment of a homogeneous synchronous mode in the disturbed power grid, while the safe connection of generators can be associated with the establishment of both homogeneous and heterogeneous synchronous modes in it.

Fig. 2 shows the division of the parameter plane (α, Δ) into subregions a_i for the case of disconnection of a generator from the power grid of 6 generators (or, accordingly, connecting a generator to the grid of 5 generators). Note that due to the specifics of the curves g and g_1, the regions a_2 and a_3 shrink with decreasing parameter μ and disappear with some values of this parameter. Also, each of the regions $a_i, i = 1, 2, 3$, decreases with an increase in the number of generators that are simultaneously disconnected/connected. At the same time, there are thresholds for the number of such generators, depending on the parameter μ, at which the regions a_2 and a_3 completely disappear.

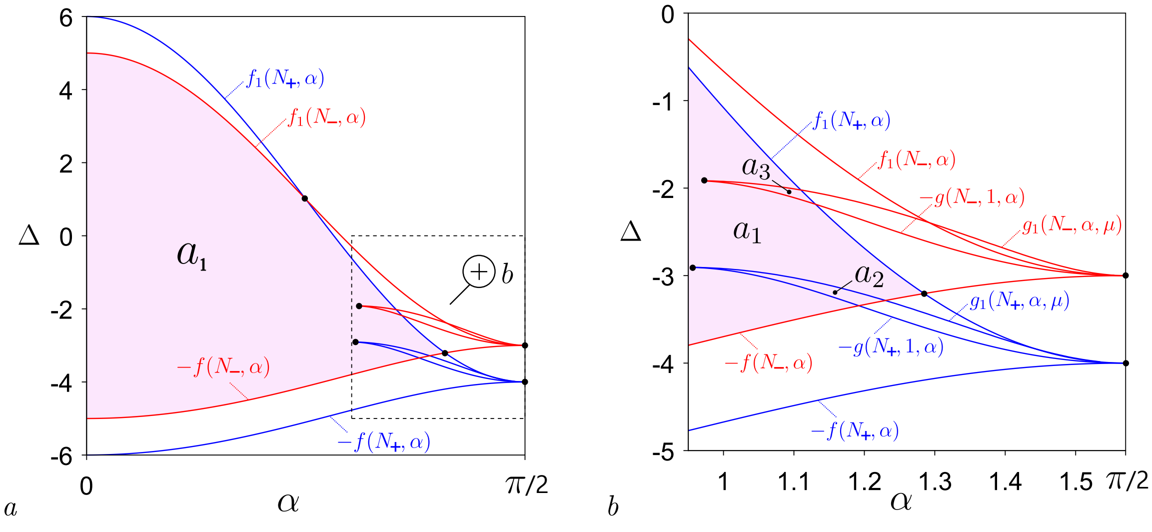

Fig. 2. Partition of the (α, Δ)-parameter plane — a and its enlarged fragment — b into the subregions $a_i \in a$, corresponding to various combinations of stable steady states of the initial and disturbed systems (1), describing the power grids before and after disconnection / connection generator. The parameter values: $N_- = 4$, $N_+ = 5$, $\mu = 3$ (color online)

3. Conditions for safe disconnection of generators

Let us now consider in more detail disconnection of generators from the power grid. In general, three different disconnection scenarios are possible. First, the initial power grid can operate in a homogeneous synchronous mode (regions $a_1 - a_3$ at $N = N_+$), before disconnection, corresponding to the steady state of $O_1^{\text{hom}}(N_+)$. Then the disturbed power grid ($N = N_-$) will be in the following homogeneous state immediately after disconnection:

$$\vec{v}^1 = \left\{\varphi_i = \varphi^{\text{hom}}(N_+),\ y_i = 0,\ i = \overline{1, N_-}\right\}. \tag{11}$$

Secondly, the power grid can initially operate in one of the heterogeneous synchronous modes (region a_2 at $N = N_+$). Let us assume that before disconnection, the power grid was in synchronous mode corresponding to the steady state $O_j^{\text{inh},1}(N_+)$. As noted in section 1, in this mode of the power grid, there is another «heterogeneous» $(j+1)$-th load supply path, including $(j+1)$-th generator, which, like the first one, differs from the others in current and transmitted power. Therefore, the state of the power grid after disconnection depends on a set of disconnected generators. If the set includes $(j+1)$-th generator, which is part of an additional «heterogeneous» load supply path, then after disconnection, the power grid will be in the following homogeneous state:

$$\vec{v}^2 = \left\{\varphi_i = -\alpha + \Phi_1^{(1)}(N_+),\ y_i = 0,\ i = \overline{1, N_-}\right\}. \tag{12}$$

If the $(j+1)$-th «heterogeneous» generator is not included in the set, then after disconnection, the power grid will be in one of the following heterogeneous states:

$$\begin{aligned} \vec{v}_k^3 = \Big\{&\varphi_k = -\alpha + \pi - \Phi_1^{(1)}(N_+), \quad y_k = 0, \\ &\varphi_i = -\alpha + \Phi_1^{(1)}(N_+), \quad y_i = 0, \quad i = \overline{1, N_-},\ i \neq k\Big\}, \\ k = \overline{1, N_- - 1}.& \end{aligned} \tag{13}$$

Note, however, that due to the permutation symmetry of the system (1), the modes established in the disturbed power grid from any of these states have identical stability characteristics. Therefore, it is sufficient to analyze the behavior of the power grid using only one of the states $\vec{v}_k^3$ as the initial one.

Next, we sequentially determine the parameter conditions under which a synchronous mode is established in the disturbed power grid from each of the initial states $\vec{v}^1 - \vec{v}^3$; these represent the conditions for the safe disconnection of generators in the corresponding initial grid.

Note that for homogeneous initial states $\vec{v}^1$ and $\vec{v}^2$, explicit analytical estimations of the regions of synchronous mode establishment can be obtained. Indeed, due to the presence of permutation symmetry in system (1), any solution for homogeneous initial conditions should have the form

$$\varphi_i(t) = \varphi(t),\ y_i(t) = y(t),\ i = \overline{1, N_-}, \tag{14}$$

where the functions $\varphi(t)$ and $y(t)$ obey the following system:

$$\begin{cases} \dot{\varphi} = y, \\ \dot{y} = \Delta - \mu y - f(N_-, \alpha)\sin\big(\varphi - \beta^{\text{hom}}(N_-, \alpha)\big). \end{cases} \tag{15}$$

After the transformations

$$\tau_{\text{new}} = f_{1/2}\tau,\ \varphi_{\text{new}} = \varphi - \beta^{\text{hom}}(N_-, \alpha),\ y_{\text{new}} = y/f_{1/2},\ \gamma = \Delta/f_{1/2}^2,\ \lambda = \mu/f_{1/2},$$

where $f_{1/2} = \sqrt{f(N_-, \alpha)}$, we get a system on the cylinder $\big(G = \mathrm{S}^1 \times \mathrm{R}^1\big)$ of the form

$$\begin{cases} \dot{\varphi}_{\text{new}} = y_{\text{new}}, \\ \dot{y}_{\text{new}} = \gamma - \lambda y_{\text{new}} - \sin\varphi_{\text{new}}, \end{cases} \tag{16}$$

where the dot indicates the derivative with respect to τ_{new}. System (16) describes, in particular, the dynamics of a damped pendulum subject to a constant external torque (γ) with a damping coefficient (λ), as well as the dynamics of a point superconducting Josephson junction. The dynamics of system (16) has been studied in detail and is widely represented in the literature (see, for example, [36–38]).

From the definition of the region $a = \{a_1, a_2, a_3\}$ it follows that $|\Delta| < f(N_-, \alpha)$, therefore, consider system (16) at $|\gamma| < 1$. It is known that the system in this case has two steady states:

$$O_1(\varphi_{\text{new}} = \varphi_1, y_{\text{new}} = 0) \text{ and } O_2(\varphi_{\text{new}} = \varphi_2, y_{\text{new}} = 0),$$

where $\varphi_1 = \arcsin\gamma$, $\varphi_2 = \pi - \arcsin\gamma$. The steady state O_1 is a stable node (or focus) and corresponds to a homogeneous synchronous mode of the disturbed power grid, and O_2 is a saddle.

If at the same time

$$\lambda > \lambda^* \approx 1.22 \left[\text{that is } \mu > \lambda^* \sqrt{f(N_-, \alpha)}\right] \tag{17}$$

or

$$\lambda \leqslant \lambda^* \text{ and } |\gamma| < \gamma^T(\lambda) \left[\text{that is } \mu \leqslant \lambda^* \sqrt{f(N_-, \alpha)} \text{ and } |\Delta| < f(N_-, \alpha)\gamma^T\left(\mu/\sqrt{f(N_-, \alpha)}\right)\right], \tag{18}$$

where $\gamma^T(\lambda)$ is the Tricomi curve, then the steady state O_1 is globally asymptotically stable and is established in system (16) from any initial conditions, including those corresponding to the states $\vec{v}^1$ and $\vec{v}^2$. Therefore, disconnecting of the generators in this case is safe, regardless of the synchronous mode initially established in the initial power grid.

If the conditions (17) and (18) are not fulfilled, then on the phase cylinder of system (16), along with the steady state O_1, there is a rotational limit cycle corresponding to the asynchronous mode of the disturbed power grid. In this case, the steady state of O_1 is established in the system only from a part of the initial conditions, and therefore disconnection of the generators may be unsafe. It is known that the function

$$V(\varphi_{\text{new}}, y_{\text{new}}) = \frac{y_{\text{new}}^2}{2} + \int_{\varphi_1}^{\varphi_{\text{new}}} \left(\sin\xi - \gamma\right) d\xi \tag{19}$$

is a Lyapunov function for system (16) [39]. It is known that along the trajectories of system (16), that is, with increasing time τ_{new}, the level curves $V(\varphi_{\text{new}}, y_{\text{new}}) = C = \text{const}$ decrease. Let us take advantage of this fact and estimate the area of attraction of O_1. Fig. 3, a shows several level curves of $V(\varphi_{\text{new}}, y_{\text{new}})$, with arrows indicating the vector field orientation of system (16). It can be seen that on the phase plane of system (16) there is the region Ω^+, which contains the steady state O_1 and inside which all the level curves are closed. Consequently, all trajectories of system (16) with initial conditions from the region Ω^+ tend to the steady state O_1. Note that the boundary of the region Ω^+ is determined by the part enclosed between the points φ_0 and φ_2 of level curve $V(\varphi_{\text{new}}, y_{\text{new}}) = V(\varphi_2^*, 0)$, passing through the saddle O_2 (see Fig. 3, a), namely

$$\Gamma_{\Omega^+} = \left\{\varphi_{\text{new}}, y_{\text{new}} \middle| \phi_0 \leqslant \varphi_{\text{new}} \leqslant \phi_2, \frac{y_{\text{new}}^2}{2} - \cos\phi_{\text{new}} - \gamma\phi_{\text{new}} = -\cos\phi_2 - \gamma\phi_2\right\},$$

where φ_0 is the smallest root of the equation $\cos\varphi + \gamma\varphi = \cos\varphi_2 + \gamma\varphi_2$. Thus, in order for the synchronous mode to be established after the generators are disconnected in the power grid, which is in state $\vec{v}^1$ $\left(\varphi_v = \varphi^{\text{hom}}(N_+)\text{, see Fig. 3, a}\right)$ or $\vec{v}^2$ $\left(\varphi_v = -\alpha + \Phi_1^{(1)}(N_+)\right)$, the following conditions must be met

$$\varphi_0(N_-) < \varphi^{\text{hom}}(N^+) < \varphi_2(N_-) \tag{20}$$

and

$$\varphi_0(N_-) < -\alpha + \Phi_1^{(1)}(N_+) < \varphi_2(N_-). \tag{21}$$

Fig. 3, b, c shows estimates of the regions ($a_{\text{syn}}^{\text{est}}$) obtained from conditions (20) and (21) for safe disconnection of one generator from the power grid of 4 generators ($N_+ = 4, N_- = 3$), which is initially in homogeneous and heterogeneous synchronous modes, respectively. Note that the conditions (20) and (21) do not depend on the parameter μ. Therefore, in the above areas, disconnection of the generator is safe for any $\mu > 0$.

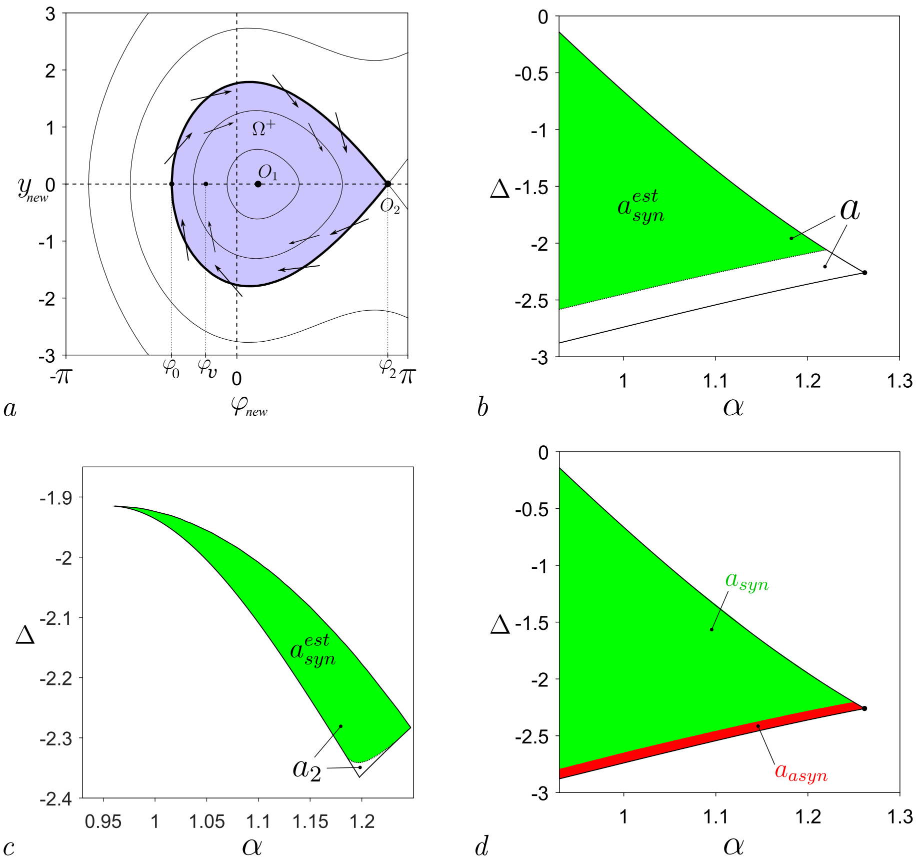

Fig. 3. Disconnection of generators associated with the establishment of homogeneous states in the disturbed power grid: a — qualitative form of the Lyapunov function for the disturbed power grid; b and c — analytical estimates of the regions of establishment of synchronous modes ($a^{\text{est}}_{\text{syn}}$, highlighted in green) from states $\vec{v}^1$ and $\vec{v}^2$, respectively; d — numerically found regions corresponding to the establishment of synchronous (a_{syn}, highlighted in green) and asynchronous modes (a_{asyn}, highlighted in red) from state $\vec{v}^1$ for $\mu = 0.5$. To compare the subregions $a^{\text{est}}_{\text{syn}}$ and a_2, the latter in (c) is depicted for $\mu = 3.0$. The parameter values: $N_+ = 4,\ N_- = 3$ (color online)

The regions of safe disconnection of one generator associated with the establishment of homogeneous initial states in the disturbed power grid $\vec{v}^1$ and $\vec{v}^2$, were also found numerically at different values of the parameter μ for the initial power grids of $N_+ = 43, 54$ and 65 generators. For this purpose, at fixed parameter values and the initial state, the corresponding system (1) was integrated at $N = N_- = N_+ - 1$ and it was determined whether its trajectory tended to a steady state corresponding to synchronous mode, or to some kind of attractor corresponding to the asynchronous mode. It has been established that for $\mu \geqslant 1.5$ in all the considered power grids, disconnection is safe for any values of the parameters α and Δ from the corresponding regions a and a_2, that is, the areas of simultaneous existence of synchronous modes in the initial and disturbed power grids. When the parameter μ is decreased, subregions appear in which asynchronous mode is set in the disturbed power grid, that is, disconnection becomes unsafe. Fig. 3, d shows the partition of the parameter plane (α, Δ) at $\mu = 0.5$ into regions corresponding to the establishment of synchronous (a_{syn}, highlighted in green) and asynchronous modes (a_{asyn}, highlighted in red), for $N_+ = 4, N_- = 3$ and the initial state of $\vec{v}^1$. A similar partition for the initial state of $\vec{v}^2$ is not given due to the smallness of the corresponding subregion of asynchronous behavior (unsafe disconnection). With a further decrease in μ, the subregion of unsafe disconnection increase, reducing the region of safe disconnection to analytically evaluated regions of $a^{\text{est}}_{\text{syn}}$ (see Fig. 3, b, c).

The case of inhomogeneous initial states $\vec{v}^3$ of the disturbed power grid was analyzed numerically. It is established that for each fixed value of μ there are two critical values of the size of the initial power grid. If the power grid is less than the minimal critical size, then at any values of the parameters α and Δ from the corresponding regions a_3, synchronous mode is set in the changed power grid, that is,

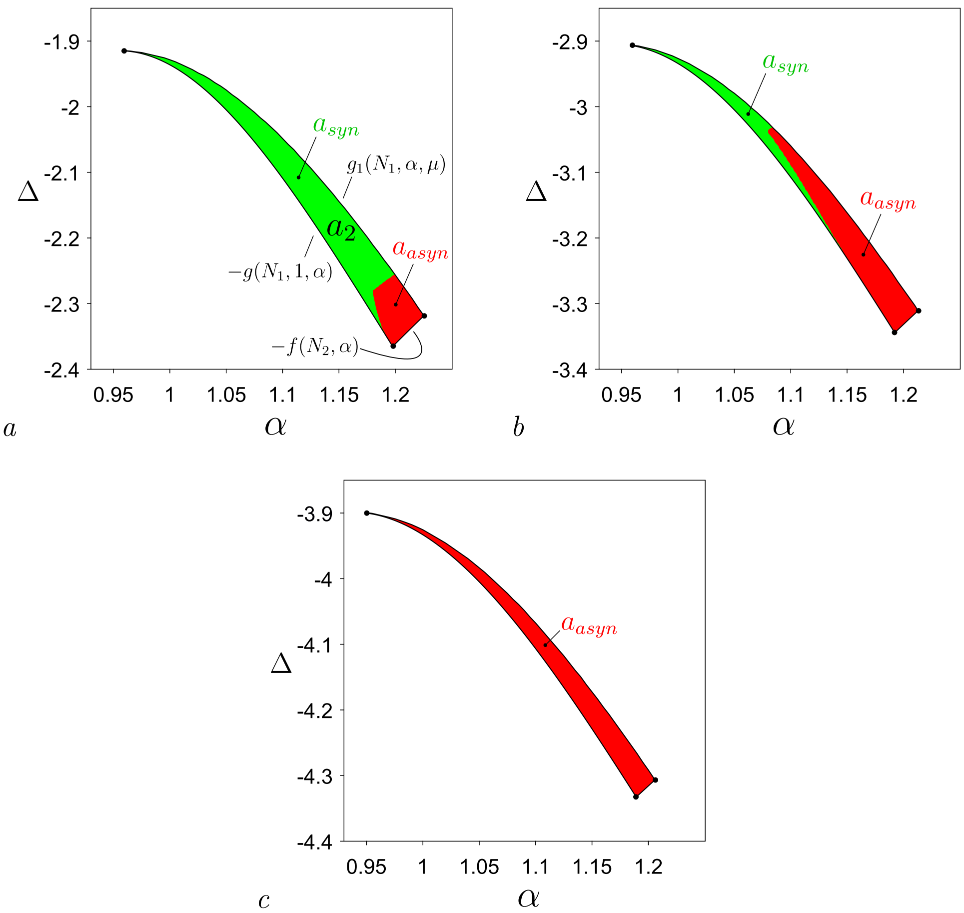

Fig. 4. Disconnection of generators associated with the establishment of one of the inhhomogeneous states $\vec{v}_k^3, k = \overline{1, N_- - 1}$, in the disturbed power grid: a–c — numerically found regions corresponding to the establishment of synchronous (a_{syn}, highlighted in green) and asynchronous modes (a_{asyn}, highlighted in red) for $N_+ = 4, N_- = 3$, $N_+ = 5, N_- = 4$, and $N_+ = 6, N_- = 5$, respectively. The parameter values: $\mu = 1.5$ (color online)

disconnection of generators will always be safe. On the contrary, if the size of the power grid is greater than the maximal critical one, then for any values of the parameters from a_3, asynchronous mode is set in the disturbed power grid, that is, disconnection of generators will always be unsafe. If the power grid has some intermediate size, then the corresponding region a_3 is divided into two subregions, corresponding respectively to the safe and unsafe disconnection of generators. Fig. 4, a–c shows the division of the parameter plane (α, Δ) at $\mu = 1.5$ into regions corresponding to the safe and unsafe disconnection of one generator in the power grid of $N_+ = 4, 5$ and 6 generators.

4. Conditions for the safe connection of generators

Let us now consider connection of generators to the power grid. We assume that the rotors of the connected generators are pre-accelerated to a rotation frequency close to the reference frequency of the power grid. In general, two different connection scenarios are possible. First, the initial power grid can operate in a homogeneous synchronous mode (regions a_1–a_3 at $N = N_-$) before connection, corresponding to the steady state $O_1^{\text{hom}}(N_-)$. Then the disturbed power grid ($N = N_+$) will be in the following state immediately after connection

$$\vec{v}^4 = \left\{\varphi_i = \varphi^{\text{hom}}(N_-), y_i = 0, i = \overline{1, N_-}; \varphi_k = \varphi_k^0, y_k = y_k^0, k = \overline{N_- + 1, N_+}\right\}, \quad (22)$$

where the coordinates $\varphi_k^0 = \delta_1(0) - \delta_k^0$ *and* $y_k^0 = \dot{\delta}_1(0) - \dot{\delta}_k^0$ *are determined by the states of both connected generators, so did the initial first generator. Since the rotor of the connected generators was pre-accelerated, we assume that these coordinates have random values from the intervals* $\varphi_k^0 \in [-\pi, \pi]$ *and* $y_k^0 = [y^-, y^+]$*, where* $y^\pm$ *are calculated using the formula (3) for* $N = N_+$*.*

If the initial power grid operates in one of the inhomogeneous synchronous modes (region a_3 *at* $N = N_-$*), then due to the symmetry of system (1), the coordinates of any of them can be used when composing the vector of the initial state, for example, corresponding to the steady state* $O_j^{inh,1}(N_-)$*. In this case, the disturbed power grid (*$N = N_+$*) will be in the following state immediately after connection*

$$\vec{v}^{\,5} = \Big\{ \quad \varphi_j = -\alpha + \pi - \Phi_1^{(1)}(N_-), \quad y_j = 0, \qquad \varphi_i = -\alpha + \Phi_1^{(1)}(N_-), \quad y_i = 0, \quad i = \overline{1, N_- - 1}, \; i \neq j, \qquad \varphi_k = \varphi_k^0, \quad y_k = y_k^0, \quad k = \overline{N_- + 1, N_+} \quad \Big\}. \tag{23}$$

To obtain the safe connection region, the following procedure was employed. For fixed parameter values based on (22) and (23), sets of initial states for the disturbed power grid were generated by randomly sampling the coordinates φ_k^0 *and* y_k^0*. These states served as initial conditions for the numerical integration of system (1) with* $N = N_+$ *to identify the resulting disturbed power grid behavior. The fraction of trajectories converging to stable steady states was then calculated to estimate the probability of establishing*

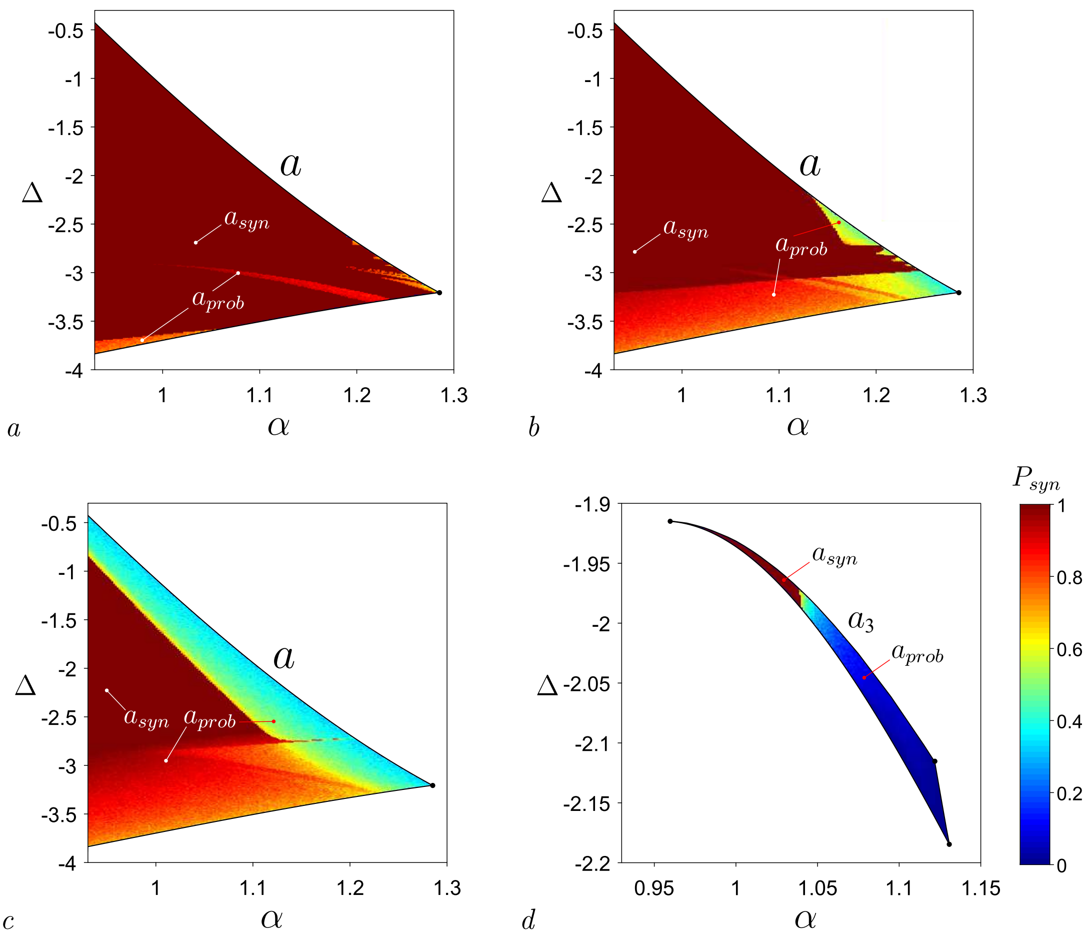

Fig. 5. Connection of generators: *a*–*c* — probability P_{syn} of establishing a synchronous mode in the disturbed power grid from a homogeneous state of the initial power grid for $\mu = 1.50$, $\mu = 1.25$ and $\mu = 1.00$, respectively; *d* — probability of establishing a synchronous mode from an inhomogeneous state of the initial power grid for $\mu = 1.00$. The parameter values: $N_- = 4$, $N_+ = 5$ (color online)

a synchronous mode (denoted as P_{syn}*). Based on values of* P_{syn}*, two distinct types of parameter subregions are identified. The first type includes the subregions in which* $P_{\text{syn}} = 1$*, which we denote by* a_{syn}*. With the parameters of these subregions, connection of the generator is safe, since synchronous operation is guaranteed in the disturbed power grid. The second type includes subregions where* $0 < P_{\text{syn}} < 1$*, which we denote by* a_{prob}*. For parameters from these subregions, the establishment of a synchronous mode is probabilistic, so connection of the generator is unsafe.*

Figure 5 shows the results for the case when one generator ($N_+ = 5$) *is connected to a power grid containing five generators* ($N_- = 4$)*. It was found that for the initial states* $\vec{v}^4$ *and* $\mu > 2.00$*, the subregion* a_{syn} *completely coincides with* a*, which guarantees the safe connection of the generator to the initial power grid. Otherwise, there are both types of subregions* a_{syn} *and* a_{prob}*. The mutual arrangements of these subregions at* $\mu = 1.50, 1.25$ *and* 1.00 *are shown in Fig. 5, a–c. When the parameter* μ *is reduced, the size of the subregion* a_{syn} *decreases. The total size of the subregions* a_{prob} *increases, and the maximum probabilities of* P_{syn} *in these subregions decrease. In turn, for the initial states* $\vec{v}^5$ *and the parameter values* $\mu \geqslant 1.25$ *the subregion* a_{syn} *completely coincides with* a_3*, which guarantees a safe connection of the generator to the initial power grid. If* $\mu < 1.25$ *(see Fig. 5, d), then along with the subregion* a_{syn} *there is a subregion* a_{prob}*, that is, the connection of the generator becomes unsafe. At the same time, it was found that for parameters from the subregions* a_{syn}*, a homogeneous synchronous mode is always established in the disturbed power grid. In parts of the subregions* a_{prob}*, where heterogeneous modes can coexist in the disturbed power grid, the probability of their establishment does not exceed* 0.13.

Conclusion

In this paper, we consider a power grid of an arbitrary number of generators operating on a common passive linear load, in the case when one of the generators is «electrically» closer to the load (due to a shorter transmission line and/or longitudinal reactance compensation). To describe the dynamics of this network, a reduced effective network model in the form of an ensemble with a hub topology (the «star» topology) is used. The problem of the stability of the power grid, initially operating in one of the synchronous modes, to disconnection and connection of generators is studied. Using an approach based on the Lyapunov direct (or second) method, conditions were found for the parameters that ensure the safe disconnection of generators, including, if any, a generator involved in a «heterogeneous» load supply path, which differs from the others in current and transmitted power. These estimates are validated numerically for networks of various sizes. The evolution of the area corresponding to the safe connection of a generator to a five-generator power grid is also numerically traced.

References

1. *Zhdanov PS. Stability Issues for Electrical Systems. M.: Energy; 1979. 456 p. (in Russian).*
2. *Venikov VA. Transient electromechanical processes in electrical systems. M.: Vysshaya shkola; 1985. 536 p. (in Russian).*
3. *Idelchik VI. Electrical systems and networks. M.: Energoatomizdat; 1989. 592 p. (in Russian).*
4. *Kundur P, Balu NJ, Lauby MG. Power System Stability and Control. New York: McGraw-Hill Education; 1994. 1176 p.*
5. *Sauer P, Pai A. Power System Dynamics and Stability. Prentice-Hall: Englewood Cliffs; 1998. 357 p.*
6. *Anderson PM, Fouad AA. Power System Control and Stability. NJ: IEEE, Piscataway; 2003. 672 p.*
7. *Horowitz SH, Phadke AG, Henville CF. Power System Relaying. New York: John Wiley & Sons; 2008. 528 p.*
8. *Machowski J, Bialek J, Bumby D. Power System Dynamics: Stability and Control. New York: John Wiley and Sons; 2008. 629 p.*
9. *Grainger JJ., Stevenson WD. Power System Analysis. New York: McGraw-Hill Education; 2016. 787 p.*

10. *Dobson I, Carreras BA, Lynch VE, Newman DE. Complex systems analysis of series of blackouts: Cascading failure, critical points, and self-organization. Chaos. 2007;17(2):026103. DOI: 10.1063/1.2737822.*
11. *Schafer B, Witthaut D, Timme M, Latora V. Dynamically induced cascading failures in power grids. Nat. Commun. 2018;9(1):1975. DOI: 10.1038/s41467-018-04287-5.*
12. *Bialek JW. Why has it happened again? Comparison between the UCTE blackout in 2006 and the blackouts of 2003. In: IEEE Lausanne Power Tech. Lausanne, Switzerland. 2007. P. 51–56. DOI: 10.1109/PCT.2007.4538291.*
13. *Li C, Sun Y, Chen X. Analysis of the blackout in Europe on November 4, 2006. In: 2007 International Power Engineering Conference (IPEC 2007). 2007. P. 939–944.*
14. *van der Vleuten E, Lagendijk V. Interpreting transnational infrastructure vulnerability: European blackout and the historical dynamics of transnational electricity governance. Energy Policy. 2010;38(4):2053–2062. DOI: 10.1016/j.enpol.2009.11.030.*
15. *Veloza OP, Santamaria F. Analysis of major blackouts from 2003 to 2015: classification of incidents and review of main causes. Electr. J. 2016;29(7):42–49. DOI: 10.1016/j.tej.2016.08.006.*
16. *Shao Y, Tang T, Yi J, Wang A. Analysis and lessons of blackout in Turkey power grid on March 31. AEPS. 2016;40(23):9–14. DOI: 10.7500/AEPS20160412004.*
17. *Coletta T, Jacquod P. Linear stability and the Braess paradox in coupled-oscillator networks and electric power grids. Phys. Rev. E. 2016;93(3):032222. DOI: 10.1103/PhysRevE.93.032222.*
18. *Khramenkov VA, Dmitrichev AS, Nekorkin VI. A new scenario for Braess's paradox in power grids. Chaos. 2023;32(11):113116. DOI: 10.1063/5.0093980.*
19. *Schafer B, Pesch T, Manik D, Gollenstede J, Lin G, Beck HP, Witthaut D, Timme M. Understanding Braess' paradox in power grids. Nat. Commun. 2022;13(1):5396. DOI: 10.1038/s41467-022-32917-6.*
20. *Sangjoon P, Kim CH, Kahng B. Optimal location of reinforced inertia to stabilize power grids. Chaos, Solitons and Fractals. 2025;199(2):116768. DOI: 10.1016/j.chaos.2025.116768.*
21. *Klinshov VV, Nekorkin VI, Kurths J. Stability threshold approach for complex dynamical systems. New J. Phys. 2015;18(1):013004. DOI: 10.1088/1367-2630/18/1/013004.*
22. *Mitra C, Kittel T, Choudhary A, Kurths J, Donner RV. Recovery time after localized perturbations in complex dynamical networks. New J. Phys. 2017;19(10):103004. DOI: 10.1088/1367-2630/aa7fab.*
23. *Mitra C, Kittel T, Choudhary A, Sinha S, Kurths J, Donner RV. Multiple-node basin stability in complex dynamical networks. Phys. Rev. E. 2017;95(3):032317. DOI: 10.1103/PhysRevE.95.032317.*
24. *Wolff MF, Lind PG, Maass P. Power grid stability under perturbation of single nodes: Effects of heterogeneity and internal nodes. Chaos. 2018;28(10):103120. DOI: 10.1063/1.5040689.*
25. *Klinshov VV, Kirillov S, Kurths J, Nekorkin VI. Interval stability for complex systems. New J. Phys. 2018;20(4):043040. DOI: 10.1088/1367-2630/aab5e6.*
26. *Halekotte L, Feudel U. Minimal fatal shocks in multistable complex networks. Sci. Rep. 2020;10(1):11783. DOI: 10.1038/s41598-020-68805-6.*
27. *Halekotte L, Vanselow A, Feudel U. Transient chaos enforces uncertainty in the british power grid. J. Phys. Complex. 2021;2(3):035015. DOI: 10.1088/2632-072X/ac080f.*
28. *Khramenkov VA, Dmitrichev AS, Nekorkin VI. Bistability of operating modes and their switching in a three-machine power grid. Chaos. 2023;33(10):103129. DOI: 10.1063/5.0165779.*

29. *Gambuzza LV, Buscarino A, Fortuna L, Porfiri M, Frasca M. Analysis of dynamical robustness to noise in power grids. IEEE Journal on Emerging and Selected Topics in Circuits and Systems. 2017;7(3):413–421. DOI: 10.1109/JETCAS.2017.2649598.*
30. *Schmietendorf K, Peinke J, Kamps O. The impact of turbulent renewable energy production on power grid stability and quality. Eur. Phys. J. B. 2017;90:222. DOI: 10.1140/epjb/e2017-80352-8.*
31. *Tumash L, Olmi S, Scholl E. Effect of disorder and noise in shaping the dynamics of power grids. EPL. 2018;123:20001. DOI: 10.1209/0295-5075/123/20001.*
32. *Haehne H, Schmietendorf K, Peinke J, Kettemann S, Tamrakar S. Propagation of wind-power-induced fluctuations in power grids. Phys. Rev. E. 2019;99:050301. DOI: 10.1103/PhysRevE.99.050301.*
33. *Arinushkin PA, Kupriyanov VD, Vadivasova TE. Influence of Gaussian noise and Levy noise on the phase dynamics of the ensemble of Kuramoto-like oscillators of first and second order // Izvestiya VUZ. Applied Nonlinear Dynamics. 2025;33(3):289–306. DOI: 10.18500/0869-6632-003145.*
34. *Khramenkov VA, Dmitrichev AS, Nekorkin VI. Multistability of synchronous modes in a multimachine power grid with a common load and their global and non-local stability. Izvestiya VUZ. Applied Nonlinear Dynamics. 2025;33(1):38–68. DOI: 10.18500/0869-6632-003128.*
35. *Kalentionok EV. Electric Power Systems Stability. Minsk: Technoperspectiva; 2008. 375 p. (in Russian).*
36. *Tricomi F. Integrazione di un' equazione differenziale presentatasi in elettrotecnica. Annali della Scuola Normale Superiore di Pisa - Classe di Scienze, Ser. 2. 1933;2(1):1–20.*
37. *Andronov AA, Vitt AA, Khaikin SE. Theory of Oscillators. Oxford: Pergamon; 1966. 848 p.*
38. *Nekorkin VI. Introduction to Nonlinear Dynamics of Oscillations and Waves. M.: Fizmatlit; 2024. 352 p. (in Russian).*
39. *Barbashin EA. Lyapunov Functions. M.: Nauka; 1979. 240 p. (in Russian).*