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Synchronization and desynchronization in ensembles of mobile agents

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Abstract. The *purpose* of this work is to analyze the mechanisms of influence and destruction, as well as the characteristics of synchronous and asynchronous modes of behavior of ensembles (swarms) of interacting mobile agents moving according to chaotic phase trajectories of Rössler and Lorentz. *Methods.* One of the main ways to obtain synchronous chaotic dynamics is the effect of chaotic phase synchronization — the process of establishing the same averaged frequency of oscillations and modulo limited phase difference in the presence of sufficiently strong coupling. Numerical integration methods of systems of differential equations are used for modeling and obtaining results. *Results.* In the context of ensembles of interacting mobile agents whose motion trajectories obey the Rössler and Lorenz systems in chaotic regimes, the influence of different types of individualities on collective dynamics was considered. The possibility of organizing sequential and parallel action of agents and various topological configurations of the organization of a swarm of agents when choosing a uniformity structure was demonstrated. The duration of transient processes in a synchronous regime in ensembles of mobile agents moving along the trajectories of the Rössler system in different regimes was considered. In addition, for cases of chaotic Rössler attractors, a method for swarm desynchronization using independent phase tuning was proposed. *Conclusion.* The article demonstrates the ability to synchronize and desynchronize ensembles of mobile agents.

Keywords: mobile agent, ensemble, chaotic phase synchronization, Rossler system, Lorenz system.

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Introduction

The concept of a mobile agent as a dynamic unit with spatial freedom and the ability to interact autonomously has become an important paradigm in modelling collective behavior [1, 2]. Such agents, unlike static nodes of network, are characterized by spatial mobility, the ability to create local rules of interaction, and emergent behavior.

As shown in [3], agent mobility can enhance synchronization in meta-populations, creating collective modes that are unattainable for static systems. Applications include swarm robotics [4], drone coordination [5], and biological modeling [6]. A fundamental role in such systems is played by *pulse coupling*, where interaction occurs discretely when agents approach. It has been shown that even simple pulse-coupled oscillators are capable of global synchronization [7]. This model has been expanded for mobile agents: spatial motion becomes an *active regulator* of synchronous states [8, 9].

Hybrid approaches combining metric (distance) and topological (neighbor selection) interactions are being developed [10]. As noted in [11], the spatial movement of agents can both suppress and enhance synchronization, depending on the system parameters.

In the first studies of the synchronization of chaotic oscillators, global coupling was mainly considered (see, for example, [12]), which is convenient for modelling and theoretical analysis, but poorly correlated with the real world. In recent studies, however, preference is given to various non-local coupling that largely reflect the actual observed phenomena. The conditions for enabling coupling between agents can also be quite diverse: elements can interact only with those that enter their field of view [13]; when located in predefined zones of space [14] or when located at a sufficiently close distance to each other [15, 16].

In this paper, we consider a coupling option in which interaction is activated only when agents enter a cylinder of a given radius. The location of the mobile agent in three-dimensional space is determined by the phase trajectory of a three-dimensional dynamic system demonstrating chaotic behavior. Due to the randomness of the movement, agents sooner or later find themselves in sufficient proximity for coupling to be activated. Using the example of chaotic Rössler and Lorentz attractors, the possibility of organizing sequential and parallel movement of agents based on the effect of chaotic phase synchronization is demonstrated.

1. The ensemble model

As a mobile agent, consider a material point moving in three-dimensional space (x, y, z) so that its trajectory completely coincides with the trajectory of the oscillator aligned with it. The generalized view describing the behavior of an ensemble of interacting particles is as follows:

$$\begin{cases} \dot{x}_i = f_x + d_x\big[\sum_{j=1}^{N}(x_j - x_i)\big], \\ \dot{y}_i = f_y + d_y\big[\sum_{j=1}^{N}(y_j - y_i)\big], \qquad i = \overline{1, N}, \\ \dot{z}_i = f_z + d_z\big[\sum_{j=1}^{N}(z_j - z_i)\big]. \end{cases} \tag{1}$$

In the absence of couplings ($d_x = d_y = d_z = 0$), the dynamics of an individual agent can be either regular or chaotic. In general, all agents are non-identical.

The couplings between the ith and jth agents is activated only when they are sufficiently close: when the agents are inside a cylinder of radius R:

$$d_k = \begin{cases} d', \ (x_i - x_j)^2 + (y_i - y_j)^2 < R^2, \\ 0, \ \text{otherwise}, \end{cases} \tag{2}$$

where $k = \{x, y, z\}$, $d' = \text{const}$ is a parameter that determines the strength of coupling. The coupling introduced in this way is called attractive for a positive d', and repellent for a negative d'. The condition for the area of agent interaction may differ from the condition (2). This may not be a cylinder, but, for example, a sphere, cube, strip, etc. The point is that due to the chaotic nature of the attractor and the relatively large area in phase space where the trajectories can lie, there is a high (100% under certain conditions on the radius r under condition (2)) probability of entering the interaction area. Coupling between the elements in time can be organized in three ways:

a) after the trajectories enter the cylinder, the coupling between the oscillators is not disabled;
b) the coupling can be disabled after one of the oscillators leaves the cylinder;
c) the coupling can only be active for a certain period of time.

Unless otherwise specified, the paper considers the first option.

The goal of the paper is to study the collective dynamics of ensembles of interacting agents moving along chaotic trajectories. In coherent dynamics, ensembles exhibit a regime of chaotic phase synchronization. For the regime of chaotic phase synchronization of two oscillators, two conditions must be met: the coincidence of the average frequencies:

$$\Omega_i = \langle \nu_i \rangle = \Omega_j = \langle \nu_j \rangle \tag{3}$$

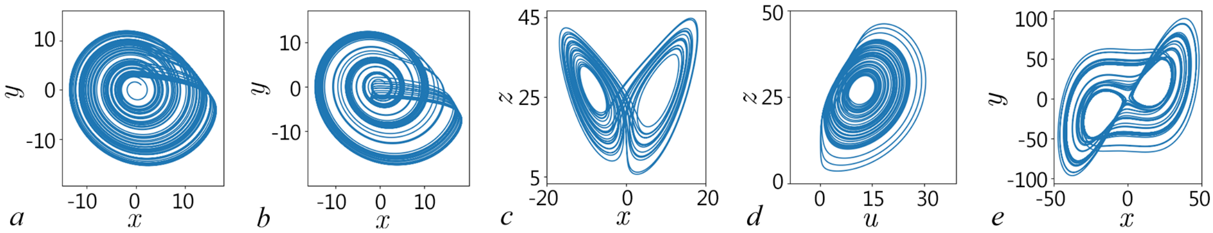

Fig. 1. Projections of chaotic attractors. a — projection onto the (x, y) plane of the Rössler attractor (system (5), $\omega = 1$) in the phase-coherent regime ($a = 0.16$, $b = 0.1$, $c = 8.5$); b — projection onto the (x, y) plane of the Rössler attractor in the funnel regime ($a = 0.22$, $b = 0.1$, $c = 8.5$); c — projection onto the (x, z) plane of the classical Lorenz attractor (system (12) with $d_y = 0$) ($\sigma = 10$, $b = 8/3$, $r = 28$); d — projection of the classical Lorenz attractor onto the plane ($u = \sqrt{x^2 + y^2}, z$); e — projection onto the plane (x, y) of the Lorentz attractor arising through intermittency $\sigma = 10$, $b = 8/3$, $r = 166.1$

and there must be a limited phase difference:

$$\mathrm{const}_1 \leqslant |\phi_i(t) - \phi_j(t)| \leqslant \mathrm{const}_2. \tag{4}$$

In ensembles, synchronization can be either global or cluster-based. In the first case, all agents follow similar trajectories with a certain spacing between them. This is achieved by introducing a phase shift $\phi_i(t) - \phi_j(t)$. In the second case , ensembles are divided into separate groups of agents that move synchronously.

The Rössler system and the Lorenz system are considered as oscillators along which the agents move. Recall that in the Rössler system, chaotic oscillations arise as a result of a cascade of bifurcations of the doubling of the period of limit cycles. Depending on the parameters of the system, the chaotic attractor can be phase-coherent (Fig. 1, *a*) or a funnel attractor (Fig. 1, *b*). In the Lorenz system, we consider the classical Lorenz attractor (Fig. 1, *c*, *d*) and the chaotic attractor that arises through intermittency (Fig. 1, *e*).

2. Synchronization of Rössler attractors

Let the agent's motion occur along a chaotic trajectory of the Rössler system [17]:

$$\begin{cases} \dot{x}_i = -w_i y_i - z_i, \\ \dot{y}_i = w_i x_i + a y_i, \quad i = \overline{1, N}, \\ \dot{z}_i = b + z_i(x_i - c_i), \end{cases} \tag{5}$$

where a, b, c are positive parameters. In the following experiments, we will assume $a = 0.16$ for the phase-coherent attractor, $a = 0.22$ and $a = 0.28$ for the funnel attractor, $b = 0.1$. The parameter w_i, which is randomly selected from the interval $[0.93; 1.07]$, characterizes the time scales of the oscillations. The parameter c_i, as will be shown later, affects the average amplitude of the oscillator. In our study, we will assume that $d' = 0.3, R = 4$. The value of the coupling parameter d' is chosen such that all interacting chaotic oscillators are phase-synchronized without introducing the parameter of trajectory proximity R [12, 18].

The system parameters are chosen so that, under initial conditions close enough to zero (in this paper, a cube with an edge length of 10 with the center at the origin was considered), the phase trajectories do not go to infinity, but are attracted to a chaotic quasi-atractor (see, for example, [19]). Formally, modeling and the specified bifurcation scenario give only a chaotic set.

Let us introduce the phase as follows:

$$\phi = \arctan \frac{\dot{y}}{\dot{x}}. \tag{6}$$

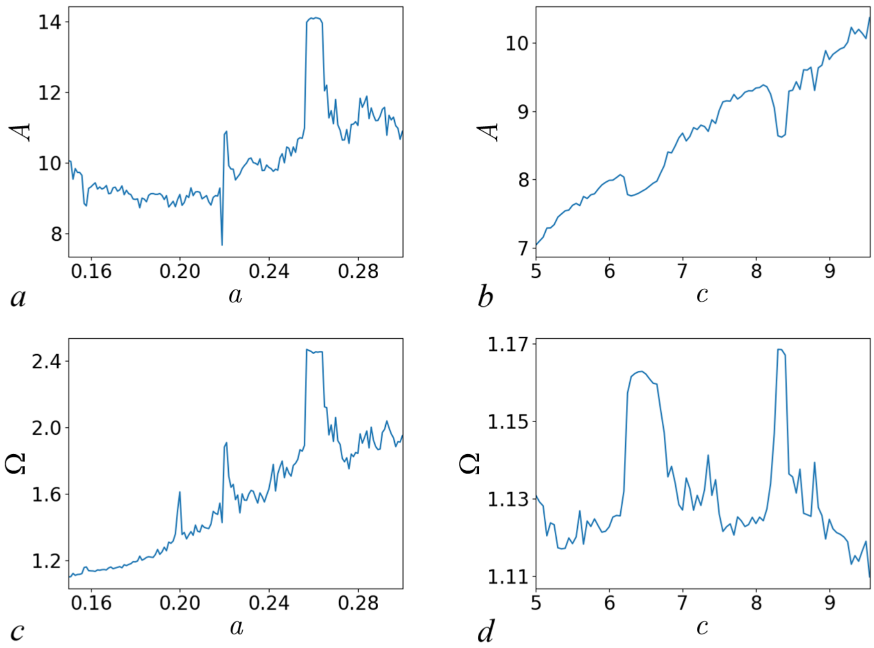

Fig. 2. Average amplitudes and frequencies of the Rössler attractor. a and c are the dependences of the average amplitude and average frequency ω on the parameter a (with the parameter $c = 8.5$). b and d are the dependences of the average amplitude and average frequency on the parameter c, the attractor is in the phase-coherent regime ($a = 0.16$). There are windows of periodic behavior of the phase trajectories

Then the formula for calculating the average frequency will take the form

$$\Omega = \left\langle \frac{\dot{y}\ddot{x} - \ddot{y}\dot{x}}{\dot{x}^2 + \dot{y}^2} \right\rangle. \tag{7}$$

The formula for calculating the average amplitude in the projection of the phase trajectory onto the plane (x, y) is as follows:

$$A = \left\langle (x^2 + y^2)^{\frac{1}{2}} \right\rangle. \tag{8}$$

To select the type of coupling between agents in order to obtain a given type of their movement, the influence of parameters a and c on the amplitude and average frequency of chaotic oscillations of system (5) was analyzed (Fig. 2). From Fig. 2, a we can only conclude that in the case of the funnel attractor, as the parameter a increases, the value of the average amplitude also increases.

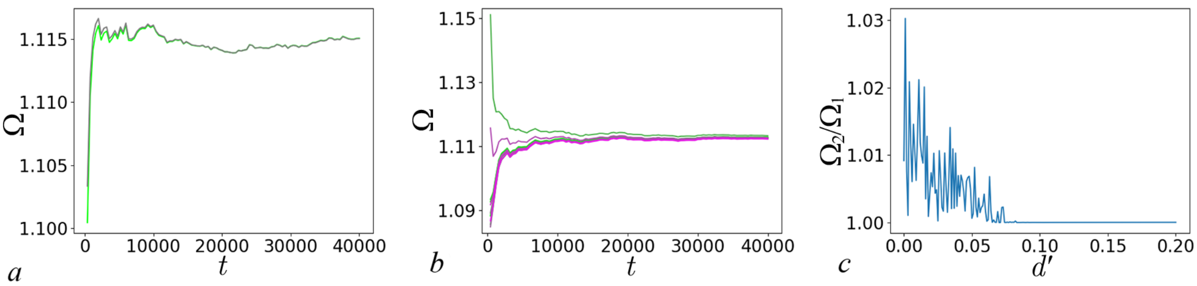

Fig. 3. Time evolution of average frequencies for a — $N = 2$ elements at $c_1 = 8.5$, $c_2 = 8.501$ and b — for $N = 10$ elements at $c_1 = 8.5, c_2 = 8.501, ..., c_{10} = 8.509$ under different initial conditions, $d' \approx 0.08$. c is the ratio of average frequencies Ω_2/Ω_1 on the coupling strength for $N = 2$. The average frequencies become the same already at $d' \approx 0.08$, which is the minimum required coupling strength for the appearance of chaotic phase synchronization. $c_1 = 8.5$, $c_2 = 8.501$, $a = 0.16$ (color online)

Fig. 2, b shows that as the parameter c, the average amplitude of the oscillator also increases. If there is a small heterogeneity in c, the average frequencies match for two or more elements (condition (3)), which is demonstrated in Fig. 3 a, b with sufficient coupling strength (Fig. 3, c).

From Fig. 2, c it can be seen that as the parameter a increases, the average oscillation frequency in the Rössler system increases. Consequently, the agent's movement speed increases. In the case of an ensemble of coupled agents, an increase in the speed of their movement at the values of the parameter a (in experiments with the ensemble, the values $a = 0.22$ and $a = 0.28$ were considered), at which the funnel attractor is realized, leads to a significant decrease in the transition time to the synchronous regime compared with the case of a phase-coherent attractor ($a = 0.16$). This was confirmed in the experiments described below.

Fig. 2, d demonstrates a weak dependence of the average frequency of Ω on the parameter c.

2.1. Two coupled elements.

Before demonstrating the possibility of controlling the behavior of a swarm of mobile agents, let us consider the synchronization of two coupled elements of unequal frequencies $\omega_{1,2}$ and values of the parameter $c_{1,2}$:

$$\begin{cases} \dot{x}_1 = -w_1 y_1 - z_1 + d_x(x_2 - x_1), \\ \dot{y}_1 = w_1 x_1 + a y_1 + d_y(y_2 - y_1), \\ \dot{z}_1 = b + z_1(x_1 - c_1) + d_z(z_2 - z_1), \\ \dot{x}_2 = -w_2 y_2 - z_2 + d_x(x_1 - x_2), \\ \dot{y}_2 = w_2 x_2 + a y_2 + d_y(y_1 - y_2), \\ \dot{z}_2 = b + z_2(x_2 - c_2) + d_z(z_1 - z_2). \end{cases} \tag{9}$$

2.1.1. Sequential motion.

A sequential motion of two agents is a motion that satisfies the following two conditions in each time interval:

- a small difference in the average amplitudes:

$$|A_2 - A_1| < \varepsilon; \tag{10}$$

- a nonzero, modulo limited difference in phases:

$$0 < \mathrm{const}_1 \leqslant |\phi_2 - \phi_1| \leqslant \mathrm{const}_2. \tag{11}$$

To obtain a sequential motion, we add a couplings in the y variable ($d_x = d_z = 0$) to a system of two elements and a phase detuning between agents $w_{1,2} \in [0.93, 1.07]$. The initial conditions of the mobile agents are different and random within the attractor. Over time, due to the chaotic nature, the depicted points will sooner or later be at a distance less than r, and with a sufficiently large value of the coupling strength (d') synchronization is achieved, in which the agents move sequentially, one after the other.

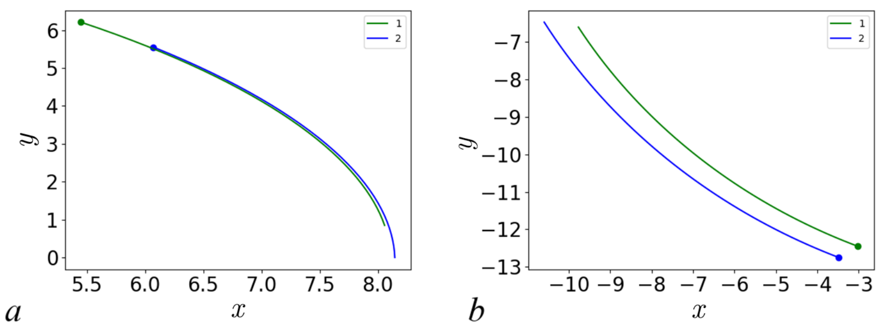

Fig. 4. Fragment of phase portraits of two coupled Rössler oscillators. a — sequential motion of mobile agents, $c_1 = c_2 = 8.5$, $\omega_1 = 1, \omega_2 = 0.94$. b — parallel motion of mobile agents, $c_1 = 8.5, c_2 = 9$, $\omega_1 = \omega_2 = 1$, $d' = 0.3$ (color online)

In the sequential version of mobile agents, the time interval of motion repetition can be adjusted by changing the coupling coefficient. For certain values of the coupling coefficient, it is possible to achieve a time interval where the phase variables are almost identical ($x_1(t) = x_2(t+\tau), ...$). This effect is known as lag synchronization, where the time shift (τ) between the states of interacting systems decreases according to a hyperbolic law [20, 21].

The results of numerical experiments are presented in Fig. 4, *a*. The depicted points almost always follow the same trajectories, but with a limited distance between them.

2.1.2. Parallel motion. Let us call the motion of two agents parallel, satisfying the following conditions:

- the modulo limited difference between the average amplitudes:

$$\text{const}_1 \leqslant |\text{A}_2 - \text{A}_1| \leqslant \text{const}_2; \tag{12}$$

- small phase difference:

$$|\phi_2 - \phi_1| < \varepsilon. \tag{13}$$

To achieve parallel motion, instead of phase detuning, we add amplitude detuning $c_1 = 8.5, c_2 = 8.51$, and a coupling in the y variable ($d_x = d_z = 0$). Due to the difference in average amplitudes, when the coupling strength is sufficient, the agents begin to move with similar phases but different amplitudes, resulting in parallel motion. The behavior of the agents under this coupling is illustrated in Fig. 4, *b*.

2.2. An ensemble of N elements. In this section, we will consider an ensemble of N mobile agents, which are Rössler attractors described by the following equations:

$$\begin{cases} \dot{x}_i = -w_i y_i - z_i + d_x\left[\sum_{j=1}^{N}(x_j - x_i)\right], \\ \dot{y}_i = w_i x_i + a y_i + d_y\left[\sum_{j=1}^{N}(y_j - y_i)\right], \quad i = \overline{1, N}, \\ \dot{z}_i = b + z_i(x_i - c_i) + d_z\left[\sum_{j=1}^{N}(z_j - z_i)\right]. \end{cases} \tag{14}$$

2.2.1. Sequential motion. Let us consider the collective behavior of an ensemble of 10 mobile agents whose motion is governed by the chaotic trajectory of the Rössler system, which is a phase-coherent attractor. Let the Rössler systems be coupled only in the y variable in the second equation of system (1), where d_y is given by (2), $d_x = 0$, $d_z = 0$. The first works on chaotic phase synchronization of coupled Rössler systems were devoted to the analysis of systems coupled by the variable y. It was found that a pair [12] and a chain [22] of coupled non-identical Rössler systems in a chaotic regime transition to a synchronous regime when a certain critical value of the coupling parameter is reached. In our case, the coupling between the elements is turned on only when the phase trajectories of a pair of oscillators enter the cylinder (2). Since the coupling is chosen to be sufficiently large, the oscillators synchronize. They have

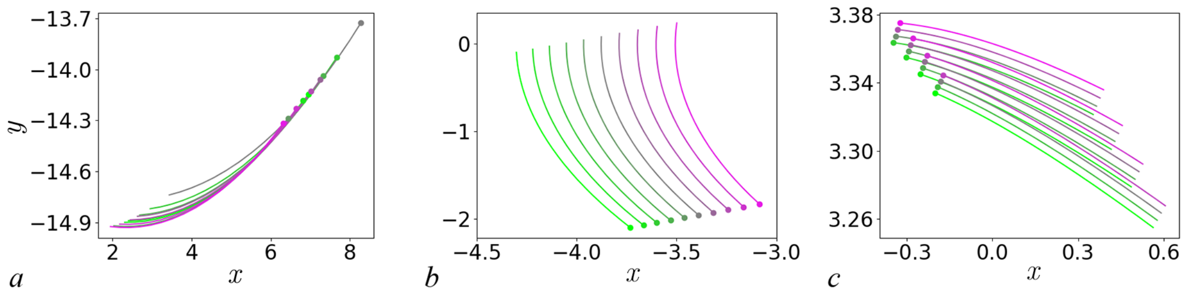

Fig. 5. Synchronization of an ensemble of mobile agents, communication by variable y. *a* — Sequential movement of agents (one after another) $w_i \in [0.93, 1.07]$, $c_i = 8.5$; *b* — parallel movement of agents (as a united front), $c_1 = 8.5$, $c_2 = 8.6$, $c_3 = 8.7, ..., c_{10} = 9.4$. $N = 10, d' = 0.3$. *c* — Movement in the form of a «square» structure of 4×4 elements ($N = 16$), $\forall i \in \{1, 2, \ldots, N\}\ \exists!\,(w_i, c_i) \in \Omega \times C$ with $\Omega = [0.9, 0.95, 1.0, 1.05]$, $C = [8.5, 8.51, 8.52, 8.53]$ (color online)

the same average frequencies, a small difference in amplitudes (10), and a limited phase detuning (11). At the same time, the corresponding mobile agents move along close trajectories at a certain distance from each other (Fig. 5, *a*) (see also [23]).

2.2.2. Parallel motion. To organize the parallel motion of an ensemble of N mobile agents whose trajectories obey the Rössler system, we use amplitude detuning in the absence of phase detuning, similar to the case of two elements. With sufficient coupling strength between the elements and sufficient experimental time, the average frequencies match (see Fig. 3, *b*), and the ensemble of agents moves with similar phases but different amplitudes (see Fig. 5).

2.2.3. Giving the swarm a structure of various geometric shapes. To form complex topological configurations of a swarm of agents, such as a square, rectangle, triangle, etc., we will use a combination of frequency and amplitude detunements. Next, without losing generality, we consider a configuration of the «square» type for $N = 16$ (structure 4×4 elements). We will select all unique combinations of sets as the values of the parameters ω_i, c_i for mobile agents

$$\omega = [0.9, 0.95, 1.0, 1.05],$$

$$c = [8.5, 8.501, 8.502, 8.503],$$

that is

$$\forall i \in \{1, 2, \ldots, N\}\ \exists!\,(w_i, c_i) \in \omega \times c.$$

If there is sufficient coupling strength, single agents, due to the chaotic nature of the attractor, begin to form groups of sequentially and parallel moving agents. Over time, the clusters will be combined in the form of the structure shown in Fig. 5 *c*. We have also conducted successful experiments on managing ensembles of mobile agents whose movements follow the chaotic trajectories of the hyperchaotic Rössler system. As the coordinates of the position of agents in the three-dimensional real space, we can use a set of three any phase variables of the four-dimensional Rössler system.

2.2.4. The effect of system parameters on the synchronization rate of the swarm. In the tasks of achieving a synchronous regime, the duration of the transition process to this regime plays an important role. For ten coupled Rössler systems, we will analyze the effect of two parameters: the parameter a, which characterizes the degree of coherence of the chaotic attractor, and the parameter τ, which affects the rate of change of the phase variables.

System under consideration (5) will take the form:

$$\begin{cases} \dot{x}_i = \tau \cdot (-w_i y_i - z_i), \\ \dot{y}_i = \tau \cdot \Big(w_i x_i + a_i y_i + d_y \cdot \big[\sum_{j=1}^{N} (y_j - y_i)\big]\Big), \qquad i = \overline{1, N}, \\ \dot{z}_i = \tau \cdot (b_i + z_i(x_i - c_i)), \end{cases} \tag{15}$$

where τ is the phase velocity multiplier of the phase variables.

To determine the synchronization moment and measure the time, we use conditions (3), (4) and formulas (6), (7). Let us consider different regimes of the Rössler system: phase-coherent ($a = 0.16$), attractor-funnel ($a = 0.22, a = 0.28$); $\tau \in \{0.1, 0.5, 1, 2, 5\}$. For each pair (a, τ) we will perform 1000 experiments with different random initial conditions, and in each experiment, we will measure the synchronization time (the integration time $t_{\text{int}} = 500$, if synchronization is not achieved within this time, we will add it to the last column of the histogram) and construct a histogram based on the results. We will collect the histograms in the following table (Fig. 6).

Based on the results presented in Fig. 6, we can draw the following conclusions:

- It is obvious that increasing the parameter τ leads to an increase in the speed of the swarm synchronization.
- In the case of a phase-coherent attractor ($a = 0.16$), the synchronization is significantly slower on average than for the funnel attractor ($a = 0.22$ and $a = 0.28$). Fig. 2, *c* shows the dependence of the average frequency Ω of chaotic oscillations on the parameter a, which characterizes the

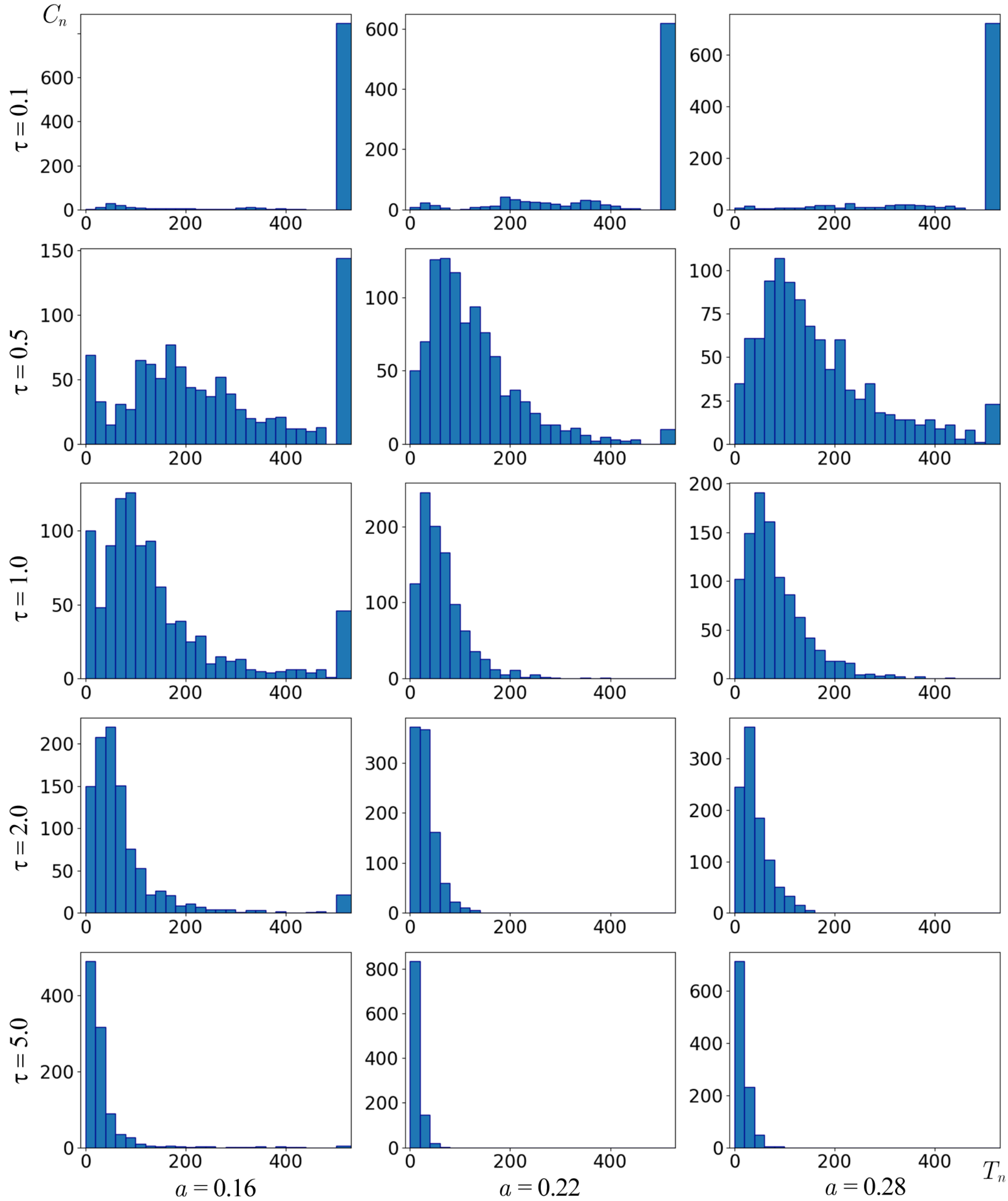

Fig. 6. Synchronization of Rössler attractors ($N = 10$). Histograms of the synchronization rate (T_n — synchronization time, C_n — number of synchronizations) for $n = 1000$ experiments with different a (columns) and τ (rows). In all experiments $c_i = 8.5$, $d' = 0.3$, $w_i \in [0.93, 1.07]$. The initial conditions in all experiments are random

degree of coherence of the oscillations. It can be seen that as the degree of coherence decreases (as a increases), the average frequency Ω increases significantly. This has an impact on the speed of transition processes to a synchronous regime. As a result, the duration of transition processes in ensembles of coupled Rössler systems with a phase-incoherent attractor is significantly shorter than in the case of systems with a phase-coherent attractor.

3. Synchronization of Lorentz attractors

Using the couplings described above, we will try to apply the same approaches to synchronize another chaotic attractor, the Lorentz chaotic attractor. The dynamic system has the following form:

$$\begin{cases} \dot{x}_i = \sigma(y_i - x_i) \\ \dot{y}_i = r_i x_i - y_i - x_i z_i + d_y \cdot \left[\sum_{j=1}^{N}(y_j - y_i)\right], \qquad i = \overline{1, N}, \\ \dot{z}_i = -bz_i + x_i y_i, \end{cases} \tag{16}$$

where $b = 8/3$, $\sigma = 10$, the values of the parameter r_i will be determined by the type of the Lorentz attractor. The value of the coupling parameter d_y is determined according to the condition (2).

Let us consider two types of Lorentz attractors:

- The classical Lorentz attractor. For a five-element system, take $r_i \in [28; 28.1]$. To determine the phase, we use the following formula:

$$\phi = \arctan \frac{z - z_0}{u - u_0}, \tag{17}$$

where $u = \sqrt{x^2 + y^2}$, $u_0 = 12$ and $z_0 = 27$.

- The Lorentz attractor, which occurs through intermittency. In this case, r_i are uniformly randomly distributed in the interval $[166.1; 166.2]$

$$\phi(t) = 2\pi \frac{t - t_n}{t_{n+1} - t_n} + 2\pi n, \quad t_n \leqslant t \leqslant t_{n+1}, \tag{18}$$

where $[t_n; t_{n+1}]$ is a cycle consisting of one laminar and one turbulent stage numbered n. This approach is described in more detail in [24].

The results of numerical simulation of the synchronization process for Lorentz systems, demonstrating both types of phase-synchronous chaotic behavior, are shown in Fig. 7. In both cases, over time, as with coupled Rössler oscillators, cluster synchronization occurs first, and then global synchronization. The coupling is selected in such a way that there is a consistent movement of mobile agents. It should be noted that the transition to global synchronization regime takes place within a short time. In this formulation, the sequential movement regime of mobile agents is implemented.

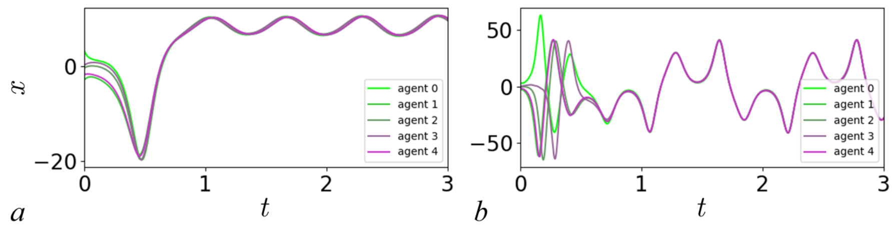

Fig. 7. Synchronization of five Lorenz systems. *a* — Synchronization in the case of the classical attractor ($r_i = [166.1, 166.11, 166.12, 166.13, 166.14]$), *b* — synchronization in the case of the attractor that emerged through intermittency ($r = [28, 28.05, 28.10, 28.15, 28.20]$). The initial conditions for each agent were generated randomly, $x \in [-5; 5], y \in [-5; 5], z \in [-1; 1]$. $d' = 0.2$, the communication radius $r = 10$ (color online)

4. Desynchronization of the swarm. Oscillatory death

If the coupling between the elements is not very small, then the interaction of an unidentical set of elements can lead not only to synchronization, but also to a phenomenon called *oscillatory death* (or *oscillation suppression*), in which oscillations are completely suppressed, and the system enters a state of static equilibrium. This effect occurs with a strong phase detuning and certain coupling parameters, when the interaction between the oscillators does not synchronize, but mutually dampens their oscillations.

The phenomenon can be demonstrated for system (5). To do this, we will rewrite the system with communication only by the variable y as follows:

$$\dot{y}_i = w_i x_i + (a - N d_y) y_i + d_y \sum_{j=1}^{N} y_j. \tag{19}$$

This form allows us to interpret the effect of coupling as a damping effect. With a significant frequency detuning, the impact of other agents becomes non-resonant and does not compensate for the increased dissipative losses. As a result, if the condition $Nd > a$ is met (in the case of synchronization of all agents), the oscillator may lose its ability to self-excite, and the oscillations fade, which is one of the ways to desynchronize the swarm, as a result of which all or part of the agents come to the equilibrium state (0, 0). The following are the time realizations for oscillatory death at $N = 10$ (Fig. 8).

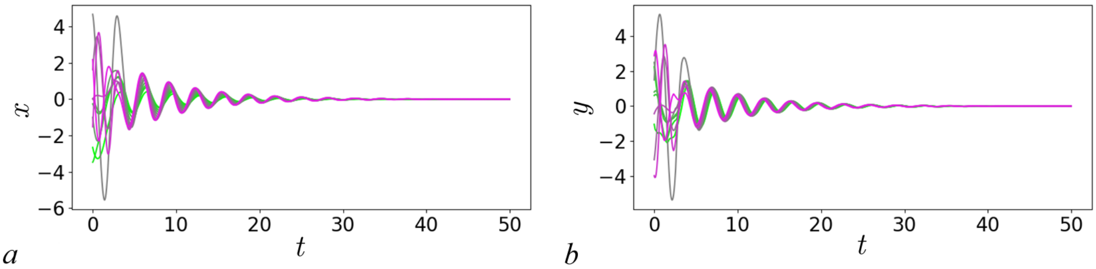

Fig. 8. Oscillatory death for the $N = 10$ connected Rössler attractor. Time realizations $x(t)$ (a) and $y(t)$ (b). In a relatively short time, all agents reach the equilibrium state (0, 0) and remain there. $d' = 0.3$, $w_1 = 1.0, w_2 = 1.2, ..., w_{10} = 2.8$ (color online)

5. Analysis of the impact of inter-element coupling disruption

In this section, we consider the results on the reliability of the existence of given geometric structures of agent ensembles. That is, we determine the dependencies of the structural stability of the ensemble when some agents are removed from it. Let us consider two ways of knocking agents out of the structure.

1. *Knocking out elements from the center of the swarm.* Through numerical experiments, it was found that for any swarm structure, there is a critical value of the number of removed agents, at which the structure breaks up into several clusters. As a result of experiments on a 10×10 structure, when 12 elements are removed, the structure already splits into two clusters (Table 1). It is interesting to note that under certain conditions, solitary agents appear, which do not join any of the clusters during the entire observation period.
2. *Knocking out random elements.* In this experiment, elements are removed from random positions. Now, the number of combined groups of agents depends not only on the number of removed elements, but also on their positions. The structure of the couplings is organized in such a way that when an entire row of agents is removed, the global coupling of the ensemble is lost, and the swarm

Table 1. The number of stable clusters as a function of the number of elements removed from an ensemble with a 10×10 square structure. All elements were removed from the center of the structure

Number of removed elements	4	8	12	16	24	32	36	44	52	60	64
Number of preserved clusters	1	1	2	2	2	3	3	2	2 and 2 separate	3 and 1 separate	4

Table 2. Dependence of the number of stable clusters on the number of elements removed from an ensemble with a 10×10 square structure. Elements were removed randomly

Number of removed elements	10	15	20	25	30	35	40	45	50	55	60
Number of preserved clusters	1	1	1	1	1	1	1	2	2	2	2

breaks up into at least two parts, before and after the removed row. The results are presented in Table 2.

According to the introduced couplings, as a result of knocking out random elements, the number of rows remains unchanged, but within each row the agents close together, occupying the space of the removed elements.

Results

As a result of the study of synchronization and desynchronization of a swarm of mobile agents, the following was obtained:

- Using the example of ensembles of interacting mobile agents, whose trajectories obey the Rössler and Lorentz systems in chaotic regimes, the influence of various types of heterogeneities on collective dynamics in the presence of inter-element couplings, in particular, on the occurrence of synchronous and asynchronous regimes, was considered. The possibility of organizing sequential (one after the other along the chain) and parallel (united front) movement of agents and the organization of various topological configurations of a swarm of agents with a certain choice of the structure of heterogeneity between the elements was demonstrated. In all cases, the interval between agents can be changed in a given direction by decreasing or increasing the strength of the coupling.
- It has been shown that the duration of transients to the synchronous regime in ensembles of mobile agents moving along the trajectories of the Rössler system in the funnel attractor regime is significantly shorter than in the case of a phase-coherent attractor.
- For the case of chaotic Rössler attractors, four methods of desynchronization of the swarm were proposed and successfully tested: a) with the introduction of additional coupling over the variable z (coupling over the variable z has a repellent character), b) with a decrease in the inter-element coupling, c) with a decrease in the coupling range, and d) with a high phase detuning and sufficient coupling strength.
- The stability of the topological configurations of a swarm of agents has been tested when agents are removed in two different ways: a) from the center of the swarm and b) randomly.

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