

## On the structure of a viscous liquid flow under periodic influences which have no predominant direction in space

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**Abstract.** *Purpose* of the work is the revealing and the researching of peculiarities of the dynamics of a viscous liquid which is undergoing by periodic in time influences possessing no predominant direction in space under possible simplest hydro-mechanical conditions (which are able to provide non-trivial behavior of the liquid). *Methods.* The analytic investigational methods for boundary problems for Navier–Stokes and continuity equations are used that are the method of perturbations, the method of Fourier. *Results.* A new problem on the flow of a viscous liquid is formulated and solved. The hydro-mechanical system consists of an incompressible viscous liquid and a moving absolutely solid wall which creates periodic influences to the liquid. A new peculiarity of the viscous liquid dynamics is revealed consisting in that in simplest hydro-mechanical conditions the liquid in the background of oscillations (in average of time) performs a motion of a new type — a stationary fading with the distance from the wall motion which is characterized by the presence of a laminate structure. A physical interpretation of the formulation of the problem under consideration is given. *Conclusion.* The obtained results can be used in particular in a scientific search of ways to control hydro-mechanical systems, under a developing methods of a creation of prescribed flows of liquid media.

**Keywords:** viscous liquid, periodic in time influences, predominant direction in space, structure of flow.

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### Introduction

Periodic processes and influences are extremely widespread and diverse in nature and technology. Hydromechanical systems, in particular, can be subject to periodic (oscillatory, vibrational) influences. A significant number of studies have been devoted to studying the dynamics of hydromechanical systems under periodic influences ([1–30] and [31–57]). To date, a number of meaningful results have been obtained in this scientific field, new hydromechanical effects have been identified, and the level of understanding of the dynamics of hydromechanical systems has been significantly increased. In particular, the effects of paradoxical behavior of a solid inclusion in a vibrating liquid were discovered [1, 2, 7, 21, 31–34, 42], «spontaneous» transition of a solid body in an oscillating viscous liquid to a position with a given

orientation in space [19], predominantly unidirectional rotation of a solid body and a viscous liquid [22]. The existence of “allowed” and “forbidden” states of a hydromechanical system subjected to periodic time effects has been established, for which the solution to the problem of the system’s motion exists and does not exist, respectively. [15]. Liquid "levitation" effect discovered [29, 52]. A mathematical model of a hydromechanical analogue of the "Kapitsa pendulum" has been constructed [12, 58]. The fundamental concepts of homogeneous and non-homogeneous oscillations of a liquid are introduced, and quantitative characteristics of the non-homogeneity of oscillations of a liquid are determined [5, 7, 11, 21]. The existence of the phenomenon of predominantly unidirectional movement of compressible inclusions in a vibrating liquid has been proven [3, 4, 7, 21, 36, 44]. The results obtained in the scientific direction under consideration may be of significant applied interest ([35, 53–57]).

In this paper, we pose and solve a new problem concerning the motion of a viscous liquid generated by periodic forces exerted on the liquid without a specific spatial direction. It is established that periodic forces on the liquid can lead to a time-averaged motion of the liquid, characterized by the presence of a periodic spatial structure.

### 1. Statement of the problem

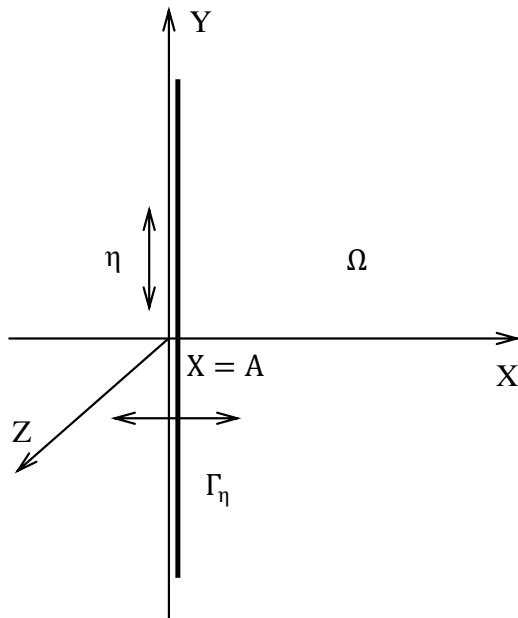


Fig. 1. The hydro-mechanical system

There is a hydromechanical system consisting of a viscous incompressible liquid and a perfectly rigid body (wall)  $\eta$  (Fig. 1). The body  $\eta$  is bounded by the  $\Gamma_\eta$  plane, perpendicular to the  $X$  axis of the inertial rectangular coordinate system  $X, Y, Z$  and intersecting the  $X$  axis at the point  $X = A$ . The liquid fills the region  $\Omega$ :  $A < X < \infty, -\infty < Y < \infty, -\infty < Z < \infty$ . The wall  $\eta$  performs translational motion along the  $X, Y$  axes; the coordinate  $A$  and the velocity  $U$  of the wall  $\eta$  motion in the direction of the  $Y$  axis change periodically with a period  $T$  in a given manner over time  $t$  ( $A = \hat{A} \sin 2\pi t/T; U = \hat{U} \sin(2\pi t/T + \varphi)$ ;  $\hat{A} > 0, \hat{U} > 0$  are constants;  $\varphi$  is a parameter that can have values  $\pi/4, -\pi/4, 3\pi/4, -3\pi/4$ ).

It is required to determine the periodic movement of a liquid.

Let  $\tau = t/T; x = X/(\hat{U}T); y = Y/(\hat{U}T); \varepsilon = \hat{A}/(\hat{U}T); a = A/(\hat{U}T); u = U/\hat{U}; \mathbf{V}, \rho$  and  $\nu$  are the velocity, density and kinematic viscosity coefficient of the liquid, respectively;  $\mathbf{v} = \mathbf{V}/\hat{U} = v_x(x, \tau)\mathbf{e}_x + v_y(x, \tau)\mathbf{e}_y$  ( $\mathbf{e}_x$  and  $\mathbf{e}_y$  are unit vectors whose directions coincide

with the directions of the axes  $X$  and  $Y$ , respectively);  $P$  is the pressure in liquid;  $p = P/(\rho\hat{U}^2) = p(x, \tau); Re = \hat{U}^2T/\nu$  is the Reynolds number.

The problem of liquid motion consists of the Navier-Stokes equation, the continuity equation, and

conditions at the boundary  $\Gamma_\eta$  and at infinity

$$\frac{\partial \mathbf{v}}{\partial \tau} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p + \frac{1}{Re} \Delta \mathbf{v} \quad \text{in } \Omega; \quad (1)$$

$$\nabla \cdot \mathbf{v} = 0 \quad \text{in } \Omega; \quad (2)$$

$$\mathbf{v} = \frac{da}{d\tau} \mathbf{e}_x + u \mathbf{e}_y \quad \text{on } \Gamma_\eta \quad (\text{at } x = a); \quad (3)$$

$$\mathbf{v} \rightarrow \frac{da}{d\tau} \mathbf{e}_x \quad \text{at } x \rightarrow \infty. \quad (4)$$

## 2. Solution of the problem

Equation (2) is equivalent to the equation

$$\frac{\partial v_x}{\partial x} = 0; \quad (5)$$

According to (5) we have

$$v_x = v_x(\tau). \quad (6)$$

From (3), (4), (6) it follows

$$v_x = 2\pi\varepsilon \cos 2\pi\tau. \quad (7)$$

Equation (1) is equivalent to the equations

$$\frac{dv_x}{d\tau} = -\frac{\partial p}{\partial x}; \quad (8)$$

$$\frac{\partial v_y}{\partial \tau} + v_x \frac{\partial v_y}{\partial x} = \frac{1}{Re} \frac{\partial^2 v_y}{\partial x^2}; \quad (9)$$

From (7), (8) it follows

$$p = 4\pi^2\varepsilon(\sin 2\pi\tau)x + c(\tau). \quad (10)$$

Note that in the liquid flow problem (in problem (1)–(4)) there are no boundary conditions that the (dimensionless) pressure  $p$  must satisfy.

According to (3), (4), (7), (9), we have

$$\frac{\partial v_y}{\partial \tau} + 2\pi\varepsilon(\cos 2\pi\tau) \frac{\partial v_y}{\partial x} = \frac{1}{Re} \frac{\partial^2 v_y}{\partial x^2} \quad \text{in } \Omega; \quad (11)$$

$$v_y = u \quad \text{at } x = a; \quad (12)$$

$$v_y \rightarrow 0 \quad \text{at } x \rightarrow \infty. \quad (13)$$

We will consider problem (11)–(13) for values of  $\varepsilon$  that are small compared to unity. We will apply the method of expansion in powers of the small parameter. [59, 60] We assume that

$$v_y \sim v_0 + \varepsilon v_1 \quad \text{at } \varepsilon \rightarrow 0. \quad (14)$$

Using (11)–(14), in the  $\varepsilon^N$ -approximation ( $N = 0, 1$ ) we obtain

$$\frac{\partial v_N}{\partial \tau} - \frac{1}{Re} \frac{\partial^2 v_N}{\partial x^2} = -2N\pi(\cos 2\pi\tau) \frac{\partial v_0}{\partial x} \quad \text{in } \bar{\Omega}; \quad (15)$$

$$v_N = (1 - N)u - N(\sin 2\pi\tau) \frac{\partial v_0}{\partial x} \quad \text{at } x = 0; \quad (16)$$

$$v_N \rightarrow 0 \quad \text{at } x \rightarrow \infty, \quad (17)$$

where  $\bar{\Omega}$  is the region  $0 < X < \infty$ ,  $-\infty < Y < \infty$ ,  $-\infty < Z < \infty$ .

Let  $N = 0$ . Problem (15)–(17) has a solution

$$v_0 = \text{Imag} (e^{i\varphi} e^{-qx} e^{2\pi i \tau}). \quad (18)$$

Here  $q = (1 + i)\sqrt{\pi R e}$ .

Let  $N = 1$ . Problem (15)–(17) has a solution

$$v_1 = \bar{v} + \tilde{v}, \quad (19)$$

where

$$\bar{v} = \frac{1}{2} \text{Real} (e^{i\varphi} q e^{-qx}), \quad \tilde{v} = -\frac{1}{2} \text{Real} (e^{i\varphi} q e^{-qx} e^{4\pi i \tau}).$$

The formulas

$$\mathbf{v} = v_x \mathbf{e}_x + (v_0 + \varepsilon v_1) \mathbf{e}_y \quad (20)$$

and (7), (10), (18), (19) determine the approximate solution of problem (1)–(4).

Let's turn to the question of the time-averaged motion of a liquid. Using (7), (18)–(20), we obtain

$$\langle \mathbf{v} \rangle = \varepsilon \bar{v} \mathbf{e}_y = \frac{\varepsilon \lambda}{\sqrt{2}} e^{-\lambda x} \cos(\lambda x - \varphi - \pi/4) \mathbf{e}_y \quad \text{in } \bar{\Omega}'. \quad (21)$$

Here

$$\langle \dots \rangle = \int_{\tau}^{\tau+1} \dots d\tau'$$

is the operator of averaging over (dimensionless) time  $\tau$  (the action of this operator on a periodic function of time  $\tau$  – the dimensionless velocity  $\mathbf{v}$  – consists in that it keeps the mean value of the function unchanged and "transforms" all its other "components" to zero; the result of the action of this operator on a periodic function  $\tau$  – the velocity  $\mathbf{v}$  – the time-averaged dimensionless velocity  $\tau$  of the liquid);  $\lambda = \sqrt{\pi R e}$ ;  $\bar{\Omega}'$  is the half-space  $0 \leq X < \infty$ ,  $-\infty < Y < \infty$ ,  $-\infty < Z < \infty$ .

Formula (21) indicates the presence of an effect, consisting of the fact that under the hydromechanical conditions under consideration, against the background of oscillations, the liquid undergoes a stationary motion.

Let us focus on the description of the properties of the time-averaged motion of a liquid. According to (21), in the half-space  $\bar{\Omega}'$ , against the background of oscillations, the following occurs.

1. If  $\varphi = \pi/4$  (Fig. 2, a), then on the planes  $P_k : x = x_k = k\pi/\lambda$ ,  $-\infty < Y < \infty$ ,  $-\infty < Z < \infty$  ( $k = 0, 1, 2, \dots$ ) the liquid velocity is zero; by the planes  $P_l$  ( $l = 1, 2, \dots$ ) the region occupied by the liquid is divided into layers  $L_l : x_{l-1} < x < x_l$  ( $-\infty < Y < \infty$ ,  $-\infty < Z < \infty$ ); the thickness of each layer  $L_l$  is  $x_l - x_{l-1} = \pi/\lambda$ ; the direction of the liquid velocity in each layer  $L_l$  coincides with the direction of the vector  $\mathbf{e}_y$  or is opposite to the direction of the vector  $\mathbf{e}_y$ ; in layer  $L_1$  the liquid moves in the direction coinciding with the direction of the vector  $\mathbf{e}_y$ ; the direction of the liquid velocity in each layer  $L_{l+1}$  is opposite to the direction of the liquid velocity in layer  $L_l$ ; in each layer  $L_l$  on plane  $P_l^* : x = x_l^* = (l - 3/4)\pi/\lambda$  the modulus of the velocity  $|\langle \mathbf{v} \rangle|$  has a maximum, which is  $(\varepsilon\lambda/2) \exp(-\lambda x_l^*)$ .

2. If  $\varphi = -\pi/4$  (Fig. 2, b), then on the planes  $P_l : x = x_l = (l - 1/2)\pi/\lambda$  ( $l = 1, 2, \dots$ ) the velocity of the liquid is zero; by the planes  $P_l$  the region occupied by the liquid is divided into layers  $L_l : x_{l-1} < x < x_l$  ( $x_0 = 0$ ); the thickness of the layer  $L_1 : 0 < x < x_1$  is  $x_1 = \pi/(2\lambda)$ ; the thickness of each layer  $L_m$  ( $m = 2, 3, \dots$ ) is  $x_m - x_{m-1} = \pi/\lambda$ ; the direction of the liquid velocity in each layer  $L_l$  coincides with the direction of the vector  $\mathbf{e}_y$  or is opposite to the direction of the vector  $\mathbf{e}_y$ ; in layer  $L_1$  the liquid moves in the direction coinciding with the direction of the vector  $\mathbf{e}_y$ ; the direction of the liquid velocity in each layer  $L_{l+1}$  is opposite to the direction of the liquid velocity in layer  $L_l$ ; in each layer  $L_m$  ( $m = 2, 3, \dots$ ) on plane  $P_m^* : x = x_m^* = (m - 5/4)\pi/\lambda$  the modulus of the velocity  $|\langle \mathbf{v} \rangle|$  has a maximum of  $(\varepsilon\lambda/2) \exp(-\lambda x_m^*)$ ; in the  $L_1$  layer, the maximum of  $|\langle \mathbf{v} \rangle|$  is absent, the largest value of the velocity modulus  $|\langle \mathbf{v} \rangle|$  is achieved on the  $P_0$  plane:  $x = x_0 = 0$  and is  $\varepsilon\lambda/\sqrt{2}$ .

3. If  $\varphi = 3\pi/4$  (Fig. 2, c), then on the planes  $P_l : x = x_l = (l - 1/2)\pi/\lambda$  ( $l = 1, 2, \dots$ ) the liquid velocity is zero; by the planes  $P_l$  the region occupied by the liquid is divided into layers  $L_l : x_{l-1} < x < x_l$  ( $x_0 = 0$ ); The thickness of layer  $L_1 : 0 < x < x_1$  is  $x_1 = \pi/(2\lambda)$ ; The thickness of each layer

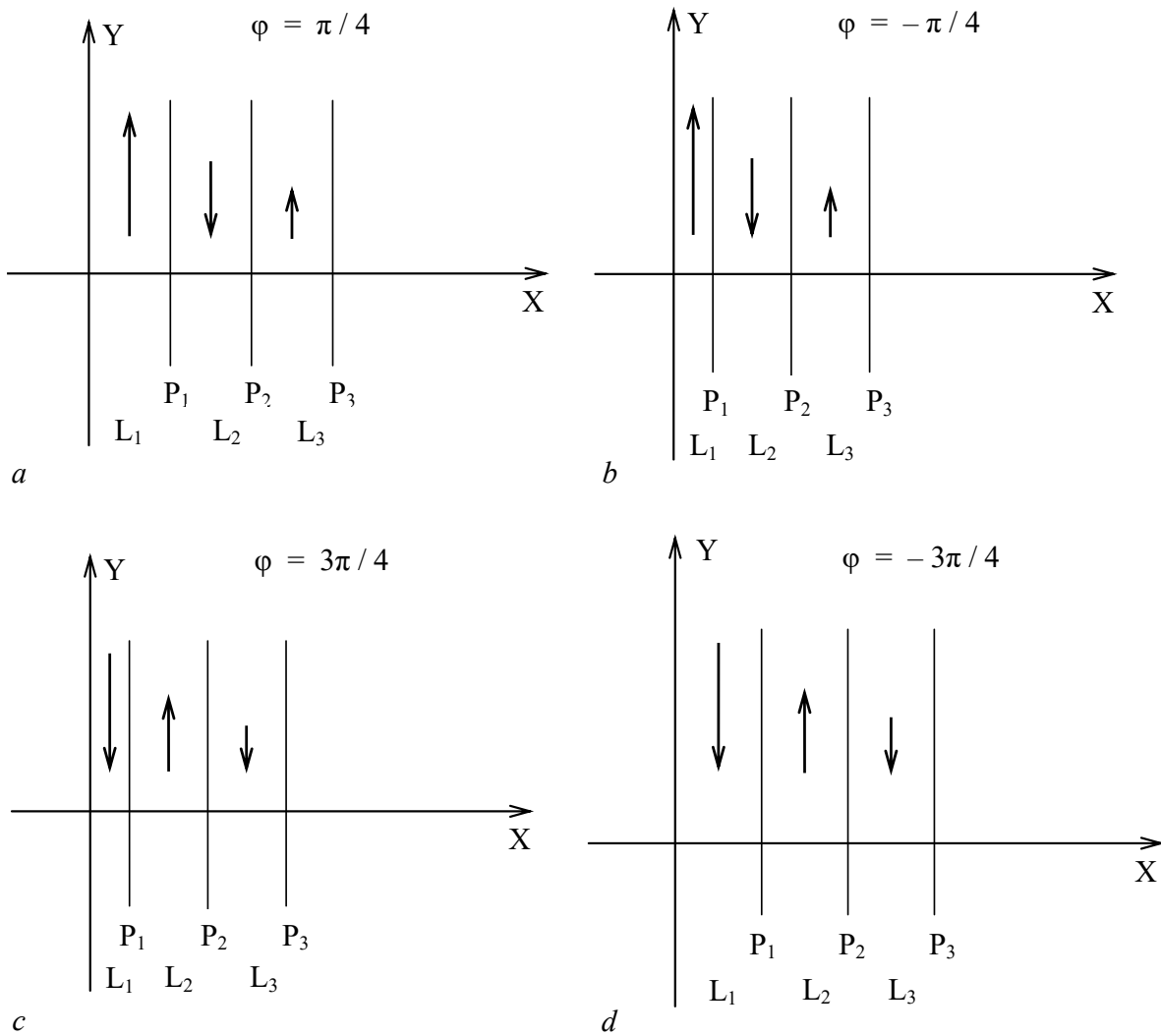


Fig. 2. Fragments of the qualitative picture of the average in time liquid motion under various values of the parameter  $\varphi$

$L_m$  ( $m = 2, 3, \dots$ ) is  $x_m - x_{m-1} = \pi/\lambda$ ; The direction of the liquid velocity in each layer  $L_l$  coincides with the direction of the vector  $\mathbf{e}_y$  or is opposite to the direction of the vector  $\mathbf{e}_y$ ; in layer  $L_1$ , the liquid moves in the direction opposite to the direction of the vector  $\mathbf{e}_y$ ; the direction of the liquid velocity in each layer  $L_{l+1}$  is opposite to the direction of the liquid velocity in layer  $L_l$ ; in each layer  $L_m$  ( $m = 2, 3, \dots$ ) on the plane  $P_m^* : x = x_m^* = (m - 5/4)\pi/\lambda$ , the absolute value of the velocity  $|\langle \mathbf{v} \rangle|$  has a maximum of  $(\varepsilon\lambda/2) \exp(-\lambda x_m^*)$ ; in layer  $L_1$ , the maximum  $|\langle \mathbf{v} \rangle|$  is absent, the greatest value of the absolute value of the velocity  $|\langle \mathbf{v} \rangle|$  is achieved on the plane  $P_0 : x = x_0 = 0$  and is  $\varepsilon\lambda/\sqrt{2}$ .

4. If  $\varphi = -3\pi/4$  (Fig. 2, d), then on the planes  $P_k : x = x_k = k\pi/\lambda$  ( $k = 0, 1, 2, \dots$ ) the liquid velocity is zero; by the planes  $P_l$  ( $l = 1, 2, \dots$ ) the region occupied by the liquid is divided into layers  $L_l : x_{l-1} < x < x_l$ ; the thickness of each layer  $L_l$  is  $x_l - x_{l-1} = \pi/\lambda$ ; the direction of the liquid velocity in each layer  $L_l$  coincides with the direction of the vector  $\mathbf{e}_y$  or is opposite to the direction of the vector  $\mathbf{e}_y$ ; in layer  $L_1$  the liquid moves in the direction opposite to the direction of the vector  $\mathbf{e}_y$ ; the direction of the liquid velocity in each layer  $L_{l+1}$  is opposite to the direction of the liquid velocity in layer  $L_l$ ; in each layer  $L_l$  on plane  $P_l^* : x = x_l^* = (l - 3/4)\pi/\lambda$  the modulus of the velocity  $|\langle \mathbf{v} \rangle|$  has a maximum of  $(\varepsilon\lambda/2) \exp(-\lambda x_l^*)$ .

Formula (21) and all the properties of the time-averaged liquid motion described in points 1-4 hold

for any value of the Reynolds number  $Re > 0$ .

Despite the differences (thickness of the first layer, direction of liquid motion in the layer), all the liquid flows considered (in each time interval, again and again) are generated and maintained by influences that are periodic in time, and thus — as a consequence — have a stationary structure, a stationary, "almost" periodic structure in space. The content of this proposition (based on the formula (21)) can be briefly expressed as follows: a periodic structure in time is capable of generating and preserving (maintaining) a stationary structure in space. Along with this point, of significant interest is also the fact that the discovered stationary spatial structure of liquid motion is "almost" periodic (periodic to the extent that the liquid motion attenuates with increasing distance from the source of influence — the moving wall). According to (21) (and as set out in paragraphs 1–4), the attenuation of liquid motion with increasing distance from the wall is weaker, and the thickness of the layers of unidirectional time-averaged liquid motion is greater, the lower the value of the Reynolds number  $Re > 0$ .

## Conclusion

This paper presents new results in the study of the dynamics of a viscous liquid subject to periodic forces characterized by the absence of a preferred spatial direction. The problem of the motion of a viscous liquid adjacent to a rigid body exerting periodic forces on the liquid is considered. New features of the mean motion of the liquid are identified.

The periodic time effects, present in this paper and in a number of other studies ([25–27, 29, 30]), characterized by the absence of a preferred direction in space, are of interest due to the fact that with such effects on hydromechanical systems (on average over time), each partial periodic effect (for example, the effect on the liquid associated with the movement of the wall  $\eta$  along the  $X$  axis (or along the  $Y$  axis) in the problem (1)–(4) is zero, «absent»; on average over time, «nothing happens» (for example, the wall  $\eta$  «stands still»), but the free parts of the hydromechanical systems (parts of the systems whose motion is not specified) perform an average motion. The reason for the effects of mean motion of free parts of systems is the consistency (with each other) of the effects exerted on the systems, which is in direct connection with the principle of mean motion [7, 21, 25, 26, 30, 61].

The obtained results can be used as a theoretical basis for organizing targeted experimental studies of nontrivial liquid dynamics under periodic influences. The results of the study indicate the fundamental possibility of effectively using periodic influences without a specific spatial direction to create specified liquid flows and control the motion of liquid media.

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