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Dynamic characteristics of spatial axisymmetric structures considering energy dissipation in the material

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Abstract. The *purpose* of the work is a comprehensive analysis of the current state of the issue concerning the dynamics of axisymmetric structures. *Results.* A mathematical model, method, algorithm, and computer program for calculations on a computer have been developed to assess the dynamic characteristics (frequency, mode, and damping ratio of vibrations) of spatial axisymmetric structures, considering energy dissipation in the material using the hereditary Boltzmann–Volterra viscoelastic model in a three-dimensional setting. The dynamic characteristics of specific spatial axisymmetric structures of the cooling tower type have been evaluated. It has been determined that for this type of structure, the lowest non-axisymmetric natural frequencies fall within the range of predominant earthquake frequencies. *Conclusion.* It has been found that: accounting for dissipation in the material results in a slight reduction in the natural vibration frequencies of the structure and a weakly frequency-independent damping ratio; the installed stiffening rings at the top of the structure somewhat increase the non-axisymmetric natural vibration frequencies, while the bending frequencies of the structures decrease slightly.

Keywords: axisymmetric spatial construction, cooling tower, dynamic properties, non-axisymmetric vibrations, complex eigenfrequencies, vibration modes and damping ratio, energy dissipation, hereditary viscoelasticity.

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Introduction

Recently, the dynamic spatial characteristics of various axisymmetric structures, taking into account both their complex geometry and inelastic properties of materials, have attracted the attention of researchers in connection with the construction of nuclear power plants and thermal power plants in areas with high seismicity. It is obvious that the dynamic characteristics (natural frequency, mode and damping ratio of oscillations) are the main parameters of structures and carry quite a lot of information about the dynamic properties of the structure as a whole. As is known, determining the dynamic characteristics of these structures is an independent and rather difficult task of mechanics.

This, in turn, requires the creation of reliable methods and algorithms for dynamic calculation for this range of structures, taking into account both elastic and inelastic properties of the material for the actual nature of the structure.

There are a large number of publications considering the dynamic behavior of structures of this type. The parameters of oscillatory processes, taking into account energy dissipation in the material, have been studied to a lesser extent. The relevance of solving such problems is primarily explained by the design, construction and operation requirements of various types of high-rise spatial axisymmetric structures.

The study aims to develop a methodology and an algorithm for solving the problem of natural oscillations of spatial structures with viscoelastic properties of their material, as well as to study the dynamics of specific structures and analyze the results obtained in terms of discovering new mechanical effects.

In recent years, a number of studies have been published on the stress-strain state and dynamic behavior of various axisymmetric structures.

These studies include the following.

• In [1–6], the dynamics of various axisymmetric structures is considered in a one-dimensional and two-dimensional setting, taking into account the elastic and viscoelastic properties of the structure's material, as well as its linear and nonlinear behavior under various loads. To assess the dynamics of these structures, a methodology and algorithm for solving the posed problems have been developed. The work focuses on the study of the natural oscillations of structures in an elastic setting, as well as the study of forced oscillations, taking into account the viscoelastic properties of the material. However, the issue of assessing the natural oscillations of structures, taking into account various types of dissipation in the material, remained open due to the complexity of solving spatial problems.

• In [7], based on the finite element method, the analysis of free oscillations of the cooling tower is given, the results of which are in good agreement with the experimental data obtained both for the real cooling tower and for the corresponding model design. The effectiveness of the method as a design tool is demonstrated. The possibility of using the obtained results for the analysis of another type of rotary-periodic structures, which are subject to seismic disturbances, missile impacts or wind forces, is shown.

• Reference [8] contains the basics of physical and geometrically nonlinear analysis of cooling towers in the form of thin shells, including the determination of instability points on the trajectory of the load movement. The numerical study consists of comparing the crack-resistant and ultimate loads of two different reinforced concrete cooling towers subjected to self-weight and quasi-static wind load with the bending loads obtained from three modes of physical linear bending analysis.

• In [9], the physical and mathematical modeling of the «cooling tower – foundation – soil» system is considered. The physical simulation was performed using a continuous 20-node isoparametric element to simulate a cooling tower, an annular rafter foundation, and ground environments. The «cooling tower – foundation – soil» system was analyzed taking into account the vertical and transverse loads resulting from its own weight and wind loads. The nonlinearity of the soil was taken into account using the hyperbolic nonlinear law of elasticity. The reaction of the structure to displacement and stresses is investigated. The influence of linear and nonlinear interactive analysis in comparison with traditional analysis is investigated. It has been established that interactive analysis of the interaction of the cooling tower, foundation and soil plays an important role in reducing stresses in the cooling tower, especially in the lower ring girder.

• In [10], provides a comprehensive review of scientific papers published in the field of cooling towers, and provides an idea of the latest developments in natural draft cooling towers. The article summarizes various methods of modeling, analysis and design, as well as discusses the problems. 118 references are given, which mainly focus on the review of published works after 2005. The paper provides a comprehensive overview of the research conducted for cooling towers and provides updated material for researchers and design engineers in the field of hyperbolic cooling towers.

• Reference [11] considers the characteristics of the hyperbolic shell of a cooling tower for free and forced oscillations, which are one of the complex real-world applications of axisymmetric structures. A 9-node harmonic ring finite element is used in the numerical model of the cooling tower. Physically, the three-dimensional problem of the cooling tower is reduced to a two-dimensional problem by expressing the seismic load as a Fourier series for a single harmonic using harmonic elements. A complete solution of the problem is obtained for one component of the load, which significantly improves the computational efficiency of the model. A parametric study of changes in the curvature of the cooling tower body is conducted. The time history analysis method is used to determine the dynamic characteristics of the cooling tower body. The study uses data on the acceleration of the Düzce earthquake (Turkey, 2022). It is concluded that the number of circular motion modes and the curvature of the body have a significant impact on the dynamic characteristics of cooling towers.

- Reference [12] shows the application of a modified Vlasov foundation model to analyze the free oscillations of hyperbolic wind towers resting on elastic foundations. The calculation uses a computational tool encoded in MATLAB that utilizes the SAP2000 open application programming interface feature to enable two-way data flow during execution. Based on numerical examples, it is concluded that the interaction between the tower, the ground, and the structure significantly reduces the system's frequency parameter compared to a fixed state, and changes in geometric parameters have a significant impact on the frequency parameters.
- In [13], the dynamic behavior of hyperbolic cooling towers with different geometric properties is investigated under the influence of an earthquake. Various cooling tower samples with different geometric dimensions are analyzed, and the influence of curvature, flexibility, thickness, and neck level on the dynamic behavior of hyperbolic cooling towers is studied. The influence of these parameters on the cooling tower's behavior is investigated by comparing lateral displacement, meridional forces, and moments. Numerical analysis is performed using software written in the MATLAB programming language, which allows for the simultaneous use of the MATLAB and SAP2000 structural analysis software packages.
- Reference [14] examines extensive review and research articles published on the modeling of hyperbolic cooling towers. The work indicates that cooling towers are doubly curved thin-walled shells with complex geometry, and their analysis and design have attracted the attention of researchers worldwide. It provides an overview of the latest developments in natural draft cooling towers in the field of modeling superstructures and substructures, and highlights aspects such as shell finite elements, experimental studies, supporting systems, and various foundation systems. The work provides a review of articles on cooling towers published since 2005 to the present.
- In [15], the seismic characteristics of a hyperbolic cooling tower resting on a soil foundation represented by the three-parameter Vlasov elastic soil model are considered. The three-parameter soil model eliminates the need for field tests to determine the soil parameters, such as the modulus of elasticity and the displacement parameter. The parameters are calculated using an iterative procedure based on the vertical deformation profile of the soil surface in the model. The calculation is performed using the SAP2000 structural analysis program, which employs a computational tool encoded in MATLAB. The numerical results show that the flexibility of the soil foundation leads to increased displacements but reduces the forces on the shell and columns. It is argued that considering the interaction between the soil and the structure in the analysis of the cooling tower's seismic response provides an efficient design process.
- In reference [16], a three-dimensional physical model is constructed for all eight stages of construction of a 210-m-high ultra-large cooling tower. The dynamic characteristics of the cooling tower are analyzed at each stage. First, information about the flow field and 3D chronology of aerodynamic forces is obtained for the entire construction process using large eddy simulation (LES). A full dynamic analysis using the finite element method was used to calculate the dynamic characteristics of the tower as the wind loads changed in real time throughout the construction process. Based on the research, the influence and mechanism of action of the vibration coefficient from the wind, the age of concrete, the load on the structure, geometric nonlinearity, and the force of internal suction on the resistance to bending and the maximum bearing capacity of cooling towers are estimated.
- In [17], a nonlinear analysis of the stability of steel hyperbolic cooling towers with a height of 150 m was carried out. Models with five structural systems have been created, including two types of mesh shells (i.e. single-layer and double-layer shells) and three forms of beams (i.e. triangular mesh, rectangular mesh and square pyramidal mesh). Geometric and material analyses of nonlinear stability

have been performed in more than 220 cases, taking into account various distributions and amplitudes of defects. The results showed that the five hyperbolic steel cooling towers have a relatively low sensitivity to defects, which distinguishes them from most other thin-walled buildings, and the sensitivity to defects of the rectangular grid is high, while the triangular grid and the square pyramid grid have low sensitivity to defects. The analysis revealed that structures with two-layer mesh shells are more sensitive to defects than structures with single-layer ones..

• In [18], the hyperbolic cooling of a 117 m high tower is considered and the behavior of the structure under earthquake and wind conditions is investigated. When calculating the wind load, a wind speed of 25 m/s was used, and three records of soil movement were applied to the structures during earthquake analysis. The wind load is calculated in accordance with the ASCE 7-10 standard and is set as angular. As a result of the analysis, the values of displacement and voltage were obtained and investigated. According to the results of the wind load analysis, as the height of the tower increases, the values of the wind load and the values of the displacements occurring in the building increase. The values of displacement and voltage vary at an angle depending on the angular wind load. The highest values of displacement and stress were obtained during the earthquake in Kobe (Japan, 1995). The displacement values are investigated depending on the height, and the largest displacement values are obtained in the upper part.

• Reference [19], presents an analysis of a number of works devoted to the design and calculation of cooling towers. Special attention is paid to the calculation of cooling towers for wind loads (static and pulsation). Based on the analysis of different types of cooling towers, the advantages of a frame-sheathed cooling tower are shown, the features of metal cooling towers in comparison with reinforced concrete ones are revealed, which consist in the fact that frame-sheathed structures can be erected in any climatic region, including construction in winter, and without the installation of greenhouses. It is shown that the spatial frame of a steel cooling tower in the form of a hyperboloid of rotation is the most appropriate design solution for nuclear and thermal power plants.

As the given analysis of the known scientific works shows, each of these works has both advantages and some disadvantages, despite this, they are all used in solving specific practical problems.

Based on this analysis, it can be noted that the development of mathematical models and solution methods for assessing the dynamic characteristics of spatial axisymmetric structures, taking into account the dissipation of energy in the structural material using the Boltzmann–Volterra hereditary viscoelasticity theory, is currently a highly relevant and important problem that requires solution.

1. Methodology

1.1. Mathematical model. The article considers the natural oscillations of a viscoelastic spatial axisymmetric system consisting of an axisymmetric structure, a foundation, and a soil base. The system under consideration (Fig. 1) occupies a volume of $V = V_1 + V_2 + V_3$, where V_1 is the volume of the structure, V_2 is the volume of the foundation, and V_3 is the volume of the base. The materials of each element of the system have different viscoelastic characteristics. The lower part of the base, Σ_Z, is rigidly fixed, while the lateral surface, Σ_R, is stress-free.

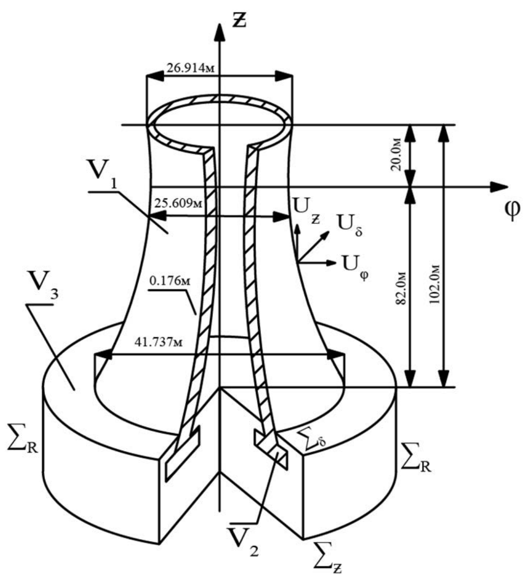


Fig. 1. Spatial axisymmetric system: 1 — structure, 2 — foundation, 3 — base

The problem is to find the natural oscillations, i.e. the most ordered motions that occur in the absence of external influences. When dissipation in the material is taken into account, all points of the system oscillate according to the same complex harmonic law with different amplitudes, i.e.

$$\vec{u}(\vec{x}, t) = \vec{u}^*(\vec{x}) e^{-i\omega t}, \tag{1}$$

here ω is the complex natural frequency; $\vec{u}^*(\vec{x})$ is the complex natural oscillation form;

$$\vec{u} = \{\vec{u}(\vec{x},t), \vec{v}(\vec{x},t), \vec{w}(\vec{x},t)\}, \quad u^* = \{\vec{u}^*, \vec{v}^*, \vec{w}^*\}\,; \quad \vec{x} = \{r, z, \varphi\} -$$

u, v, w are longitudinal, tangential, and radial displacements of the system point under consideration.

In the case of non-conservative systems, that is, when energy dissipation is taken into account, ω and $\vec{u}^*$ are complex quantities. At the same time $\omega = \omega_R + i\omega_I$ and $\vec{u}^* = \vec{u}^*_R + i\vec{u}^*_I$.

In the physical sense, the real part of ω, that is ω_R, is the frequency of the system's natural damped oscillations, while the imaginary part, ω_I, carries information about the rate of damping of oscillations and is sign-accurate to the damping coefficient. The damping coefficient, as a quantitative measure of the oscillation damping rate, reflects the dissipative properties of the system as a whole.

Usually, when using the Kelvin or Voigt model, dissipation is taken into account only during creep or relaxation. In this paper, to describe the dissipative processes in the system (see Fig. 1) the hereditary theory of Boltzmann–Volterra viscoelasticity is used, which simultaneously takes into account dissipative processes in both creep and relaxation [20–24].

For the mathematical formulation of the problem, the principle of virtual displacements is used, according to which the work of all active forces, including inertial forces on an arbitrarily virtual displacement, is zero, that is [20, 21]:

$$\delta A = \int_V \sigma_{ij}\delta\varepsilon_{ij}dV - \int_V \rho\frac{\partial^2\vec{u}}{\partial t^2}\delta\vec{u}dV = 0, \tag{2}$$

$$\vec{x} \in \sigma_z : \vec{u} = 0, \quad \delta\vec{u} = 0. \tag{3}$$

$\vec{u}, \delta_{ij}, \varepsilon_{ij}$ are the components of the displacement vector, stress and strain tensors; $\delta\vec{u}, \delta\varepsilon_{ij}$ are variations of displacements and deformations; $V = V_1 + V_2 + V_3$ is the sum of the volumes of the body, foundation and base; ρ is the density of the material; $\vec{x} = \{r, z, \varphi\}$ are the cylindrical coordinates; $i, j = r, z, \varphi$.

In this case, the complex natural frequency ω and the oscillation form $\vec{u}^*$ of the viscoelastic spatial system are to be determined (see Fig. 1), satisfying variational equation (2) under kinematic conditions (3).

Substitution of (1) into (2) and (3) reduces the problem under consideration to a complex variational problem of eigenvalues:

$$-\int \sigma^*_{ij}\delta\varepsilon_{ij}dV + \omega^2\int_V \rho_n\vec{u}*\delta\vec{u}*dV = 0, \tag{4}$$

$$x \in \sigma_z : \delta\vec{u}^* = 0, \tag{5}$$

where σ^*_{ij} is the amplitude of the components of the stress tensors.

The natural form of the oscillation $\vec{u}^*$ is determined from (4) up to a constant factor. To eliminate this, an additional normalization condition for the natural forms is introduced:

$$\int_V \rho_n\vec{u}^{*2}dV = 1. \tag{6}$$

Now, the problem of the natural oscillations of a viscoelastic system (see Fig. 1) is reduced to finding the constant ω^2 and the function $\vec{u}^*\,(\vec{x})$, that satisfy the equations (1), the normalization conditions (6) and the kinematic conditions (5) for any $\delta\,\vec{u}^*\,(\vec{x})$.

Here

$$\omega, \vec{u}* = \{u^*_3(r,z,\varphi), u^*_\varphi(r,z,\varphi), u^*_z(r,z,\varphi),\}$$

are, respectively, the complex natural frequency and natural form of oscillation of the system.

The strain tensor and displacement vector are related by the Cauchy relations [21, 25]:

$$\begin{aligned}
&\varepsilon_{rr} = \frac{\partial u_r^*}{\partial r}; \quad \varepsilon_{\varphi\varphi} = \frac{1}{r}\frac{\partial u_\varphi^*}{\partial \varphi} + \frac{u_r^*}{r}; \quad \varepsilon_{zz} = \frac{\partial u_z^*}{\partial z};\\
&\varepsilon_{r\varphi} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r^*}{\partial \varphi} + \frac{\partial u_\varphi^*}{\partial r} - \frac{u_\varphi^*}{r}\right);\\
&\varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_z^*}{\partial r} + \frac{\partial u_r^*}{\partial z}\right);\\
&\varepsilon_{\varphi z} = \frac{1}{2}\left(\frac{\partial u_\varphi^*}{\partial z} + \frac{1}{r}\frac{\partial u_z^*}{\partial \varphi}\right).
\end{aligned} \tag{7}$$

The relationship between stresses and deformations is as follows [21, 26]:

$$\begin{aligned}
&\sigma_r = K_m(\varepsilon_r + \varepsilon_\varphi + \varepsilon_z) + \frac{2}{3}\tilde{\mu}_m(2\varepsilon_r - \varepsilon_z - \varepsilon_\varphi);\\
&\sigma_z = K_m(\varepsilon_r + \varepsilon_\varphi + \varepsilon_z) + \frac{2}{3}\tilde{\mu}_m(2\varepsilon_z - \varepsilon_r - \varepsilon_\varphi);\\
&\sigma_\varphi = K_m(\varepsilon_r + \varepsilon_\varphi + \varepsilon_z) + \frac{2}{3}\tilde{\mu}_m(2\varepsilon_\varphi - \varepsilon_r - \varepsilon_z);\\
&\tau_{rz} = \tilde{\mu}_m\gamma_{rz}; \quad \tau_{r\varphi} = \tilde{\mu}_m\gamma_{r\varphi}; \quad \tau_{z\varphi} = \tilde{\mu}_m\gamma_{z\varphi}.
\end{aligned} \tag{8}$$

It is assumed that volumetric deformation occurs according to an elastic law, and shear deformation occurs according to a viscoelastic law, i.e., [21, 26], where

$$\tilde{\mu}_m\varphi = \mu_m\left[\varphi(t) - \int_0^t \Gamma_m(t-\tau)\varphi(\tau)d\tau\right], \tag{9}$$

where μ_m is the shift modulus; K_m is the volume compression modulus; $\Gamma_m(t-\tau)$ is the relaxation kernel; φ is an arbitrary function of time; m is the body to which the mechanical characteristics relate ($m = 1$ is the structure, $m = 2$ is the foundation, $m = 3$ is the base).

If the integral terms are in ratio (9) are small, then the function $\varphi(t)$ into (5) can be represented as

$$\varphi(t) = \psi(t)e^{-i\omega_R t},$$

where ψ is a slowly varying function of time, i is an imaginary unit, ω_R is a real constant, then assuming that the integral terms are small compared to $\varphi(t)$, and using the freezing method [27], we can reduce the integral relation to a complex one, where ω_R is a real constant; φ is a slowly varying function of time; i is an imaginary unit

$$\begin{aligned}
&\tilde{\mu_m}\varphi = \mu_m\left[1 - \Gamma_{\mu_m}^c(\omega_R) - i\Gamma_m^S(\omega_R)\right]\varphi,\\
&\Gamma_m^c(\omega_R) = \int_0^\infty \Gamma_{\mu_m}(\tau)\cos\omega_R\tau d\tau,\\
&\Gamma_m^S(\omega_R) = \int_0^\infty \Gamma_{\mu_m}(\tau)\sin\omega_R\tau d\tau.
\end{aligned} \tag{10}$$

$\Gamma_{\mu_m}^S$, $\Gamma_{\mu_m}^C$ are the sines and cosines of the Fourier image of the kernel $\Gamma_{\mu_m}(\tau)$.

Thus, the problem of finding complex natural frequencies and natural forms of oscillations of a viscoelastic system (see Fig. 1) was reduced to finding the constant ω^2 and the vector of the function $\vec{u}^*(\vec{x})$ satisfying variational equation (4), taking into account the relations (7)–(10), and the conditions of (5) and (6) with any virtual movement of $\delta\vec{u}*$.

1.2. The method and algorithm for solving the problem. When solving the above variational problem on the dynamic characteristics of a spatial axisymmetric viscoelastic system (see Fig. 1) a semi-analytical version of the finite element method [28] is used. In this case, the solution along one coordinate (in the circumferential direction) is represented as separate harmonics, that is, through n in the form

$$\begin{aligned} &\vec{u} = \{u_r, u_\varphi, u_z\},\\ &u_r = u_r^*(r,\varphi,z)\cos\omega t,\\ &u_\varphi = u_\varphi^*(r,\varphi,z)\cos\omega t,\\ &u_z = u_z^*(r,\varphi,z)\cos\omega t,\\ &u_r^* = u_n(r,z)\cos n\varphi,\\ &u_\varphi^* = \mathrm{w}_n(r,z)\sin n\varphi,\\ &u_z^* = \mathrm{v_n}\cos n\varphi. \end{aligned} \tag{11}$$

Discretization of the system under consideration (see Fig. 1) is performed using a finite element in the form of an annular finite element of triangular cross section [20, 28].

The use of finite elements of complex shapes complicates the solution of a system of tens of thousands of integro-differential equations. Therefore, simple triangular elements are used, and the required accuracy is ensured by their large number.

Using the finite element method procedure allows us to reduce the variational problem (4)–(5) to the solution of complex algebraic eigenvalue equation, that is

$$\left(\left[\bar{K}\right] - \omega^2\left[M\right]\right)\left\{\bar{X}\right\} = 0, \tag{12}$$

where $\left[\bar{K}\right]$ is the complex system stiffness matrix; $[M]$ is the mass matrix of the system;; $\omega = \omega_R - i\omega_I$, $\left\{\bar{X}\right\} = \{X_R - iX_I\}$ are complex natural frequency and the eigenvector of the system under consideration (see Fig. 1) respectively.

Kinematic boundary conditions are taken into account when forming the equations (12), that is, if the displacement components or deflection of a node are zero, then the corresponding lines of the equations (12) are not formed.

Complex roots of the characteristic determinant of problem (12) are searched by the Muller method [29], and the eigenvector is searched by the Gauss method [30].

In this case, the solution to a homogeneous system of algebraic equations with complex coefficients (12), that is, the real part ω_R of frequency ω is the frequency of free damped oscillations of the system (see Fig. 1), and the imaginary part ω_I carries information about the rate of oscillation damping, and is equal to the damping coefficient, up to a sign.

1.3. Results and conclusions. As an example for assessing the spatial dynamic characteristics of axisymmetric structures, taking into account the dissipative properties of the material, the cooling tower of the Novo-Angren thermal power plant with a height of 102 m with a variable slope and variable thickness is considered, which allows taking into account their real geometry.

To account for the dissipative properties in the material of the structure, the linear hereditary Boltzmann-Volterra theory is used, which provides a good description of the viscoelastic properties of concrete. The Koltunov–Rzhanitsyn kernels [21, 31] are used as the relaxation kernel, which is

$$\Gamma(t-\tau) = \frac{Ae^{-\beta(t-\tau)}}{(t-\tau)^{1-\alpha}}. \tag{13}$$

Parameters of the relaxation kernel (13) are determined using the method of M. A. Koltunov [21] from the experimental creep curves for concrete given in [32], that is, $A = 0.0194$, $\beta = 0.00000014$, $\alpha = 0.075$.

For different soils, the parameters of the relaxation kernel (13), obtained using this technique, are determined from the experimental creep curves [33], the results are given in [34].

Other physical and mechanical parameters of concrete are accepted in the following form: $\rho = 2.5t/m^3$; $K = 1.510^5$ MPa; $\mu = 1.3610^5$ MPa, and the geometric parameters of the structure were taken from the design documentation.

Computer programs [35,36], registered with the Intellectual Property Agency under the Ministry of Justice of the Republic of Uzbekistan on 03/04/2025, have been developed to determine complex natural frequencies and oscillation modes.

In subsequent calculations, to evaluate the dynamic characteristics of the structure, the rigidity of the foundation and soil base was assumed to be significantly greater than that of the structure.

Table 1 shows, in an elastic formulation, five natural frequencies of the Novo-Angren cooling tower corresponding to different harmonics, that is, $n = 0, 1, 2, 3, 4, 5, 6$.

Table 2 shows the values of complex natural frequencies and logarithmic decrements of cooling tower oscillations at various harmonics, i.e. $n = 0, 1, 2, 3, 4, 5, 6$. The natural frequencies and waveforms at $n = 0$ correspond to torsional, axisymmetric, at $n = 1$ correspond to bending, and at $n = 2, 3, 4, 5, 6$

Table 1

Harmonic number	Natural frequencies, rad/s				
	ω_1	ω_2	ω_3	ω_4	ω_5
n=0 (torsional)	14.0296	32.9882	54.2194	75.5951	97.0503
n=0 (axisymmetric)	18.2147	30.4233	33.7759	37.1874	40.2305
n=1	7.6475	16.0506	25.4146	28.9566	31.3514
n=2	4.1310	8.6557	16.5660	22.8386	26.9128
n=3	3.3395	4.7183	10.3443	16.2120	21.3067
n=4	3.1419	3.7494	6.8003	11.4990	16.3899
n=5	3.2486	4.3613	5.2828	8.6026	12.8673
n=6	3.9139	4.7766	5.6844	7.2035	10.7321

Table 2

Harmonic number	Natural frequencies, rad/s				
	ω_1	ω_2	ω_3	ω_4	ω_5
n=0 (torsional)	$13.2364 - 0.1771i$ $\delta_1 = 0.084$	$31.3441 - 0.3867i$ $\delta_2 = 0.077$	$51.7152 - 0.6088i$ $\delta_3 = 0.074$	$72.2778 - 0.8250i$ $\delta_4 = 0.072$	$92.9605 - 0.1037i$ $\delta_5 = 0.070$
n=0 (axisymmetric)	$16.2283 - 0.2473i$ $\delta_1 = 0.095$	$27.3264 - 0.3852i$ $\delta_2 = 0.086$	$31.3398 - 0.3873i$ $\delta_3 = 0.078$	$33.5728 - 0.3994i$ $\delta_4 = 0.074$	$38.1766 - 0.4332i$ $\delta_5 = 0.071$
n=1	$6.8208 - 0.1053i$ $\delta_1 = 0.097$	$14.4436 - 0.2074i$ $\delta_2 = 0.090$	$23.2058 - 0.3250i$ $\delta_3 = 0.088$	$27.4214 - 0.3814i$ $\delta_4 = 0.086$	$32.6175 - 0.4370i$ $\delta_5 = 0.084$
n=2	$3.6661 - 0.0618i$ $\delta_1 = 0.105$	$7.8060 - 0.1181i$ $\delta_2 = 0.095$	$15.0946 - 0.2180i$ $\delta_3 = 0.091$	$21.1896 - 0.3015i$ $\delta_4 = 0.089$	$25.8517 - 0.3587i$ $\delta_5 = 0.087$
n=3	$3.0718 - 0.0528i$ $\delta_1 = 0.107$	$4.4524 - 0.0719i$ $\delta_2 = 0.101$	$9.6549 - 0.1444i$ $\delta_3 = 0.094$	$15.3610 - 0.2225i$ $\delta_4 = 0.091$	$20.7157 - 0.2931i$ $\delta_5 = 0.088$
n=4	$2.9875 - 0.0500i$ $\delta_1 = 0.109$	$3.2864 - 0.0546i$ $\delta_2 = 0.103$	$6.0604 - 0.0921i$ $\delta_3 = 0.096$	$10.3025 - 0.1498i$ $\delta_4 = 0.091$	$14.7369 - 0.2078i$ $\delta_5 = 0.089$

correspond to non-axisymmetric oscillations.

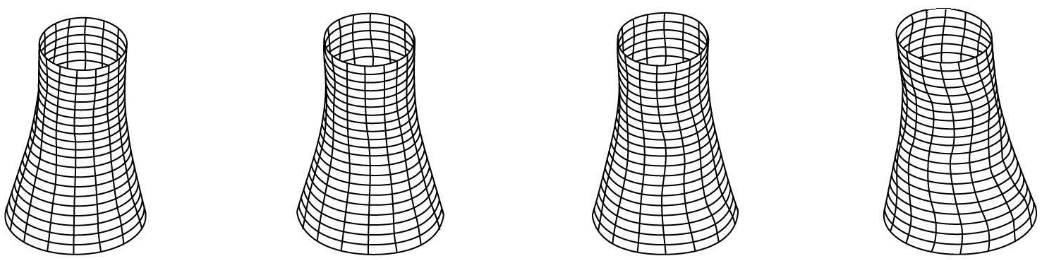
Comparison of the values of natural frequencies obtained in the elastic formulation (Table 1) and taking into account the energy dissipation in the material (Table 2), shows that the value of the natural oscillation frequencies obtained taking into account dissipation is approximately 8–12% less. It should be noted that taking into account the dissipation in the material leads the oscillation decrement to weakly frequency-dependent results, that is, an increase in the natural oscillation frequencies of structures leads to a slight decrease in the oscillation decrement.

Figures 2–7 demonstrate, as an example, the found first and fifth eigenforms of oscillations (that is, the real parts of X_R eigenforms of X) corresponding to various harmonics: $n = 0, 1, 2, 3, 4$. An analysis of the obtained natural eigenforms of oscillations at $n = 0$ shows that the first form of axisymmetric oscillations is a longitudinal deformation of the cooling tower, and at other higher frequencies ($\omega_2 \ldots \omega_5 \ldots$) of oscillations are an axisymmetric deformation of cooling towers, and at the same time, the lower part of the cooling tower is mainly deformed, while the upper part is practically not deformed.

In the modes of oscillations corresponding to the lower frequencies (with harmonics $n = 1, 2, 3, 4, 5$), the upper part of the cooling tower is significantly deformed compared to the lower part.

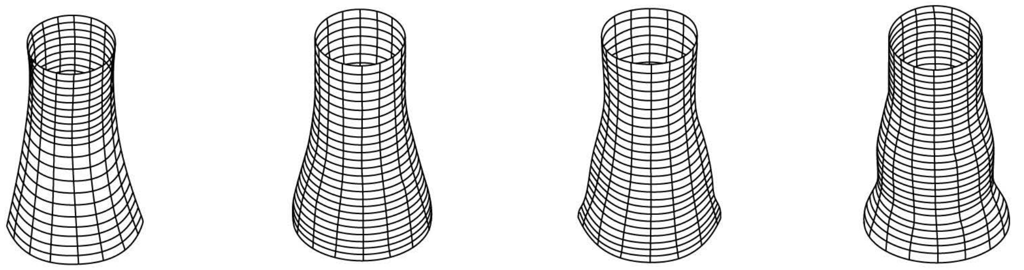
An analysis of the values of the obtained lower natural frequencies and oscillations patterns of the structure, corresponding to various harmonics, shows that the first natural frequencies of non-axisymmetric oscillation patterns fall within the prevailing frequency range of earthquakes.

Along with this, the natural frequencies and oscillation patterns of the cooling tower were investigated, taking into account the upper reinforcing rib (i.e., the ring of rigidity). A comparison of the obtained results of the natural frequencies of a cooling tower without a stiffener and with a stiffener showed that the upper stiffener affects a slight increase in the frequency of axisymmetric and non-axisymmetric oscillations of the structure due to an increase in stiffness in the upper part of the cooling tower. At the same time, the values of the bending oscillation frequencies are slightly reduced due to an increase in the mass of the structure.



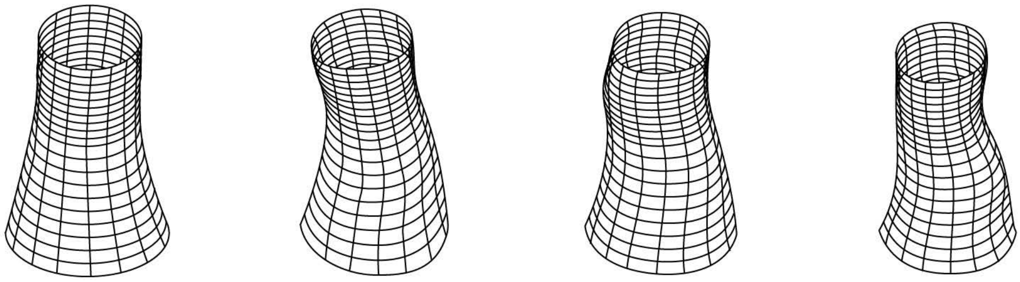
$\omega_1 = 13.2364$ $\omega_2 = 31.3441$ $\omega_3 = 51.7152$ $\omega_4 = 92.9605$

Fig. 2. Torsional modes of cooling tower vibrations ($n = 0$)



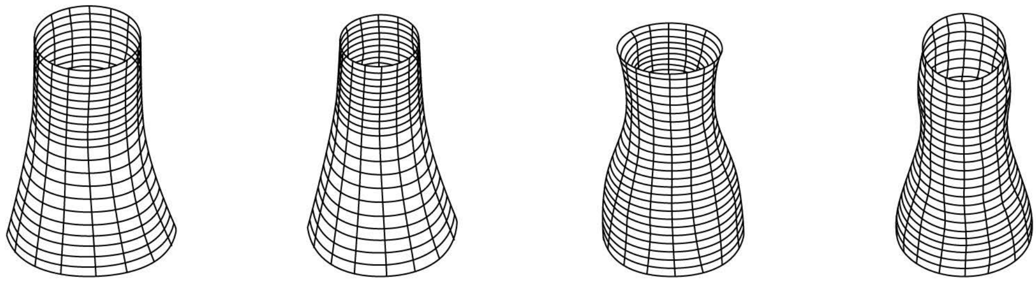
$\omega_1 = 16.2283$ $\omega_2 = 27.3264$ $\omega_3 = 31.3398$ $\omega_4 = 33.5728$

Fig. 3. Axisymmetric modes of cooling tower oscillations ($n = 0$)



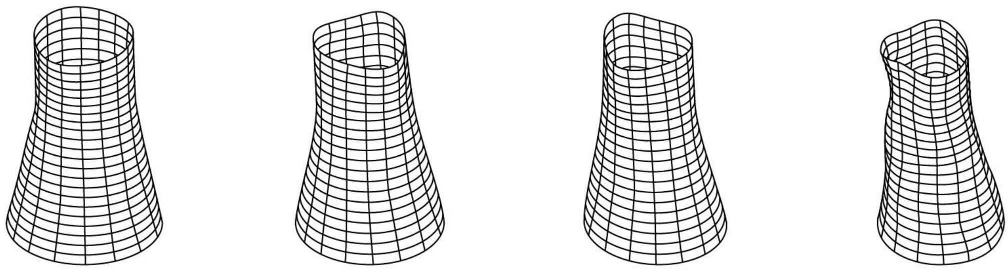
$\omega_1 = 6.8208$ $\omega_2 = 14.4436$ $\omega_3 = 23.2058$ $\omega_4 = 27.4214$

Fig. 4. Bending modes of cooling tower vibrations ($n = 1$)



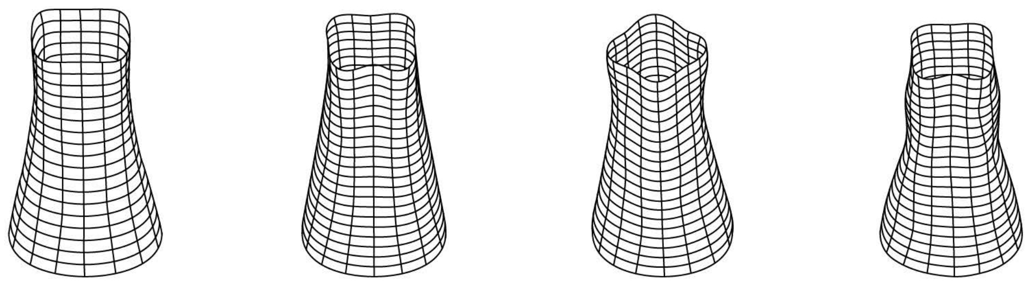
$\omega_1 = 3.6661$ $\omega_2 = 7.8060$ $\omega_3 = 15.0946$ $\omega_4 = 21.1896$

Fig. 5. Non-symmetrical modes of cooling tower oscillations ($n = 2$)



$\omega_1 = 3.0718$ $\omega_2 = 4.4524$ $\omega_3 = 9.6549$ $\omega_4 = 15.3610$

Fig. 6. Non-symmetrical modes of cooling tower oscillations ($n = 3$)



$\omega_1 = 2.9875$ $\omega_2 = 3.2864$ $\omega_3 = 6.0604$ $\omega_4 = 10.3025$

Fig. 7. Non-symmetrical modes of cooling tower oscillations ($n = 4$)

Conclusion

1. The mathematical model has been developed to evaluate the dynamic characteristics of spatial axisymmetric structures, taking into account energy dissipation in the material using the hereditary

theory of viscoelasticity.

2. The method, algorithm, and computer calculation program have been developed for estimating complex natural frequencies, oscillation modes, and oscillation decrements of spatial axisymmetric structures, taking into account the viscoelastic properties of the material.

3. The dynamic characteristics (complex frequencies, decrement, and oscillation form) of a specific spatial axisymmetric cooling tower-type structure are estimated.

4. It is established that for this spatial axisymmetric structure, the lowest non-axisymmetric natural frequencies fall within the range of prevailing earthquake frequencies.

5. Some mechanical effects related to energy dissipation in the material and taking into account the actual geometry of the structure have been identified, that is:

- taking into account the dissipation in the material using the hereditary theory of viscoelasticity leads to a slight decrease in the natural oscillation frequencies of structures and to a weakly frequency independent oscillation decrement;
- rings of rigidity installed at the top of spatial axisymmetric structures slightly increase the non-axisymmetric natural oscillation frequencies, while the bending frequencies of structures decrease slightly.

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