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# DYNAMICAL CHAOS - MODELS AND EXPERIMENTS

## Appearance Routes and Structure of Chaos in Simple Dynamical Systems

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### Preface

Dear reader! You are faced with a further book on the problem of dynamical chaos in dissipative systems. This subject has already been covered in a number of notable books. An impression might come to mind that the problem has been exhausted. However, let us not hasten this conclusion. In studies of most of the books dedicated to dynamical chaos in dissipative systems, a group of questions can be separated which, in my opinion, deserve a more detailed analysis. The following issues are the most important:

1. The role of characteristic homoclinic trajectories and Poincaré structures which define general bifurcational mechanisms of the appearance and the main properties of chaotic attractors in the appropriate class of nonlinear systems.

2. Regularities and interplay of the bifurcational mechanisms of transition to chaos, as well as the statistical properties of chaotic attractors which are revealed under the multi-parameter bifurcational study of dynamical systems.

3. The influence of external and internal fluctuations on typical bifurcations, structure transformation, the scenarios of development of chaotic attractors and their statistical properties.

The use of the theory of robust hyperbolic attracting sets (the theory of «truly» strange attractors) is found to be insufficient for investigation of the above problems. The application of the concept of quasiattractors which include regular (periodic and quasiperiodic) subsets of trajectories, along with the «strange» ones appears to be more fruitful. The analysis of dynamical systems from the viewpoint of this quasiattractor conception is more constructive for the examination of experimental results.

In low-dimensional dynamical systems, the occurrence of quasiattractors is mainly due to three types of homoclinic trajectories, and namely: 1) a saddle-focus separatrix loop of the equilibrium state; 2) a homoclinic structure in the form of «Smale's horseshoe» that is realized under intersection between stable and unstable manifolds of a saddle limit cycle; and 3) homoclinic trajectories appearing from breakdown the of a resonant two-dimensional torus.

The study of the typical properties of quasiattractors engendered by the above types of homoclinic trajectories in the basic content of this book.

In Chapters 1 and 2, the elements of the stability and bifurcation theories are briefly outlined, as well as the methods for experimental research of dynamical chaos. The incorporation of this material pursues the goal of assisting young scientists in understanding of the main part of the book and to spare them the necessity of referring to additional literature on the first stage of examination.

Chapter 3 attends to an original system with chaotic dynamics (a modified oscillator with inertial nonlinearity) along with its mathematical model. A simple electronic circuit with one-and-a-half degree of freedom due to a number of its attributes has proved to be extremely convenient for numerical and experimental investigation of a low-dimensional chaos.

The objective of Chapters 4 and 5 is to analyze in more detail, both numerically and experimentally, the dynamics of an autonomous oscillator under variation of the control parameters of the system.

In Chapters 6 and 7, the typical hierarchies of instabilities are discussed which accompany nonlinear phenomena as the regimes of quasiperiodic oscillations with two or three independent frequencies are destroyed. For a basic model the inertial nonlinearity oscillator is used. The successive increase in dimension is achieved for the models under study by introducing an external periodic force or using a system of coupled oscillators.

Chapter 8 is devoted to the problem of synchronization of chaotic oscillations. A more simple class of chaotic attractors is considered for which a pronounced basic frequency in the power spectrum is typical (Shilnikov's attractors). The feasibility of generalization for the concepts of the classical oscillation theory on the external and mutual synchronization of periodic oscillations is shown for the case when these attractors are synchronized.

Recent findings on chaotic oscillations in the well-known Chua's circuit are presented in Chapter 9. Unlike the inertial nonlinearity oscillator, Chua's circuit is described by the equations with distinct symmetry properties and is characterized by three equilibrium states. These properties are responsible for the appearance of attractors with more complicated structure permitting a number of novel effects (e.g. the phenomenon of stochastic resonance) to be observed in this system. Chapter 9 has been written in collaboration with A.B.Neiman and M.A.Safonova.

Chapters 10 and 11 deal with the results of research into the influence of fluctuation on the bifurcations of regular and chaotic attractors. The foremost conclusion is here a justification of a strong sensitivity of particular chaos regimes in quasihyperbolic systems to small external perturbations. These chapters has been written together with A.B.Neiman.

Finally, Chapter 12 is devoted to the problem of reconstruction of dynamical systems with account of homoclinic trajectories and noise. This chapter is written together with M.A. Safonova.

The exposition and discussion of most of the results obtained are performed by starting from a detailed comparison between the data of theory, numerical simulation and full-scale experiment. This allows many typical regularities to be obviously interpreted from the viewpoint of physics and the role of fluctuations, involved in a physical experiment, to be evaluated.

Finally, a distinctive feature of this book is that it has been written based on the results of original studies performed in the Laboratory of Nonlinear Dynamics of the Saratov State University for the past decade under my supervision.

The book's peculiarities briefly listed above, which make it different from books in print, lead me to hope that you, dear reader, will read this book with particular interest and to your benefit.

I express my profound thanks to V.V.Astakhov, T.E.Vadivasova-Letchford, M.A.Safonova, D.E.Postnov and A.B.Neiman, my students and colleagues, and collaborators in most of the works which have provided the basis for this book.

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