Applied problems of nonlinear oscillation and wave theory

Official English translation

Chaos and order in atmospheric dynamics
Part 1. Chaotic weather variations

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Received 19.05.2017

Ideas of temporary energy distributions of large-scale atmospheric motions are made more accurately in the range of scales from days to one year in order to solve the problem of the chaos and order co-existence in the weather and climate dynamics. Spectra of the Blinova's mean and shifted zonal extratropical flow indices as well as spectra of the tropical Southern Oscillation and El Niño indices are used for this purpose. Unlike earlier had ideas, it is found for the Blinova indices that transitions between the parts of the spectrum ranges having different average inclinations happen smoothly so there is no «synoptic maximum» of the spectral density near the period about one week and no «index cycle maximum» near the period of about two-three weeks. It confirms a chaoticity of the extratropical weather variations. As for the tropical indices, a break of the spectral density curve is found at the period of 5 days, which has been earlier noticed only in the dynamics of some local characteristics of tropical weather. The second break is found at the period of about 45 days for the modified index of the Southern Oscillation where a peak in the spectrum of the Madden-Julian Oscillation has been earlier found. These breaks indicate the existence of an «order» in the tropical weather dynamics, which also is chaotic, in general. Spectra of the monthly and seasonal weather variations everywhere on the Earth are found composed from a seemingly continuous background and some delta peaks imposed on this background. As a result, the dynamics consists of a mix of partly chaotic and partly ordered weather variations.

Key words: Temporal power spectra of weather and its seasonal variations, chaos and order in the weather variations.

DOI:10.18500/0869-6632-2017-25-4-4-22

References: Serykh I.V., Sonechkin D.M. Chaos and order in atmospheric dynamics. Part 1. Chaotic weather variations. Izvestiya VUZ. Applied Nonlinear Dynamics. 2017. Vol. 25. Issue 4. P. 4–22. DOI:10.18500/0869-6632-2017-25-4-4-22.

Introduction

The latter half of the 20th century has witnessed a significant advance in the study of atmosphere, ocean and climate dynamics due to computer technology development. This led to the development of numerical methods for short- and medium-term weather forecasts. In 2003, the World Meteorological Organization (WMO) launched THORPEX (THe Observation System Research and Predictability EXperiment), a ten-year international research program to increase accuracy and utility of high-impact weather forecasts up to two weeks ahead. Great efforts have been made to improve prognostic models, both computationally and physically. However, the goal was not fully achieved.

For surface air temperature the limit of useful predictability in the best models, such as the model of the European Center for Medium-Range Weather Forecasts (ECMWF), is now not more than a week. Quantitative precipitation forecasts exhaust their utility even faster.

The result obtained is not unexpected due to efforts of several forecast analysts, primarily E.N. Lorenz, who developed in the middle of the 20th century the theory of weather predictability (unpredictability, to be more precise). This theory pointed out that inaccuracy of atmosphere's initial state data, combined with the instability of weather-forming atmospheric processes to small disturbances, is an insurmountable obstacle to weather forecasts for longer lead time. Initially, Lorenz [1] estimated that monthly achievable integration for detailed predictive models of atmospheric hydrothermodynamics is the maximum achievable. Over the coming years, this estimate was slightly reduced [2, 3], since the growth of forecast errors in the newly developed more detailed models turned out to be faster than in the model considered by Lorenz. In addition, ensemble forecasts, initially accompanied by high expectations, showed a progressive smoothing of calculated meteorological fields after about the tenth day. According to L. Bengtsson [4], the lead time of reliable forecasts achieved so far is almost equal to the theoretically possible lead time if we use the traditional system of predictive equations for atmospheric hydrothermodynamics. However, this does not necessarily mean that there are no other possibilities. Application of mathematics to solving problems in Earth Sciences is under active consideration (see, for example, [5]).

Lack of obvious ways to further increase the lead time of useful weather forecasts was one of the reasons that the World Meteorological Organization has focused on the problem of modern climate change since the late 20th century. To solve this problem, it was proposed to use the same models as for weather forecasts. Indeed, these models needed to be supplemented by taking into account the ocean-atmosphere interaction and distinctly describe all currently known external influences on the heat balance of the climate system.

This approach has led to the concept of unified or so-called seamless prediction [6, 7]. Within this concept, each individual change in the climate system state is considered to be limitedly predictable. Thus, the concept considers weather and short-period climate fluctuations to be limitedly predictable (chaotic). However, climate change has a much higher predictability limit due to the so-called Second-kind predictability determined by slowly changing external influences on the climate system.

In fact, the evidence of weather changes being random is only empirical, but not rigorously theoretical. There is no evidence of such randomness at all. Indeed, the modern theory of dynamical systems indicates the positivity of one or several Lyapunov exponents quantitatively describing instability of atmospheric processes due to small perturbations as a necessary condition for deterministic «chaos» [8].

Another sign of such «chaos» is a non-integral dimension of the attractor in the dynamics under consideration. Regrettably, verification of these conditions is possible only for «toy-like» atmospheric models. Therefore, we can only focus on a sufficient condition of «chaos», consisting in the continuity of the energy spectra in atmospheric processes.

Dynamics of the real atmosphere (and its numerical hydrothermodynamic models) is characterized by two types of energy spectra: spatial and temporal. Spatial spectra of atmospheric motions are obviously discrete due to the finite horizontal size of the atmosphere. Calculations of such spectra began to be carried out from the beginning of the latter half of the 20th century. Examples can be found in [9, 10], an early review of these works is provided in [11]. Usually, spectral density of kinetic and available potential energies was estimated as functions of the zonal wave number. There were found two ranges of these wave numbers, and inclinations of the spectra on a double logarithmic scale approximately followed the first-digit law.

Similar laws were previously derived for homogeneous and isotropic two-dimensional turbulence from simple dimensional analyzes, and then confirmed as predominantly observed in numerical experiments with models of quasi-two-dimensional flows, including atmospheric and oceanic flows, and also found in many field measurements of turbulent flows [12–14]. It should be noted, however, that numerical models of those years, for example, the pioneering experiment of D.K. Lilly [15], were executed on very slow (in hindsight) computers. Grid regions considered were very small; therefore, boundary conditions could not but affect the calculation results. Assumptions of homogeneity and isotropy that underlie the theory of turbulence do not allow this. It is also important that integral times of the models were very short, and, in fact, the results obtained relate to some transient processes arousing in models due to random initial conditions. By this line of reasoning, it seems that it would be useful to carry out similar numerical simulations anew using today's computer technology and taking into account both deficiencies noted.

Energy dependence of atmospheric oscillations as a function of a two-dimensional wave number was studied less frequently. In addition, all calculations of this kind published so far have been performed on very small samples of initial meteorological data. As an example, we mention the only indigenous Russian calculation of the two-dimensional spectrum for wave amplitudes in 500 hPa flow patterns for the northern hemisphere in [16]. According to this work, amplitudes are greatest for those waves where differences between meridional and zonal wave numbers are equal to four. These amplitudes decrease during the transition from ultra-long waves to synoptic waves.

Since it is impossible to determine the chaotic or non-chaotic nature of the planetary atmosphere dynamics by its spatial energy spectra, one has to examine temporal energy spectra which, basically, can be either discrete or continuous. However, if guided by the well-known Taylor's hypothesis of «frozen» turbulence [17], we can take into account the aforementioned results of the quasi-two-dimensional (and, more specifically, geostrophic) turbulence theory, where energy spectral density depends on spatial wave numbers. Corresponding considerations for geophysical flows are available, for example, in [12,13,18].

The purpose of this two-part paper is to verify whether the chaos condition (continuity of the temporary energy spectrum) is fulfilled separately for weather changes (the first part of this paper) and for climate changes (the second part of this paper). We found that weather is chaotic while climate is not. This depreciates the concept of weather and climate predictions using the same model in favor of the concept of weather and climate predictions using different models.

1. Energy spectra of extratropical weather variations

Temporal energy spectra of the Blinova indices at 500 hPa geopotential height in the northern hemisphere were calculated by many researchers at the beginning of the latter half of the 20th century. However, the initial data of radiosonde measurements of the geopotential heights that were available from operational analyzes of prognostic meteorological centers back then existed for only one or two dozen years and were imperfect. The most recent estimate was made in [20] by applying the Fourier Transform to various segments of H500 daily values during 1949–1983, followed by summing the results of all transformations.

In fact, any evaluation of the temporal energy spectrum is not continuous but discrete, since it is calculated from a temporary archive of finite length at a finite number of frequencies. However, the resulting spectrum may look continuous if the spectral density for all considered frequencies is noticeably different from zero. This is exactly what happened when evaluating the spectrum in [20]. This could have given a positive answer to the question on continuity of temporary energy spectrum of the Blinova index, if not for the breaks in the course of spectral density visible in this evaluation. For periods from 2 to about 10 days, the average inclination of the seemingly continuous spectrum approximately followed the «-3» law, and for longer periods, from 10 to about 45 days, it followed the «$-5/3$» law. Break at the junction of these inclinations, as it seemed, confirmed the existence of so-called «synoptic maximum», where energy is transferred into atmosphere by resolving baroclinic instability of cyclonic waves. An important promoter of this viewpoint in our country was A.S. Monin [19].

Note that in the theory of geostrophic turbulence, that usually operates not with temporal, but with spatial spectra of atmospheric movements, it is considered that the inclination «-3» corresponds to the direct (to large wave numbers) cascade transfer of enstrophy, first described by Y. Ogura [21]. The inclination «$-5/3$» corresponds to the inverse (to small wave numbers) energy transfer. However, subject to the adoption of J. Taylor's «frozen» turbulence hypothesis [17], such a reverse transfer is considered observable in atmospheric time spectra. Physically, this transfer corresponds to the so-called «negative viscosity» [18], in which energy of synoptic formations (cyclones and anticyclones) is transferred to larger objects, such as «blocking» anticyclones and stationary regions of low atmospheric pressure.

Practice of meteorologists-forecasters confirms this assumption. On daily synoptic maps, one can see how cyclonic waves arising in the vicinity are enlarged and combined into larger and longer-lived areas of low atmospheric pressure. In this case, high-pressure wedges that originally existed between cyclonic waves are forced out to the boundaries of emerging low-pressure regions, forming vast anticyclones that «block» the west-east transfer at moderate latitudes and are very important for medium-term weather forecasts.

Referring to the estimation of the Blinova index spectrum in [20], we note that the break seen in it during spectral density for a period of about 5 days corresponds well to the idea of a specific scale in which energy is introduced into atmospheric dynamics due to resolution of cyclones' baroclinic instability. In the spectra obtained by numerical experiments, a break or even a peak in spectral density was always visible on this scale over a period of about a week and at wavenumbers of 5–8 synoptic waves. Therefore, many researchers of the general atmospheric circulation (GAC) in the latter half of the 20th century suggested that there is a so-called «synoptic maximum» in the GAC spectra.

Many researchers also suggested that in the real atmosphere there is a characteristic alternation time of «blocking» anticyclones and mainly zonal circulation at moderate latitudes – the so-called zonal circulation index cycle [22]. Presence of another peak of spectral density over a period of about two to three weeks was associated with this cycle [23]. It may be added that in [24] it was stated that there is a peak at a period of 45 days in the spectrum of atmospheric angular momentum, a characteristic strongly correlating with the zonal circulation index. In spectrum [20] this hypothetical peak is seen as a break in the spectral density over a period of about 40–50 days, that is, in the vicinity of the superharmonic period, 1:8 annual cycle.

For periods longer than a season, in spectrum estimation [20] there was a «spectral plateau», that is, an inclination becoming almost zero, as, probably, newly found in the spectrum of variations in average hemispheric air temperature in [25]. Powerful peaks of annual and semi-annual periods, as well as a break in the spectral density in the vicinity of the quarter-year period, were superimposed on this plateau. Blinova and her assistants explained the existence of this peak within the framework of the linear model of the general circulation for the baroclinic atmosphere and also knew about the six-month peak [26, 27]. Of course, it is now easy to criticize these works, for it is known that no linear system is capable of producing peaks at the super-harmonics of a periodic external force effecting this system. For that to happen, non-linearity must be present in the system. These harmonics appear if the response of the nonlinear system to the effecting periodicity becomes unstable. This was indicated already in [20].

In this paper we have carried out new calculations of the energy spectrum of the Blinova index using daily data from modern re-analyzes, where assimilation of observational data is performed using models of general atmospheric circulation (GAC). On the one hand, use of models has greatly facilitated the analysis of meteorological fields in areas with a sparse observation network. On the other hand, it may have caused some distortions of real atmospheric dynamics that are almost impossible to identify. Calculations on the data of all re-analyses were made using multiple fast Fourier transform (dark lines in the figures), as well as using preliminary calculation of time correlation function in the range of time shifts from zero to 10 years (light lines in the figures). First, NCEP/NCAR re-analysis was used. However, it turned out that there are significant inaccuracies. Thus, in the spectrum of the Blinova index, calculated by re-analysis of NCEP/NCAR for 500 hPa level in the southern hemisphere, the annual peak is generally lacking, although for 300–700 hPa level this peak occurred. It's hard to believe. Then other re-analyses

(ERA-20C, ERA-INTERIM, NOAA CIRES 20th Century Reanalysis) were used. All of them resulted in almost identical estimates of the time energy spectrum of the Blinova index for both hemispheres. Therefore, in this article these estimates are illustrated only for the 20th Century Reanalysis during 1871–2012 [28]. In this re-analysis the GAC model is used to assimilate atmospheric pressure data at sea level, while monthly ocean surface temperature and sea ice distribution data are used as boundary conditions.

Figure 1 (above) shows estimates of the temporal energy spectrum of the Blinova index at the 500 hPa level in the northern hemisphere, which confirm the general character of this spectrum described above in [20], but with differences that are significant for concluding that this spectrum is continuous. Most notably, no traces of «synoptic maximum» of spectral density are visible, that is, transition from «-3» inclination to «$-5/3$»

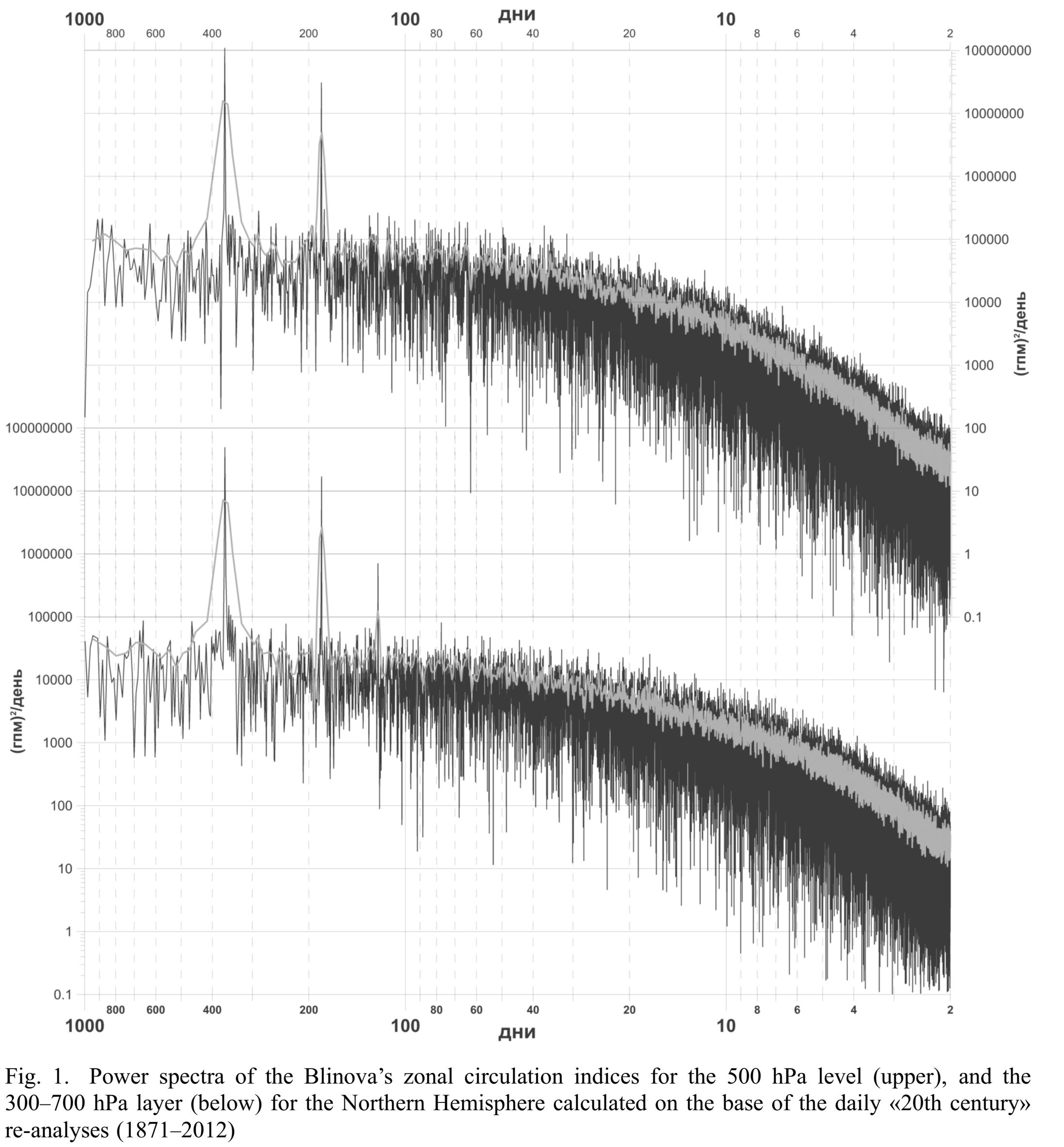

Fig. 1. Power spectra of the Blinova's zonal circulation indices for the 500 hPa level (upper), and the 300–700 hPa layer (below) for the Northern Hemisphere calculated on the base of the daily «20th century» re-analyses (1871–2012)

inclination occurs smoothly. Furthermore, maxima at periods of superharmonics 1:4 and 1:8 are not visible, and «$-5/3$» inclination smoothly transforms into a «spectral plateau». This, as it seems, excludes the assumption from [24] on a 45-day peak that limits range of «negative viscosity» from large periods. Perhaps, this range continues up to a peak in the six-month period, that is the main peak in the spectra of the Blinova index for the southern hemisphere (Figure 2, above).

It should be mentioned here that, unlike peaks of spectral density, for the significance/insignificance of which it is proposed to check the null hypothesis of «brown noise», there are no tests to check statistical significance of the breaks during the spectral density. Concluding their reality, we could be guided only by indirect signs. Such a sign can serve as an indication obtained many years ago in the works of a famous American meteorologist B. Saltzman and his staff [29, 30]. They showed that real atmosphere has no single scale localized in the space of wave numbers, where the energy is introduced into the

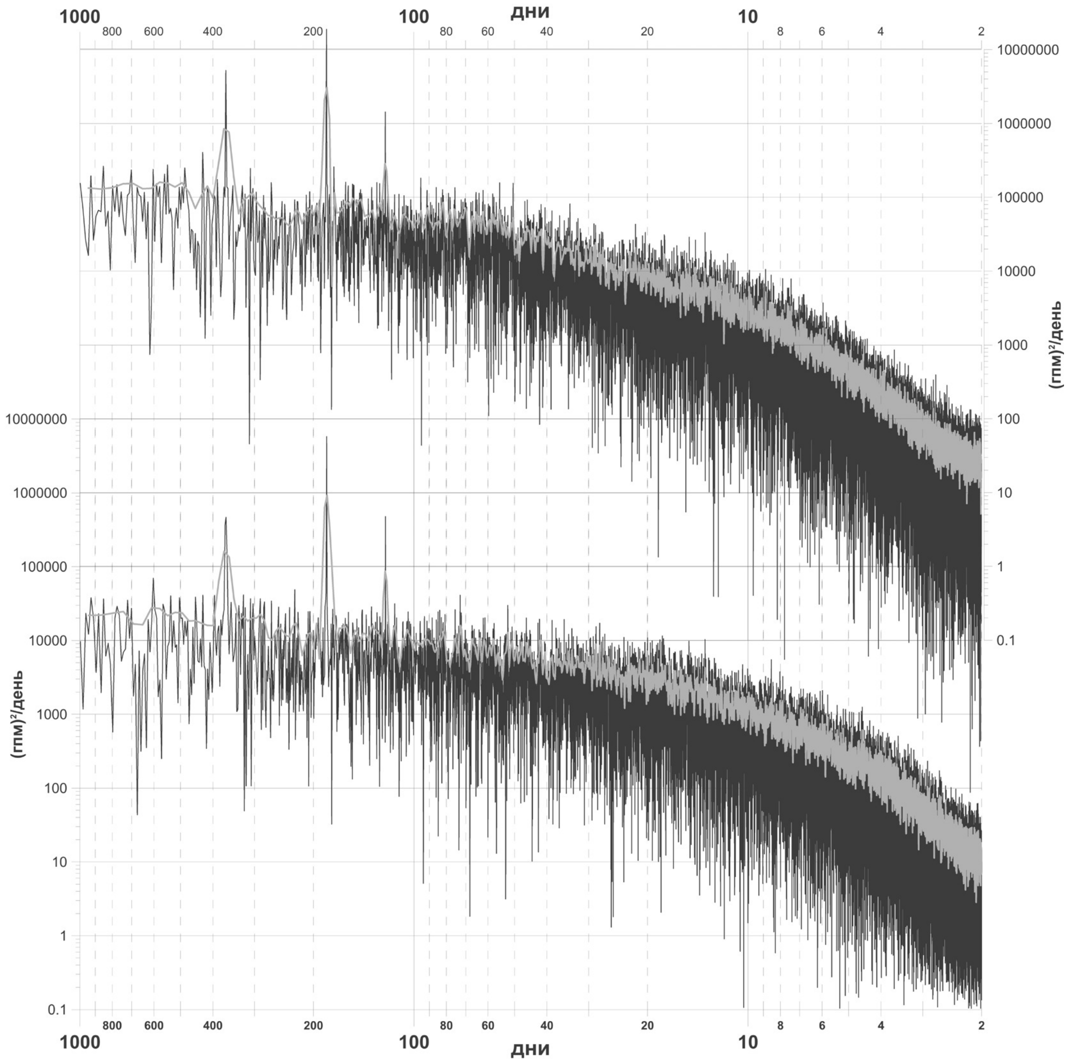

Fig. 2. The same that is shown in Fig. 1, but for the Southern Hemisphere

GAC. In fact, there is one rather wide range of zonal wave numbers from 5 to 11, that receives energy at resolution of baroclinic instability of cyclones, as well as another (zonal wave number 2), which receives energy from resolution of instability that occurs in the West-East transfer of temperate latitudes due to configuration of continents and oceans. As a result, both of these ranges are energy sources for the GAC.

Moreover, energy exchanges between GAC perturbations corresponding to different zonal wave numbers are not local in scale. Thus, energy comes to long-wave perturbations of the west-east transfer, corresponding to zonal wave numbers 3 and 4, from all perturbations of the synoptic scale, although the greatest contribution is made by those with wave numbers 6–8. The same synoptic disturbances make the largest contribution to the energy increase in the zonal flow. It is unlikely that in the absence of one selected scale, in which energy is transferred into the atmosphere, any peaks or breaks would appear in spectra of the GAC indices.

These works of Saltzman and his assistants used a nine-year archive of all radiosonde observations available at the time network. This is quite a lot even from modern positions, therefore, the conclusions based on redistribution of kinetic energy of atmospheric motion seem plausible. However, their likelihood is further increased by theoretical consideration of the spectral form of the atmospheric dynamics equations, that is a system of ordinary differential equations for the expansion coefficients of meteorological fields (flow functions, etc.) in spherical harmonics. Quadratically nonlinear terms of these equations, for example, the barotropic vorticity equation, describe interactions of a zonal flow component with two waves, or interactions of three waves [31]. In order for contribution of such a term to the dynamics to be nonzero, linear combinations of wave number triples corresponding to the zonal component under consideration and two waves (corresponding to three waves) satisfy certain rules called «selection principles» [32]. According to these principles, effective interaction requires at least two sets of wave numbers distributed along the spectrum of wave numbers, including that at least two zonal wave numbers are not adjacent. In other words, «selection principles» oblige redistribution of energy over the spectrum of wave numbers to be non-local. This is exactly what Saltzman and his assistants achieved. Note that such non-locality is mentioned in the most recent Russian review of papers on quasi-two-dimensional turbulence [14].

Thus, character of energy redistribution in the GAC system is different from that assumed in theoretical works on geostrophic turbulence. All the more surprising is that in this case laws «-3» and «$-5/3$» justified only for local nonlinear interactions are observed while calculating energy spectra of real atmospheric motions.

Spectrum of the Blinova index calculated for a relative topography of 300–700 hPa level in the northern hemisphere (Figure 1, below), contains a peak over a period of one third of the year, although its power is approximately an order lower than that of a semi-annual peak. In the spectra of the southern hemisphere (Figure 2), peaks for a period of one third of the year are clearly marked for both the 500 hPa level and the 300–700 hPa level. They are not less powerful than corresponding peaks in the annual period. Eventually, it may be assumed that the most high-frequent of all these peaks (for a period of one third of the year) in the spectra limits the range of «negative viscosity» from low frequencies.

Again, the most powerful peak in the southern hemisphere is the semi-annual peak. This is understandable if we accept the above statement that superharmonics of the annual period arise due to the instability of the atmosphere response to the annual solar flux. Presently, in the southern hemisphere perihelion occurs in summer, and apogee occurs in winter. As a result, annual course insolation amplitude in the southern hemisphere is larger than in the northern hemisphere. This more powerful periodic forcing leads to stronger destabilization of the atmosphere dynamics, which gives rise to a powerful peak in the half-year period.

Position of the low-frequency edge of the «negative viscosity» range is extremely important from the point of view of predictability theory. If the forecasting model is perfect enough, it should have the same aggregation of forecast errors that exists for real weather fluctuations. Due to limitation of the range of «negative viscosity» by large periods, forecast errors should not, on average, enter into even longer time scales. Therefore, within the framework of the Lorenz predictability (unpredictability) paradigm, it is impossible to explain the failure of modern seasonal weather forecasts. Actual causes of this are to be sought in the lack of proper parameterization of the actual weather variations and insufficiently accurate modeling of its seasonal changes. Therefore, special models for seasonal weather predictions need to be created.

2. Energy spectra of tropic weather variations

Weather in tropics is strikingly different from weather in middle latitudes, both in its subjective perception and in physical laws. In middle latitudes weather variations are determined by frequent change of cyclones and anticyclones – formations of so-called synoptic scales (many hundreds and several thousand kilometers in size). In tropics cyclones (tropical hurricanes) are relatively rare and smaller-scale phenomena, accordingly, local weather and mesoscale formations (ranging in size from several kilometers to many tens of kilometers) determine the usual weather regime. Therefore, physics of hydrometeorological phenomena in middle latitudes and in tropical latitudes is different.

The most famous hydrometeorological processes in the Earth's tropical belt are those combined under the name El Niño – Southern Oscillation (ENSO). In line with the objective of this work, it is appropriate to consider energy spectra of these processes, as they are represented by the Equatorial Southern Oscillation Index (ESOI) and the Extended Oceanic Niño Index (EONI). The first index characterizes the atmospheric component of ENSO processes. It is calculated as a difference in surface pressure at sea level between Indonesia (5°N–5°S, 90°E–140°E) and the equatorial Pacific Ocean (5°N–5°S, 80°W–130°W). The second index characterizes the oceanic component of ENSO, and is calculated by the temperature of the ocean surface averaged in the region (5°N–5°S, 80°W–170°W). Since the reanalysis «20th Century» does not contain data on ocean surface temperature, EONI was calculated from the mean surface air temperature.

It may be noted that energy spectra of ENSO indices in the scale range of less than a year have never been analyzed or even calculated. The ESOI and EONI spectra, calculated in the range from two to a thousand days, are shown in Figure 3. They are similar to the spectra of Blinova indices since they have a seemingly continuous base.

Thus, these spectra indicate chaotic dynamics of tropical weather. Average inclinations of these spectra follow the «$-5/3$» law over a wide range of scales. Therefore, it is possible that dynamics of tropical weather also has a «negative viscosity» where relatively small-scale disturbances become larger. For the EONI index (Figure 3, below), the range «$-5/3$» extends from five days to a quarter of a year (on a period of about ninety days one can see a small break) or even up to six months. However, the exact position of the low-frequency edge of the range «$-5/3$» cannot be determined, as it was noted for the spectra of the Blinova index.

At a high-frequency edge (about five days), an analogue of the «synoptic maximum» is visible in the EONI spectrum. To the right of this maximum there is an area where the spectrum inclination follows the law of minus third degree. The main difference between the EONI spectrum and the Blinova index spectra is that in the first spectrum

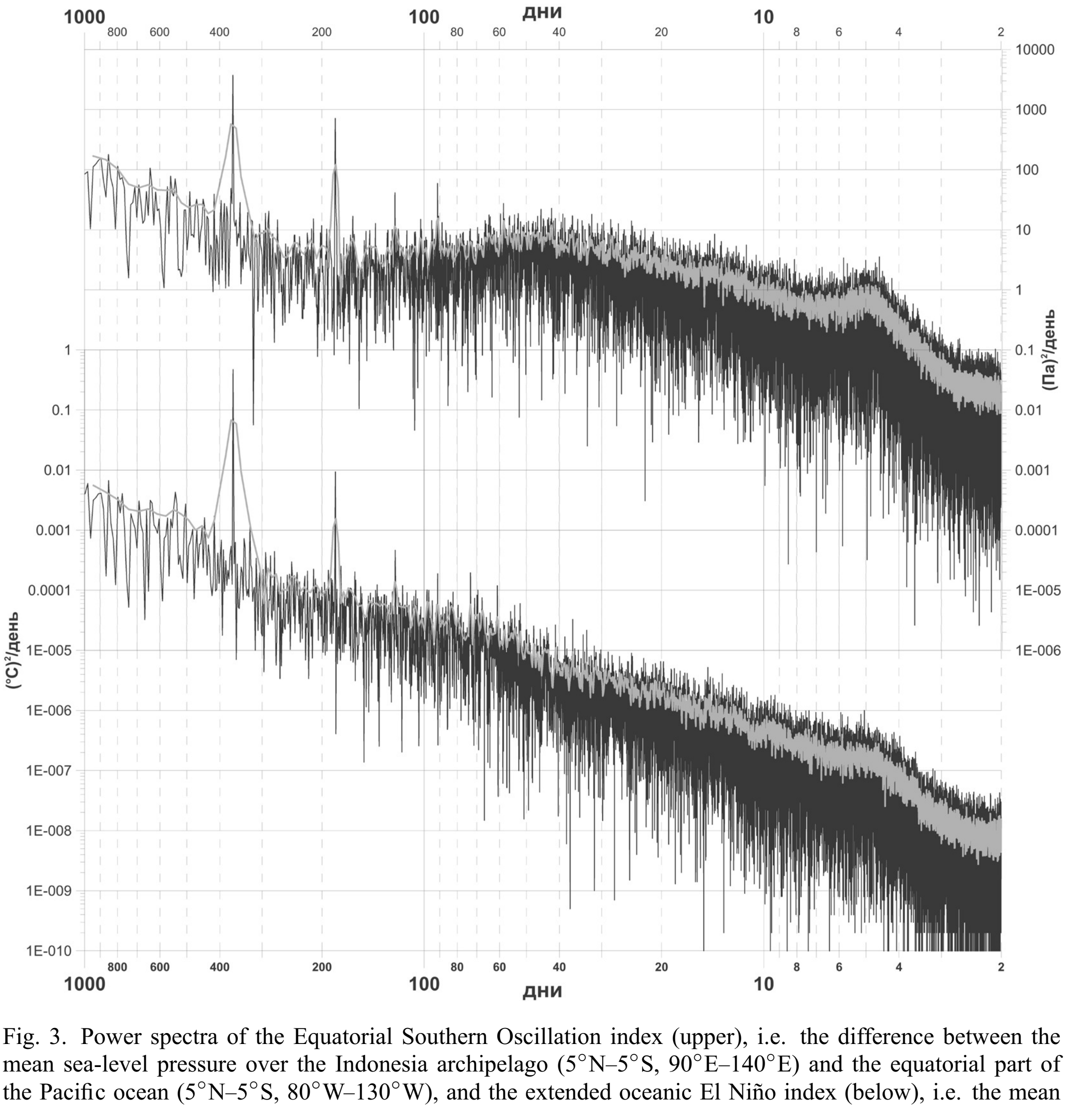

Fig. 3. Power spectra of the Equatorial Southern Oscillation index (upper), i.e. the difference between the mean sea-level pressure over the Indonesia archipelago (5°N–5°S, 90°E–140°E) and the equatorial part of the Pacific ocean (5°N–5°S, 80°W–130°W), and the extended oceanic El Niño index (below), i.e. the mean near-surface air temperature over the region (5°N–5°S, 80°W–170°W) calculated on the base of daily «20th century» re-analyses (1871–2012)

there is practically no range of zero inclination. Its presence can be suspected only for the area from a quarter to a third of the year. On an even larger scale, instead of the «spectral plateau», there is an even steeper increase than in the range «$-5/3$» of the spectral density up to the peak of the annual period and beyond. A more detailed speculation of this inclination will be given in the second part of this paper. The ESOI index spectrum (Figure 3, above) differs from the Blinova index spectra much more significantly. The range «$-5/3$» is clearly limited from the low-frequency side by a break during a period of 45 days. Recall that such a position of the low-frequency edge of the «negative viscosity» cascade was assumed in the estimation of the Blinova index spectrum in [20], but it is clearly lacking in those shown in Figure 1 and 2 new estimations. The 45-day spectral density maximum is well known in tropical weather characteristics. Perhaps for the first time this maximum was discovered by R.A. Madden and P.R. Julian [33] in the study of temporal evolution of mesoscale convection foci over the waters of the Indian and West Pacific oceans. Since then, this evolution has been called by their names: Madden – Julian oscillation (MJO). The Madden – Julian oscillation is now being actively studied as one of the possible precursors of El Niño, on the one hand, and as a result of the accumulation of diverse weather fluctuations in tropics, on the other hand (see, for example, [34, 35].) However, so far no attention has been paid to the coincidence of the peak in the MJO spectrum with the low-frequency edge of the inverse energy cascade of weather fluctuations and, at the same time, with superharmonics of 1:8 of the annual period. This double coincidence is important because it indicates that in tropics variations in the atmosphere state with periods of more than 45 days have a different origin compared to shorter-period variations.

Returning to the ESOI spectrum, it should be added that for periods of more than 45 days the spectral density noticeably decreases up to a half-year period. Moreover, rather obvious peaks of superharmonics 1:4 and 1:3 of the annual period are superimposed on the continuous base of the spectrum. Density increases again over periods longer than half a year, as is the case with the EONI spectrum. It is possible that the ESOI spectrum also has a 1:6 superharmonic break. However, it is not worth interpreting it and peaks 1:4, 1:3 as the results of resolving instability of the atmospheric reaction to the annual course of solar flux, because corresponding peaks are visible in the spectrum of solar radiation coming to the Earth (see Figure 2 in [36]). It is easier to interpret them as the results of a simple linear response of the tropical atmosphere to insolation external influence.

At the high-frequency end of the «$-5/3$» scale range, the ESOI spectrum has a downward convex break of spectral density (local minimum). Further, the spectral density increases, but not decreases, up to a period of 5 days. It is worth to say that Madden and Julian have already pointed out in their first publication the presence of a five-day wave in the convective activity in tropics. The ESOI spectrum in Figure 3 shows that actually this is not a wave of a fixed 5-day period, because the maximum near this period is not a peak, but an upward convex break. Thus, actually there should be a whole package of waves with periods close to five days.

Toward periods shorter than 5 days' density in the ESOI spectrum decreases sharply, following the law of minus fourth degree. Such a law was described by P.G. Saffman for fields of two-dimensional turbulence, where concentration of vorticity occurs in thin boundary layers between large vortices [37]. However, A.S. Monin and A.M. Yaglom (see section 26.3 in the book [12]) at their time doubted that such a concentration is actually possible. Nevertheless, it was precisely this kind of concentration of convective activity foci in tropics discovered by Madden and Julian. As far as we know, none of the meteorologists has previously indicated the presence of the «-4» law in the high-frequency parts of the ENSO index spectra.

Summary and Conclusions

- Based on the data of modern re-analyzes of meteorological observations, we have estimated the energy spectra of several indices for the general atmospheric circulation in the range of time scales from two days to one thousand days. The obtained spectra characterize weather variations of extratropical and tropical latitudes. Those are obtained from an incomparably larger amount of initial data than all previously published estimates.
- Spectra of daily, monthly, and seasonal variations in weather outside the tropics are continuous and, moreover, smooth; that is, they lack not only spectral peaks, but even breaks in the course of spectral density. This confirms randomness and, consequently, time-limited predictability of these changes in extratropical weather.
- For changes in extratropical weather from day to day, the average inclinations of all four estimated spectra (for the average and vertical shear flow of Blinova indices at moderate latitudes for both hemispheres) were found to be equal to minus three. For changes with periods of more than a week and up to one or two months – equal to minus five-thirds. This corresponds to the inclinations deduced from the theory of geostrophic turbulence, despite the fact that conditions of homogeneity, isotropy and locality of nonlinear interactions of motions for different scales that are imposed by this theory are not met in a real atmosphere. Parts of all spectra in the period range from one or two months to a year have a seemingly continuous base with a zero inclination, on which superharmonics peaks of the annual period 1:2 and 1:3 are superimposed, and the 1:2 superharmonic for the southern hemisphere is even more powerful than the annual harmonic itself. This indicates presence of some internal «order» in seasonal changes of extratropical weather, and those, on the whole, are also chaotic.
- Spectra of high-frequency variations of tropical weather are estimated using examples of specifically defined El Niño – the Southern Oscillation indices. These spectra are also found to be continuous. The high-frequency part of one of them has an average inclination minus third degree, as in the spectra of extratropical weather. The high-frequency part of the other spectrum has an average inclination minus fourth degree, which was not obtained in any previously performed study of tropical weather. The average inclinations of both spectra' parts in the range from about five to about forty-five days are minus five-thirds. In the range of seasonal changes, the pattern of these spectra is mixed (partly chaotic, partly ordered), since continuous peaks with zero (or even positive) inclination are superimposed on the peaks of numerous superharmonics of the annual period, up to 1:8 superharmonics (period of about 45 days).

- Parts of tropical spectra with different average inclinations are separated from each other by breaks in the course of spectral density. These breaks are visible at periods of five days and forty-five days, those were previously identified only for some local characteristics of tropical weather, such as the Madden–Julian oscillation.

Summing up the results of this research, it is important to emphasize that the low-frequency edge of the reverse energy transfer, that exists (judging by the available empirical data) in the real atmosphere, serves as a barrier to the spread of fast weather changes over longer time scales both outside and inside the tropics. Herein, there is a disconnection of links between the actual weather and its seasonal changes. If forecasts are made by integrating detailed models of atmospheric hydrothermodynamics that actually reproduce the energy spectra of weather variations, then the same barrier should be inherent in the consolidation of weather forecast errors. Therefore, the idea of applying the Lorenz randomness paradigm to seasonal predictions, i.e. inevitably consolidating forecast errors as the lead time increases, seems rather doubtful.

For this reason, the concept of «seamless weather and climate prediction» using the same universal model is unlikely to succeed. Instead, it appears to be promising to develop specific models for different scales of motion in order to exclude from explicit consideration the greatest instabilities of atmospheric processes typical for these scales.

This research was carried out partially under the financial support by the grant of Russian Science Foundation (project No. 14-50-00095).

References

1. *Lorenz E.N.* Atmospheric predictability experiments with a large numerical model // Tellus. 1982. Vol. 34. P. 505–513.
2. *Bengtsson L., Hodges K.I.* A note on atmospheric predictability // Tellus A. 2005. Vol. 58. No. 1. P. 154–157.
3. *Bengtsson L., Hodges K.I., Froude L.S.R.* Global observations and forecasting skill // Tellus A. 2005. Vol. 57. No. 4. P. 515–527.
4. *Froude L.S.R., Bengtsson L., Hodges K.I.* Atmospheric predictability revisited // Tellus A. 2013. Vol. 65i0, doi:10.3402/tellusa.v65i0.19022
5. *Bunimovich L.A.* Short- and long-term forecast for chaotic and random systems // Nonlinearity. 2014. Vol. 27. P. R51–R60.
6. *Cullen, MJ.P.* The Unified Forecast/Climate Model // Meteorol. Magazine. 1993. Vol. 122. P. 81–94.
7. *Palmer T.N., Doblas-Reyes F.J., Weisheimer A., Rodwell M.J.* Toward seamless prediction: calibration of climate change projections using seasonal forecasts // Bull. Amer. Meteorol. Soc., 2008. Vol. 89. No. 4. P. 459–470.
8. *Arnold V.I., Afraimovich V.S., Ilyashenko Yu.S., Shilnikov L.P.* Teoriya bifurkatsiy // in the book: Dinamicheskiye sistemy–5, Itogi Nauki i Tekhniki. Ser. Sovrem. probl. mat. fund. napr., 5, M.: VINITI. 1986. P. 5–218 (in Russian).
9. *Eliasen E., Machenhauer B.* On the observed large-scale atmospheric wave motions // Tellus. 1969. Vol. 21. P. 149–166.

10. *Wiin-Nielsen A.* On the annual variation and spectral distribution of atmospheric energy // Tellus. 1967. Vol. 19. P. 540–558.
11. *Wiin-Nielsen A., Chen T.-C.* Fundamentals of Atmospheric Energetics // New York, NY: Oxford University Press, 1993. 376 p.
12. *Monin A.S., Yaglom A.M.* Statisticheskaya gidromekhanika // 2nd ed., St. Petersburg: Gidrometeoizdat, 1996. Vol. 2. 742 p. (in Russian).
13. *Mirabel A.P., Monin A.S.* // Uspekhi Mekhaniki. 1979. Vol. 2. No. 3. P. 47–95 (in Russian).
14. *Danilov S.D., Gurarie D.* Quasi-two-dimensional turbulence // Phys. Usp. 2000. 43. P. 863–900.
15. *Lilli D.K.* Numerical simulation of two-dimensional turbulence // Phys. Fluids, Suppl. II. 1969. Vol. 12. II-240 – II-249.
16. *Shapovalova N.S.* // Proceedings of the Hydrometeorological Center of the USSR. 1987. Issue. 278. Pp. 90–99 (in Russian).
17. *Taylor G.I.* The spectrum of turbulence // Proc. R. Soc. London. Ser A. Vol. 164. P. 476–490.
18. *Starr V.P.* Physics of phenomena with negative viscosity // Moscow: Mir, 1971. 130 p.
19. *Monin A.S.* Hydrodynamic theory of short-range weather forecasts // Sov. Phys. Usp. 1969. 11. P. 746–767.
20. *Vinogradskaya A.A., Vlasova I.L., Datsenko N.M., Sonechkin D.M.* // Proceedings of the USSR Hydrometeorological Center. 1988. Issue. 297. P. 150–165 (in Russian).
21. *Ogura Y.* Energy transfer in a normally distributed and isotropic turbulent velocity field in two dimension // Physics Fluids. 1962. Vol. 5. No. 4. P. 395–401.
22. *Namias J.* The index cycle and its role in the general circulation // J. Meteorol. 1950. Vol. 8. P. 131–140.
23. *Webster P.J., Keller J.L.* Atmospheric variations: vacillation and index cycle // J. Atmos. Sci. 1975. Vol. 32. P. 1283–1300.
24. *Dickey J.O., Ghil M., Marcus S.L.* Extratropical aspects of the 40-50 day oscillation in length-of-day and atmospheric angular momentum // J. Geophys. Res.: Atmosphere. 1991. Vol. 96. No. D12. P. 22643–22658.
25. *Lovejoy S., Schertzer D.* Scale invariance in climatological temperatures and the spectral plateau // Annales Geophysicae. 1986. Vol. 4B. P. 401–410.
26. *Blinova E.N., Marchuk G.I.* // Proceedings of the Institute of Atmospheric Physics of the Academy of Sciences of the USSR. 1958. No. 2. P. 106–113 (in Russian).
27. *Marchuk G.I.* // Proceedings of the Institute of Atmospheric Physics of the Academy of Sciences of the USSR. 1958. No. 2. P. 114–118 (in Russian).
28. *Compo G.P., Whitaker J.S., Sardeshmukh P.D. et al.* The Twentieth Century Reanalysis Project // Quarterly J. Roy. Meteorol. Soc. 2011. 137. P. 1–28.
29. *Saltzman B., Fleisher A.* The exchange of kinetic energy between larger scales of atmospheric motions // Tellus. 1960. Vol. 12. P. 374–377.
30. *Saltzman B., Teweles S.* Further statistics on the exchange of kinetic energy between harmonic components of the atmospheric flow // Tellus. 1964. Vol. 16. P. 432–435.

31. *Silberman I.* Planetary waves in the atmosphere // J. Meteorol. 1954. Vol. 11. P. 27–34.
32. *Elsaesser H.W.* Evaluation of spectral versus grid methods of hemispheric numerical weather prediction // J. Appl. Meteorol. 1966. Vol. 5. P. 246–262.
33. *Madden R.A., Julian P.R.* Observations of the 40-50 day tropical oscillation – a review // Mon. Wea. Rev. 1994. Vol. 122. P. 814–837.
34. *Donald A., Meinke H., Power N., de Maia H.N., Wheeler M.C., White N., Stone R.C., Ribbe J.* Near-global impact of the Madden – Julian Oscillation on rainfall // Geophys. Res. Lett. 2006. Vol. 33. L09704.
35. *Zhang C.* Madden – Julian Oscillation. Bringing weather and climate // Bull. Amer. Meteor. Soc. 2013. Vol. 94. P. 1849–1870.
36. *Huybers P., Curry W.* Links between annual, Milankovitch and continuum temperature variability // Nature. 2006. Vol. 441. P. 329–332.
37. *Saffman P.G.* A note on the spectrum and decay of random two-dimensional vorticity distributions at large Reynolds number // Studies in Applied Mathematics // 1971. Vol. 50. No. 4. P. 377–383.