Izvestiya Vysshikh Uchebnykh Zavedeniy. Applied Nonlinear Dynamics. 2022;30(1)

Article
DOI: 10.18500/0869-6632-2022-30-1-37-56

Correlation of attracting sets of tool deformations with spatial orientation of tool elasticity and regeneration of cutting forces in turning

D. V. Zakovorotny, V. Gvindjiliya✉

Don State Technical University, Rostov-on-Don, Russsia
E-mail: vzakovorotny@dstu.edu.ru, ✉sinedden@yandex.ru
Received 14.10.2021, accepted 05.12.2021, published 31.01.2022

Abstract. Nowadays, the dynamic cutting system is represented in the form of two subsystems — tool and workpiece, interacting through a nonlinear relationship formed by the cutting process. Such a representation determines the importance of studying the dynamics of the cutting process as the main factor influencing the efficiency of machines, the trajectories of the executive elements of which are set by CNC and are provided with high accuracy. However, in order to improve the efficiency of cutting, it is necessary to align the trajectories of the executive elements are defined by CNC with the changing dynamics of cutting, which introduces deviations in the program-defined trajectories. *Purpose* of this article is to consider the dependence of the dynamics of the cutting process on the spatial orientation of the cutting tool elasticity and the regenerative effect, and to find out the effect of the proposed dependence on the efficiency of the cutting process. All the issues discussed in the article are analyzed using the example of external shaft turning. *Methods*. The study is based on the methods of mathematical modeling and experimental dynamics. In contrast to the known studies, the dependence of the turnover lag time on the oscillatory displacements in the direction of the cutting speed, as well as the influence of the positive feedback formed in this case, is taken into account. In addition, changes in the sign of the internal feedback from the direction of deformations, as well as the influence of the regenerative effect on the generated attracting sets of deformations are taken into account. *Results*. Dependence of the system evolution on the elements of the stiffness matrix at different spindle speeds is disclosed. The properties of the system evolution depending on the ratio of the spindle rotation frequency and the eigenfrequencies of the tool subsystem, as well as the spatial distribution of the stiffness are studied. *Conclusion*. The frequency and time characteristics of the system are discussed. Conclusion is made about the possibility of efficiency increasing of the cutting process based on the coordination of the CNC program with the dynamic properties of the system.

Keywords: effect of regeneration of cutting forces, stability and attracting sets of deformations, cutting process efficiency.

Acknowledgements. This work was supported by Russian Foundation for Basic Research, grants No 19-08-00022 and No 20-38-90074.

For citation: Zakovorotny VL, Gvindjiliya VE. Correlation of attracting sets of tool deformations with spatial orientation of tool elasticity and regeneration of cutting forces in turning. Izvestiya VUZ. Applied Nonlinear Dynamics. 2022;30(1):37–56. DOI: 10.18500/0869-6632-2022-30-1-37-56

Introduction

The dynamic cutting system (DSR), in which the main attention is paid to the stability of movement and the formed attractive sets of deformations, has attracted the attention of researchers since the middle of the last century [1–9]. This is due to the fact that the properties of DSR affect the processing efficiency [10–16]. They depend on the dynamic coupling, which is modeled by the forces $\mathbf{F} = \{F_1, F_2, F_3\}^{\mathrm{T}} \in \mathfrak{R}^{(3)}$, represented as their dependence on deformations

$\mathbf{X} = \{X_1, X_2, X_3\}^{\mathrm{T}} \in \mathfrak{R}^{(3)}$ and $\mathbf{dX/dt} = \mathbf{v}_X = \{v_{X,1}, v_{X,2}, v_{X,3}\}^{\mathrm{T}} \in \mathfrak{R}^{(3)}$, trajectories of machine actuating elements (TIES) and uncontrollable disturbances. TIES, as a rule, are set by a CNC computer system and, when turning, represent a set of movement trajectories $\mathbf{L} = \{L_1, L_2, L_3\}^{\mathrm{T}} \in \mathfrak{R}^{(3)}$ and speeds $\mathbf{V} = \{V_1, V_2, V_3\}^{\mathrm{T}} \in \mathfrak{R}^{(3)}$ of longitudinal, transverse calipers and spindle rotation. Thus, the trajectories of the tool tip relative to the workpiece, which we call the trajectories of shaping movements, are determined by the sum of $\mathbf{L}^{(\Phi)} = \mathbf{L} - \mathbf{X}$ and $\mathbf{V}^{(\Phi)} = \mathbf{V} - \mathbf{dX/dt}$. Deformations are considered in a moving coordinate system, the movement of which is determined by the TIEP. At the same time, the principle of dividing the movements of [16] into "slow"(TIES) is used and "fast"(tool deformations) [17, 18]. The following factors causing the loss of stability are considered. The forces vary when changing $S(t)$ — the area of intersection of the front face of the tool by the workpiece. The change of forces is delayed with respect to the variations of $S(t)$ [1, 2, 7, 19–25]. The effects of the regeneration of forces caused by the trace from the vibrations of the instrument on the previous revolution are considered. The studies were carried out under the assumption that the turnover time $T = \mathrm{const}$ [26–35] , and $v_{X,3}$ were not taken into account. Note that the very kinematics of the formation of the $S(t)$ feed leads to the need to use a ratio that takes into account the effect of force regeneration

$$S(t) = \int\limits_{t-T}^{t} \{V_2(\xi) - v_{X,2}(\xi)\} d\xi. \tag{1}$$

Finally, the nonlinear characteristics of the change of forces from velocity are taken into account. Modified Rayleigh, van der Pol equations and models of reversible friction are used to explain the loss of stability and the formation of attracting sets [36–43]. The parametric self-excitation [44, 45] is taken into account, which is formed starting from a certain critical velocity. The influence of periodic disturbances of [46–48], which form such effects as synchronization, asynchronous interaction, vibration stabilization in the low-frequency region, etc., was also considered.

The analysis shows that the forces depending on deformations and TIES form an intra-system feedback, the properties of which affect the attractive sets of deformations formed during cutting. The study of stability and attracting sets, taking into account the noted features, complements the knowledge of nonlinear cutting dynamics, which allows improving the output properties of processing, which determines the content of the article.

1. Mathematical modeling

The properties of the system can be disclosed based on the use of the following model (рис. 1) [10, 22, 44, 49]:

$$\mathbf{m}\frac{d^2\mathbf{X}}{dt^2} + \mathbf{h}\frac{d\mathbf{X}}{dt} + \mathbf{cX} = \mathbf{F}_\Sigma. \tag{2}$$

Here $\mathbf{m}$, $\mathbf{h}$, $\mathbf{c}$ are symmetric, positive definite matrices of inertial, velocity and elastic coefficients, respectively: $\mathbf{m} = [m_s]$ in [kgf^2/mm], $m_s = m$, $s = 1, 2, 3$; $\mathbf{h} = [h_{s,l}]$ in [kgf/mm]; $\mathbf{c} = [c_{s,l}]$ in [kg/mm], $s, l = 1, 2, 3$; $\mathbf{F}_\Sigma = \mathbf{F} + \mathbf{\Phi} + \mathbf{\Phi}^1$; $\mathbf{F} = \{F_1, F_2, F_3\}^{\mathrm{T}} \in \mathfrak{R}^{(3)}$ — the force formed in the area of the front face; $\mathbf{\Phi}, \mathbf{\Phi}^1$ — forces acting on the back faces. The projections $\mathbf{F}$ are determined by the coefficients χ_i satisfying the condition $\sum\limits_{i=1}^{i=3} (\chi_i)^2 = 1$, that is, $\mathbf{F(t)} = \{F_1, F_2, F_3\}^{\mathrm{T}}$. For further it is necessary, in addition to (1), to link the TPP with technological modes: depth $t_p(t)$ and speed $V_p(t)$ cutting

$$V_p(t) = V_3(t) - v_{X_3}, \quad t_p(t) = d/2 - \int\limits_0^t \{V_1(\xi) - v_{X_1}(\xi)\} d\xi, \tag{3}$$

where $V_3(t) = \pi D \Omega$, Ω — the rotation frequency of the workpiece in [c^{-1}]. If in (1) and (3) $DX_i/dt = 0$ and $V_i = \text{const}, i = 1, 2, 3$, then we denote: $S_p^{(0)} = V_2 T$, $t_p^{(0)} = d/2 - L_1(0)$, $V_p^{(0)} = V_3$.

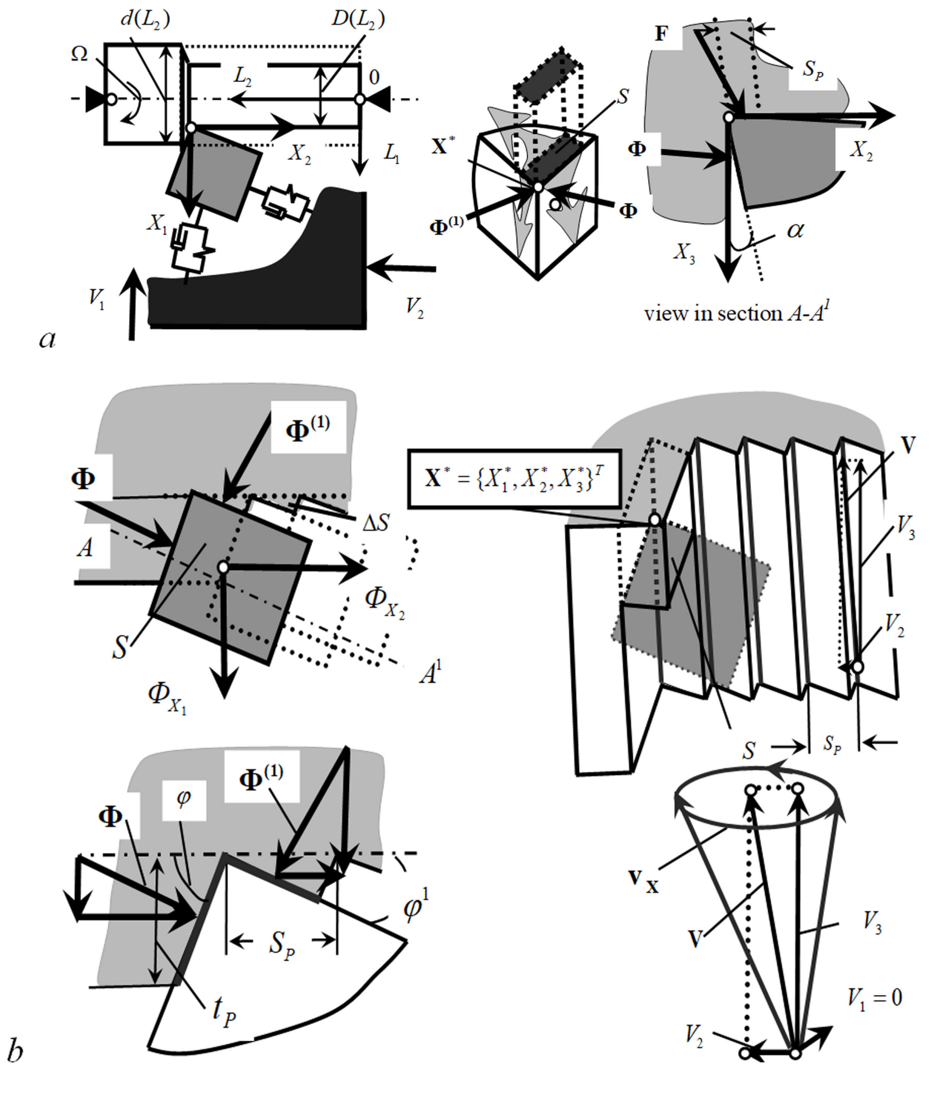


Fig. 1. Схема преобразования траекторий формообразующих движений в силы резания: *a* — схема взаимосвязи сил и деформационных смещений; *b* — силы в области задних граней

Fig. 1. Scheme of transformation of trajectories of the shape-generating movements into cutting forces: *a* — scheme of the interrelation of forces and deformation displacements; *b* — forces in the area of the rear edges

Let's reveal the dependence of $F^{(0)}(t)$ on deformations and thermal power plants

$$T^{(0)}dF^{(0)}/dt + F^{(0)} = \rho\{1 + \mu\exp[-\zeta(V_3 - v_{X_3})]\}[t_p^{(0)} - X_1]\int\limits_{t-T}^{t}\{V_2(\xi) - v_{X_2}(\xi)\}d(\xi), \tag{4}$$

where ρ — pressure in [kg/mm^2]; ζ — steepness parameter in [s/mm]; μ — dimensionless coefficient; $T^{(0)}$ — parameter, determining the lag of forces. If the equilibrium is $\mathbf{X}^* = \{X_1^*, X_2^*, X_3^*\}^{\mathbf{T}}$ (fig. 1, *b*) is asymptotically stable, then after transients in the workpiece, a stationary direction of movement and the corresponding trajectory of the tool trace are formed. Then variations in the position of the tool relative to the trace cause the formation of forces $\Phi(t)$ and $\Phi(t)^{(1)}$ acting on the main and auxiliary back faces. Consider turning with tetrahedral plates made of a hard alloy. Then the connection between the main φ and the auxiliary φ^1 angles $\varphi^1 = \pi/2 - \varphi$ is obvious. Since longitudinal turning is considered, the velocity $V_1 = 0$ (see Fig. 1, *b*). First, we calculate the orientation angles of the velocity vector with respect to the front and rear faces. Here, the positive direction is determined by the direction of convergence of the front α_Σ and the rear α_Σ^1 faces. From the geometric relations we determine

$$\begin{cases} \alpha_\Sigma(t) = \alpha + \Delta\alpha(t) = \alpha + \operatorname{arctg}\left\{\dfrac{v_{X,1}(t)\cos\varphi + [V_2 - v_{X,2}(t)]\sin\varphi}{V_3 - v_{X,3}(t)}\right\}, \\ \alpha_\Sigma^1(t) = \alpha^1 + \Delta\alpha^1(t) = \alpha^1 + \operatorname{arctg}\left\{\dfrac{v_{X,1}(t)\sin\varphi + [V_2 - v_{X,2}(t)]\cos\varphi}{V_3 - v_{X,3}(t)}\right\}. \end{cases} \tag{5}$$

Modules $\Phi, \Phi^{(1)}$ can be approximated by the law of the exponent increasing with decreasing $\alpha_\Sigma(t)$, $\alpha_\Sigma^1(t)$. Moreover, the angles are determined in sections $(A - A^1)$ normal to the cutting blades (Fig. 1, *a*). Since the back angles are small quantities, we have expressions for additional forces $\mathbf{\Phi}_X = \{\Phi_{X_1}, \Phi_{X_2}\Phi_{X_3}\}^{\mathrm{T}} \in \Re^{(3)}$ in the function TIES and $\mathbf{X}$

$$\begin{cases} \Phi_{X_1}(t) = \rho_0\{(t_p^{(0)} - X_1)\operatorname{ctg}\varphi\exp(-a\alpha_\Sigma) + \exp(-a\alpha_\Sigma^1)\displaystyle\int\limits_{t-T}^{t}[V_2(\xi) - v_{X_2}(\xi)]d\xi\}, \\ \Phi_{X_2}(t) = \rho_0\{(t_p^{(0)} - X_1)\exp(-a\alpha_\Sigma) - \exp(-a\alpha_\Sigma^1)\displaystyle\int\limits_{t-T}^{t}[V_2(\xi) - v_{X_2}(\xi)]d\xi\operatorname{ctg}\varphi\}, \\ \Phi_{X_3}(t) = k_{\mathrm{fr}}\{\Phi_{X_1}(t) + \Phi_{X_2}(t)\}, \end{cases} \tag{6}$$

where α is the coefficient of steepness; ρ_0 is a parameter that has the meaning of rigidity; k_{fr} is the coefficient of friction. The forces of $\mathbf{\Phi}_X$ limit the development of deformations. The systems (2), (4), (6) allow us to explore the trajectories of $X, F, \Phi, \Phi^{(1)}$, as well as analyze the attracting sets of deformations and their evolution during the transition from one stationary state to another. These properties of evolution differ from those considered in [48, 50].

2. Stability of equilibrium

The interaction depends on the trajectories of $L^{(\Phi)}$ and $V^{(\Phi)}$. The system will be considered stable if the stable equilibrium point is X^* in a moving coordinate system. Let us confine ourselves to the consideration of an undisturbed system on constant modes. To analyze the stability, it is necessary to determine the linearized equation in variations [51]. For the point X^* it is true:

$v_{X_i} = 0,\ i = 1,2,3,\ dF^{(0)} = 0$. A direction is formed on the workpiece in which $\Phi \Rightarrow 0,\ \Phi_{(1)} \Rightarrow 0$. Then X^*:

$$c_\Sigma X^* = \rho^{(0)} S_p^{(0)} t_p^{(0)} \{\chi_1, \chi_2, \chi_3\}^{\mathrm{T}}, \tag{7}$$

где

$$c_\Sigma = \begin{bmatrix} c_{1,1} + \chi_1 \rho^{(0)} S_p^{(0)} & c_{2,1} & c_{3,1} \\ c_{1,2} + \chi_2 \rho^{(0)} S_p^{(0)} & c_{2,2} & c_{3,2} \\ c_{1,3} + \chi_3 \rho^{(0)} S_p^{(0)} & c_{2,3} & c_{3,3} \end{bmatrix}, \quad \rho^{(0)} = \rho\{1 + \mu \exp[-\zeta V_3]\}.$$

The solution (7) is unique and $X^* = \mathrm{const}$. Then the linearized equation in variations has constant parameters, and the system can be considered as a subsystem of an instrument with feedback. Therefore, the Nyquist frequency criterion [52] can be used for stability analysis, and the amplitude-phase frequency response (AFC) of the system in the open state $W_\Sigma(jw)$ represent as

$$W_\Sigma(jw) = \frac{\rho}{(1 + T^{(0)}\omega j)} \Big\{ g_1 S_p^{(0)} W_1(j\omega) + g_2 t_p^{(*)} [1 - \exp(-T\omega j)] \times \\ \times W_2(j\omega) - g_3 S_p^{(0)} t_p^{(*)} \mu(\zeta)^{(-1)} W_3(j\omega) j\omega \Big\}, \tag{8}$$

где

$$t_p^{(*)} = t_p^{(0)} - X_1^*; \quad g_1 = \Delta_{g_1}/\Delta_g; \quad g_2 = \Delta_{g_2}/\Delta_g; \quad g_3 = \Delta_{g_3}/\Delta_g; \quad \Delta_g = [c_{i,s}], \quad i,s = 1,2,3;$$

$$W_{0,i}(p)_{p=j\omega} = \Delta_i(p)/\Delta(p); \quad W_i(p)_{p=j\omega} = g_i W_{0,i}(p)_{p=j\omega}; \quad i = 1,2,3;$$

$$\Delta_{g_1} = \begin{bmatrix} \chi_1 & c_{2,1} & c_{3,1} \\ \chi_2 & c_{2,2} & c_{3,2} \\ \chi_3 & c_{2,3} & c_{3,3} \end{bmatrix}; \quad \Delta_{g_2} = \begin{bmatrix} c_{1,1} & \chi_1 & c_{3,1} \\ c_{1,2} & \chi_2 & c_{3,2} \\ c_{1,3} & \chi_3 & c_{3,3} \end{bmatrix}; \quad \Delta_{g_3} = \begin{bmatrix} c_{1,1} & c_{2,1} & \chi_1 \\ c_{1,2} & c_{2,2} & \chi_2 \\ c_{1,3} & c_{2,3} & \chi_3 \end{bmatrix}; \tag{9}$$

$$\Delta_1 = \begin{bmatrix} \chi_1 & h_{2,1}p + c_{2,1} & c_{3,1} \\ \chi_2 & mp^2 + h_{2,2}p + c_{2,2} & c_{3,2} \\ \chi_3 & mp^2 + h_{2,3}p + c_{2,3} & c_{3,3} \end{bmatrix};$$

$$\Delta_2 = \begin{bmatrix} mp^2 + h_{1,1}p + c_{1,1} & \chi_1 & h_{3,1}p + c_{3,1} \\ h_{1,2}p + c_{1,2} & \chi_2 & h_{3,2}p + c_{3,2} \\ h_{1,3}p + c_{1,3} & \chi_3 & mp^2 + h_{3,3}p + c_{3,3} \end{bmatrix};$$

$$\Delta_3 = \begin{bmatrix} mp^2 + h_{1,1}p + c_{1,1} & h_{2,1}p + c_{2,1} & \chi_1 \\ h_{1,2}p + c_{1,2} & mp^2 + h_{2,2}p + c_{2,2} & \chi_2 \\ h_{1,3}p + c_{1,3} & mp^2 + h_{2,3}p + c_{2,3} & \chi_3 \end{bmatrix};$$

$$\Delta = \begin{bmatrix} mp^2 + h_{1,1}p + c_{1,1} & h_{2,1}p + c_{2,1} & h_{3,1}p + c_{3,1} \\ h_{1,2}p + c_{1,2} & mp^2 + h_{2,2}p + c_{2,2} & h_{3,2}p + c_{3,2} \\ h_{1,3}p + c_{1,3} & mp^2 + h_{2,3}p + c_{2,3} & mp^2 + h_{3,3}p + c_{3,3} \end{bmatrix}.$$

Transfer functions $W_i(p)$ can be represented as

$$W_i(p) = \frac{(1 + 2\xi_1^{(i)} T_1^{(i)} p + (T_1^{(i)})^2 p^2)(1 + 2\xi_2^{(i)} T_2^{(i)} p + (T_2^{(i)})^2 p^2)}{(1 + 2\xi_1 T_1 p + (T_1)^2 p^2)(1 + 2\xi_2 T_2 p + (T_2)^2 p^2)(1 + 2\xi_3 T_3 p + (T_3)^2 p^2)}, \quad i = 1,2,3.$$

Moreover, $\Omega_i = (T_i)^{(-1)},\ i = 1,2,3,$ — resonances, and $\Omega_i^{(s)} = (T_i^{(s)})^{(-1)},\ i,s = 1,2,3,$ — antiresonances. In traditional lathes of the turning group, the condition $\Omega = T^{(-1)} \ll \Omega_i$ is met. Therefore, it makes sense to consider two modes: $t < T$ and $t > T$. Usually $S_p^{(0)} \ll t_p$, $\mu(\zeta)^{(-1)} \to 0$. Then $W_\Sigma(j\omega) \approx \frac{\rho}{(1+T^{(0)}\omega j)} \left\{ g_2 t_p^{(*)} \left[1 - \exp(-T\omega j)\right] \right\}$. For this case, in Fig. 2 an

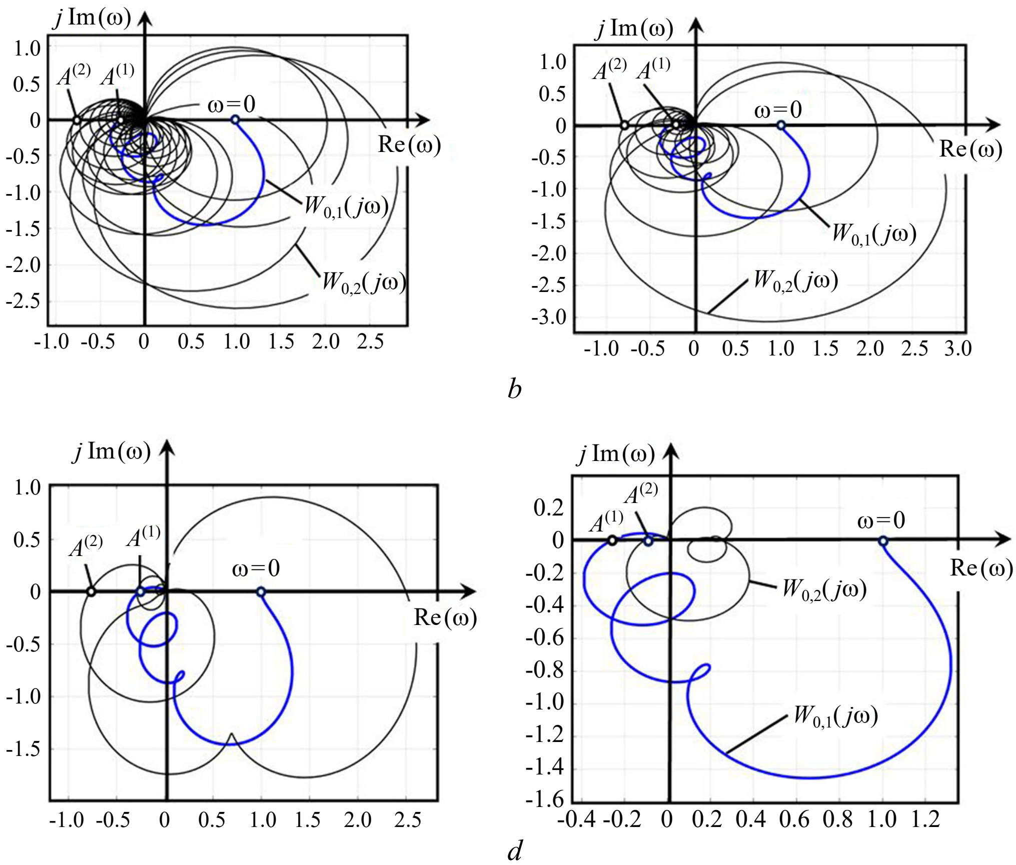


Fig. 2. АФЧХ $W_{(0,1)}(jw)$ и $W_{(0,2)}(jw)$ при параметрах $T_1 = 1$, $T_2 = 3$, $T_3 = 5$, $T_{(1)}^{(2)} = 2$, $T_{(2)}^{(2)} = 4$, $\xi_i = 0.08$, $i = 1, 2, 3$; $\xi_s = 0.08$, $s = 1, 2$: $T = 100$ (*a*), 50 (*b*), 1 (*c*), 0.5 (*b*)

Fig. 2. Nyquist plot $W_{(0,1)}(jw)$ и $W_{(0,2)}(jw)$ with parameters $T_1 = 1$, $T_2 = 3$, $T_3 = 5$, $T_{(1)}^{(2)} = 2$, $T_{(2)}^{(2)} = 4$, $\xi_i = 0.08$, $i = 1, 2, 3$; $\xi_s = 0.08$, $s = 1, 2$: $T = 100$ (*a*); 50 (*b*); 1 (*c*); 0.5 (*b*)

example of AFC $W_{0,1} = \frac{\rho}{(1+T^{(0)}\omega j)} \left\{ g_2 t_p^{(*)} W_2(j\omega) \right\}$ and $W_{0,2} = W_\Sigma(j\omega)$. The illustrations in Fig. 2, *a, b* refer to the case of $\Omega \ll \Omega_i$, in Fig. 2, *c, d* — to the case of $\Omega > \Omega_i$. If $\Omega \ll \Omega_i$, then the AFC can allocate frequencies $\omega_i = 2\pi(T_i)^{(-1)}$, on which $\{1 - \exp(-T\omega j)\} = 0$. Cycloid-type curves are formed in the vicinity of ω_i points. Due to the rapid rotation of the phase in the vicinity of ω_i, variations of Ω practically do not change the tendency to self-excitation. If $\Omega > \Omega_i$, then reducing T can increase the stability margin (compare Fig. 2, *a, b, c, d*). Even small variations of T can affect stability. However, the increase in Ω is limited by the parametric self-excitation of [44].

Let's analyze the factors affecting sustainability. Usually, the pulse response time of the instrument subsystem is significantly less than T. Two options can be considered for this case. *The first* option is related to the behavior of the system without taking into account the regenerative effect, that is, on the site $t \in (0, T)$. When processing large diameter parts, we can assume that $T \to \infty$. Here, the loss of stability is caused by phase shifts between the variations of the area $S(t)$ and the forces **F**. Regardless of the method of modeling the phase shift (by introducing an aperiodic link, a "falling"characteristic of the change in forces with increasing speed, etc.), the reaction from the cutting side leads to the transformation of symmetric matrices **c** and **h** into asymmetric ones. Then, firstly, the skew-symmetric components of the matrix formed form circulating forces leading to precessional oscillations, which is always observed in practice. Secondly, the symmetric components of **h** can become negatively defined, that is, accelerating. Then the equilibrium becomes unstable, and it is not possible to stabilize it by gyroscopic forces formed by the skew-symmetric component of the matrix **h**. To increase stability, it is always necessary to increase the elements of the matrices **c** and **h**. *The second* option: the regenerative effect is additionally considered and the stability at $t \to \infty$ is analyzed. Here the regenerative effect depends on the frequency of $\boldsymbol{\Omega}$. It promotes self-excitation in the low-frequency region. However, its influence changes as Ω approaches one of the frequencies of the instrument subsystem. When they are close, the stability becomes sensitive to variations of $\boldsymbol{\Omega}$, and in the region of $\Omega > \Omega_i$, the regenerative effect stabilizes the equilibrium.

3. Evolution of attracting sets

For practice, properties are important not at $t \to \infty$, but in the course of evolution due to the transition from one stationary state to another, for example, when cutting a tool, when processing with changing modes [44, 51] and others. By summing trajectories with a shift of T, synchronous and asynchronous interactions are possible. The circulating forces cause changes in the direction of deformation, which together with the summation of $(\mathbf{X}(t) - \mathbf{X}(t-T))$ form attracting sets that "slowly"rearrange over time. The formation of attracting sets is influenced not only by nonlinear connections (5) and (6), but also by a change in the sign of the force feedback. Finally, the time T depends not only on the frequency $\boldsymbol{\Omega}$, but also on the speed $v_{X,3}/\pi D$, that is, $T = (\Omega - v_{x,3}/\pi D)^{-1}$. The change in T causes a time shift of the trajectories, which introduces additional uncertainty in the sum of $\mathbf{X}(t) - \mathbf{X}(t-T)$. This leads to the need to use digital modeling methods to analyze attractive sets of deformations. Therefore, the study was carried out by direct digital modeling methods using the example of longitudinal turning of a non-deformable shaft made of 45 steel with a diameter of $D = 40$ mm with a constant feed of $S_p^{(0)} = 0.1$ mm. The system parameters are given in Table. 1 and table. 2 [45, 48]. The generalized mass is $m = 0.5 \cdot 10^{-3}$ kg·with ^{2}/mm. When the frequency of $\boldsymbol{\Omega}$ was varied, its matching with V_2 was performed.

ρ, кг/мм^2	ρ_0, кг/мм	ζ, м/с	$\alpha_1 = \alpha_2$, рад$^{-1}$	$T^{(0)}$, с	μ	k_{fr}
50–1200	50	0.1	120	0.0008	0.5	0.2

$h_{1,1}$, кг·с/мм	$h_{2,2}$, кг·с/мм	$h_{3,3}$, кг·с/мм	$h_{1,2} = h_{2,1}$, кг·с/мм	$h_{1,3} = h_{3,1}$, кг·с/мм	$h_{2,3} = h_{3,2}$, кг·с/мм
1.3	1.1	0.8	0.6	0.5	0.4
$c_{1,1}$, кг/мм	$c_{2,2}$, кг/мм	$c_{3,3}$, кг/мм	$c_{1,2} = c_{2,1}$, кг/мм	$c_{1,3} = c_{3,1}$, кг/мм	$c_{2,3} = c_{3,2}$, кг/мм
2000	1500	200–1500	200	150	80

Here are examples of $X_i(t)$ (Fig. 3, fig. 4) when embedding the tool ($T = \text{const}$). Under other unchanged conditions, the stability is affected by the gain factor $k = \rho g_2 t_p^{(*)}$, that is, the cutting depth $t_p^{(0)}$ also affects. A system that is stable at $t_p^{(0)} = 1.0$ mm, becomes unstable at $t_p^{(0)} = 2.0$ mm, and then an attracting set of the type of a two-dimensional invariant torus is formed in the vicinity of equilibrium.

A more visual picture of the transformation is given by the phase portrait (Fig. 4). It is found that the time of establishing a stationary state can significantly increase with an increase in g_2 due to the functional connectivity of forces and deformations. The situation changes if we take into account the dependence of

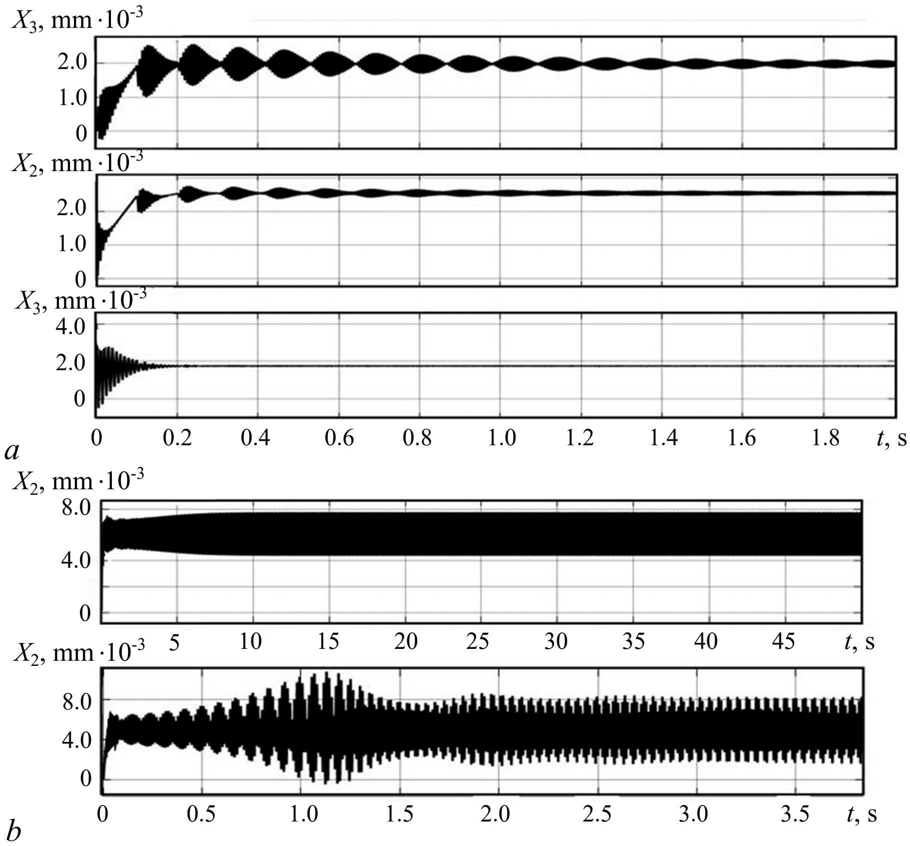


Fig. 3. Примеры переходных процессов деформационных смещений при врезании инструмента в заготовку (ρ = 400 кг/мм^2): $t_p^{(0)} = 1.0$ мм; $T = 1$ (a), 0.5 (b)

Fig. 3. Examples of transient deformation displacements when tools are plunged into the workpiece (ρ = 400 kg/mm^2): $t_p^{(0)} = 1.0$ mm; $T = 1$ (a), 0.5 (b)

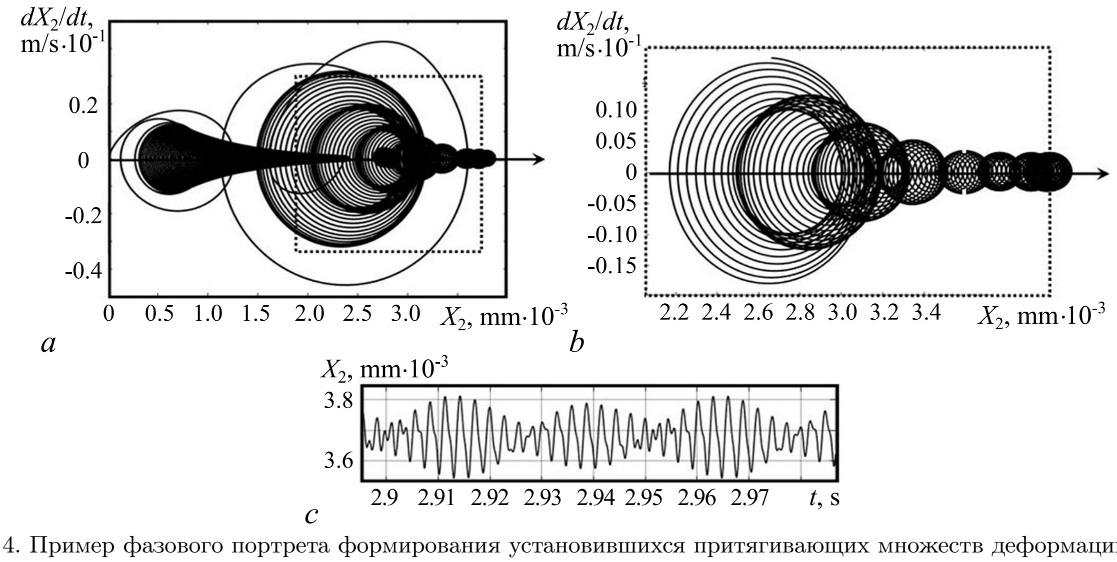


Fig. 4. Пример фазового портрета формирования установившихся притягивающих множеств деформаций (a, b) и фрагмента временной траектории для деформаций $X_2(t)$ (c)

Fig. 4. Example of the phase portrait of the formation of the steady-state attracting sets of deformations (a, b) and fragment of the time trajectory for deformations $X_2(t)$ (c)

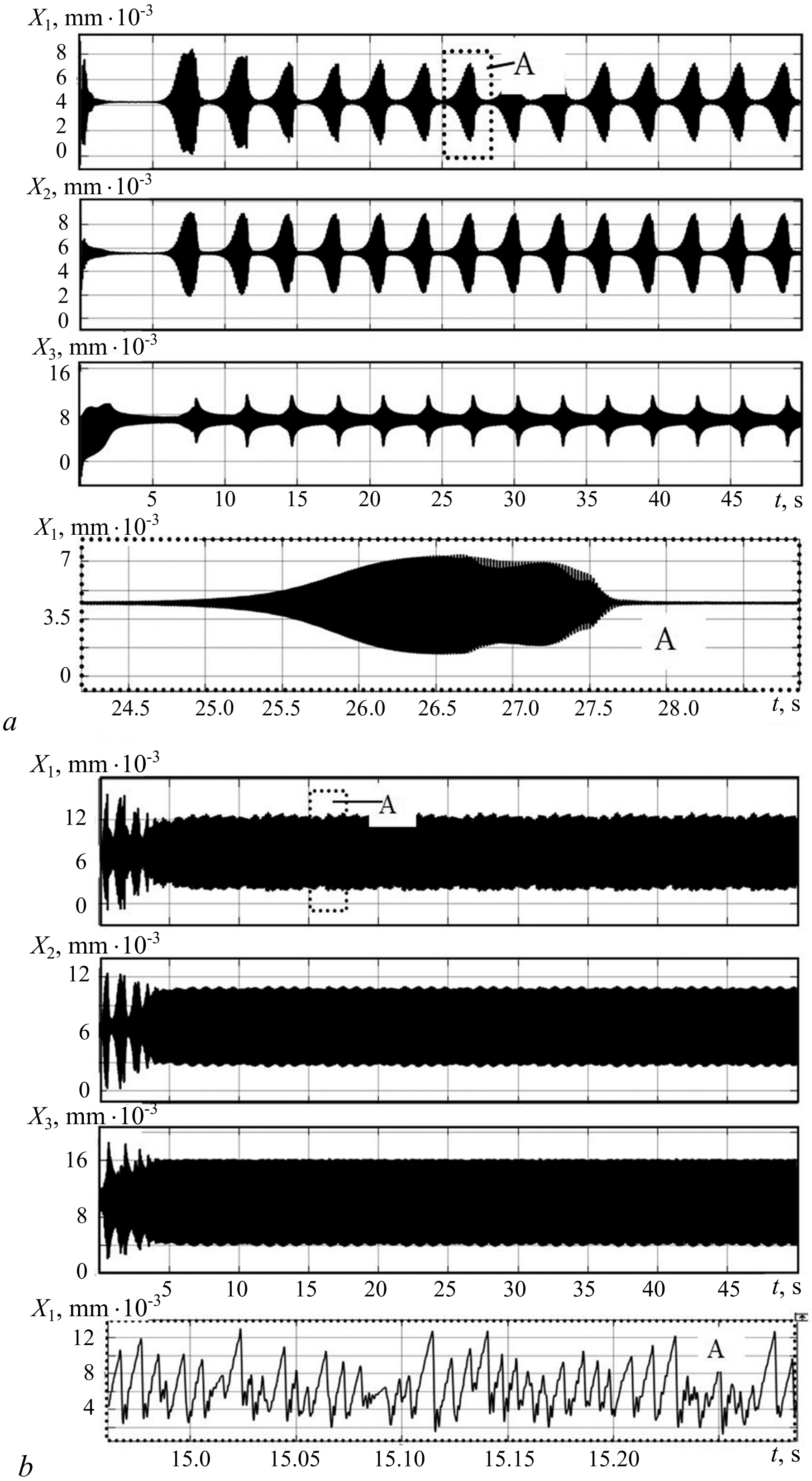


Fig. 5. Примеры переходных процессов деформационных смещений в случае $T = T(\Omega, v_{X,3})$: $k = 1.0$ (a); 2.0 (b)

Fig. 5. Examples of transient deformation displacements in the case of $T = T(\Omega, v_{X,3})$: $k = 1.0$ (a); 2.0 (b)

T not only on Ω, but also on $v_{X,3}$ (Fig. 5). In this case, as $k = \rho g_2 t_p^{(*)}$ increases, the system initially loses stability, then attracting sets of deformations of a complex structure are formed, periodically repeating at a super-low frequency (Fig. 5, *a*, insert "A"). The structure of each fragment of these periodic deformations represents a set of limit cycles with tunable parameters and two-dimensional tori. Perestroika is observed both in time and in space. Therefore, variations in the modulus of deformations are not as noticeable as their periodically recurring changes, for example, in the direction of X_1. With a further increase of $k = 2$ in the system, super-low-frequency attracting sets disappear and chaotic deformations are formed (Fig. 5, *b*). The forces Φ and $\Phi^{(1)}$ take part in the formation of attracting sets. In Fig. 6 an example of a diagram of bifurcations of attracting sets of deformations is given, clearly showing changes in their properties. The points "a" and "b" in Fig. 6 correspond to the trajectory in Fig. 5, *a, b*.

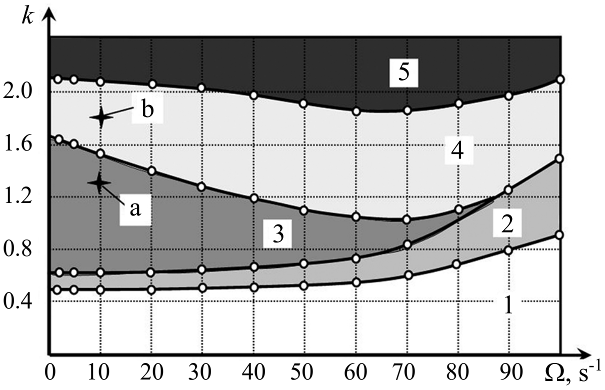
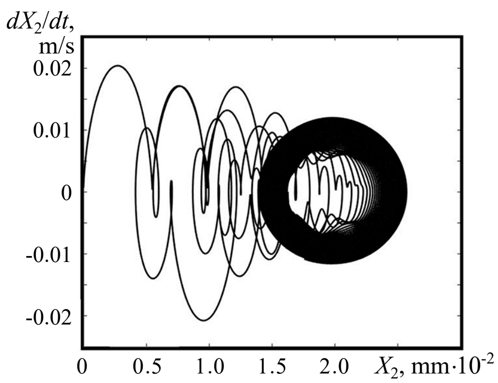


Fig. 6. Диаграмма бифуркаций притягивающих множеств деформационных смещений вершины инструмента: 1 — точка равновесия асимптотически устойчива; 2 — область формирования предельного цикла; 3 — область супернизкочастотных притягивающих множеств; 4 — область хаотической динамики; 5 — область неустойчивой системы в целом

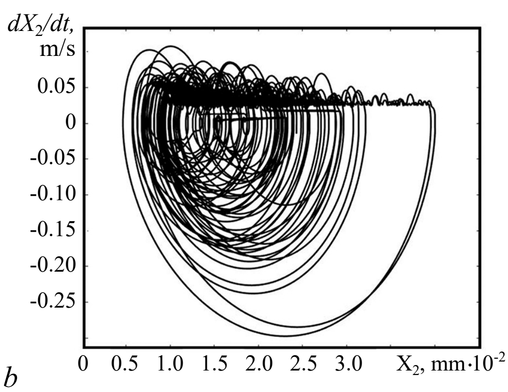
Fig. 6. Bifurcation diagram of the attracting sets of deformation displacements of the tool tip: 1 — asymptotically stable equilibrium point; 2 — region of limit cycle formation; 3 — region of super low-frequency attracting sets; 4 — region of chaotic dynamics; 5 — region of unstable system as a whole

Studies show that the area "3" in Fig. 6 can be leveled if g_3 decreases. Thus, a change in the orientation of deformations in space affects not only the stability, but also the attracting sets. In that case, if the dependence of T on the velocity $v_{X,3}$ is not taken into account when modeling dynamics, then the region "3" in Fig. 6 is not formed.

In connection with the development of wear-resistant tool materials [53] and the improvement of spindle assemblies [54], it became possible to consider modes in which the conditions $\Omega \geqslant (T_i)^{-1}$ are met. They are carried out when drilling holes with low-rigid rods, etc. When stability is lost, a limit cycle is immediately formed here (Fig. 7, *a*). After a certain cascade of doubling of the period, a chaotic dynamics is formed (Fig. 7, *b*), the formation of which is influenced by both the ratio of g_2 and g_3, and the parameters of the equations (6). The formation of super-low-frequency attracting sets and their slow evolution is not observed in this case.



a



b

Fig. 7. Примеры изменения фазовых траекторий при вариации соотношения: *a* — g_2 и *b* — g_3

Fig. 7. Examples of changes in phase trajectories when the ratio: *a* — g_2 and *b* — g_3

4. Analysis of results

The representation of cutting forces in the coordinates of elastic deformations of the tool tip relative to the workpiece, as well as in technological modes determined by the TIES, fundamentally change the paradigm of the analysis of the cutting process. It is considered as a single dynamic system, the coordinates of the state in which the forces are functionally related, and the mathematical modeling of forces reveals the internal feedback in the dynamic cutting system. Depending on the direction of deformations, the sign of the feedback formed by cutting changes. It can promote self-excitation of the system, as well as stabilize the equilibrium. It is shown that an increase in dynamic compliance in the direction of the cutting speed, leading to an increase in the oscillations of $v_{X,3}$ in the same direction, causes a change in the direction of the total speed (see Fig. 7, *a, b*). If the velocity $v_{X,3}$ is formed in a certain segment in the direction of V_3, then after its integration in time, the area of the cut layer increases, which leads to the formation of a positive feedback. However, it must be taken into account that the velocity $v_{X,3}$ in stationary mode cannot have one sign. If the processing time is t_Σ, then the condition $\int_0^{t_\Sigma} v_{X,3}(\xi)d\xi \to 0$ is obvious. In the class of systems under consideration, the function $v_{X,3}$ is periodic. Therefore, temporary areas of self-excitation should be replaced by areas of stabilization. Studies have shown that the noted self-excitation factor causes additional nonlinear interactions, leading to the formation of super-low-frequency attracting sets of a complex time structure. An increase in the excitation coefficient leads to the formation of limit cycles, and with an increase in the excitation coefficients — to the formation of chaotic dynamics. It is formed, as a rule, after cascades of doubling of the oscillation period.

Reducing the malleability of g_1 and g_2 leads to the formation of other effects. Firstly, deformations in these directions reduce the area of the cut layer, that is, they form a negative feedback that can potentially stabilize the equilibrium. Secondly, a decrease in the total stiffness in the direction of X_2 leads to a delay in the transition process of establishing a stationary state. When the tool is embedded in the workpiece, there is a functional coherence of forces and deformations. Therefore, with an increase in g_2, the transition time increases, which limits the possibilities of controlling elastic deformations.

The change in the total velocity of deformation displacements and spindle rotation changes the time window, that is, $T = (\Omega - v_{X,3}/\pi D)^{-1}$ in integral operators of systems (3), (4) and (6), which was considered constant in all previously performed studies [1–7, 10–13, 20–32, 44, 49]. Its change not only contributes to the loss of stability, but also largely determines the topology of the phase space of the system in a steady state, introducing an irregularity in T. Changing the direction of the total velocity leads to the manifestation of additional forces Φ, $\Phi^{(1)}$. They provide nonlinear damping of vibrations and contribute to the formation of various attracting sets in a steady state, including the formation of chaotic attractors (see Fig. 7). The attracting sets of deformations are considered in the mobile coordinate system of the TPP, which are set and provided by the CNC systems of the machine. The analysis of the relationship between TES and deformations is based on the motion separation method based on the asymptotic properties of solutions of nonlinear differential equations with small parameters at higher derivatives [17, 18]. In [16] it is shown that the TIES are characterized by “slow” movements. It is these movements that are provided with high precision in modern CNC machines. "Fast"movements, considered in variations relative to “slow” ones, in metal-cutting machines characterize elastic deformation displacements. They, along with TIES, determine the geometric topology of the surface formed by cutting.

If a geometric topology is analytically defined, then from it, based on the use of various statistical functionals, it is possible to estimate most of the accuracy indicators used in engineering practice for linear dimensions, longitudinal and transverse undulation and in some cases roughness [17, 54]. The performed studies allow us to link the geometric estimates of the part formed by cutting with the attracting sets of deformation displacements. In particular, it is shown that the change in the diameter of the workpiece is determined not only by the displacement of the equilibrium point of the system, but also by the attracting sets formed, since nonlinear functions (6) do not have the property of central symmetry relative to the equilibrium point. Therefore, the effects of dynamic, oscillation-dependent displacement of the equilibrium point are observed, affecting the diameter of the workpiece. When considering the effect of vibrations on the geometric topology, it should be noted that not all vibrations leave a trace on the treated surface. Fluctuations in the directions X_2 and X_3 are collinear to the surface formed by cutting and practically do not directly affect its formation. However, they can stabilize deformations in the orthogonal direction. Moreover, high-frequency fluctuations in the direction of the cutting speed cause the effect of vibrational stabilization of equilibrium in the low-frequency region [13, 45]. The issues of mapping attractive sets of deformation displacements in the geometric topology of the surface formed by cutting are described in detail by us earlier in [16].

The formed attracting sets of deformation displacements affect the power of irreversible energy transformations in the areas of interface of the back faces of the tool and the workpiece. This changes the physical phenomena in the contact area, affecting the wear rate of the tool. As the power of irreversible transformations increases, there is a change in the prevailing physical interactions: the process of mechanical interaction is replaced by molecular adhesion and, finally, molecular diffusion. For example, in [52] it is shown that the minimum wear intensity is observed during the transition from adhesive to diffusion interaction. The attracting sets of deformation displacements formed during cutting, changing the prevailing mechanisms of interactions along with technological modes, affect the optimal parameters of technological modes, primarily the cutting speed. In other words, the wear characteristics also become dependent on the phase trajectory of the power of irreversible transformations in the interface areas of the back faces of the tool and the workpiece, which is affected by the attracting sets. They can be modeled by Volterra integral operators of the second kind with respect to the phase trajectories of the power of irreversible transformations according to the work performed [55, 56]. Therefore, depending on the attracting sets, a correction of the cutting speed is required, at which the wear intensity is minimal.

In the last decade, world scientific centers have developed research on the creation of mathematical and software tools in the direction of virtual research of machining processes on machines [57, 58]. They include algorithms and programs for dynamic analysis, including studies of attracting sets of deformation displacements and forces in the areas of interface of tool and workpiece faces. The use of this toolkit is aimed at developing algorithms for matching the CNC program and cutting dynamics. The methods of correction of the CNC program include the following steps. At the first stage, the cutting speed trajectory is determined, at which the wear intensity is minimized along the trajectory. At the second stage, deformation displacements are provided along the trajectory, at which the geometric topology of the part formed by cutting meets the technical requirements. At the same time, information exchange and adjustment of both the CNC program and control algorithms and identification of dynamic communication parameters are assumed. The purpose of adjustment (approval) is to manufacture a batch of parts while minimizing the above costs and ensuring the required quality of parts. The new knowledge given in the article on the dynamics of cutting, the laws of formation of attracting sets of deformations, their evolution and bifurcations are an integral part of solving this general problem.

Conclusions

Mathematical modeling and experimentally proved that variations in spatial dynamic compliance change the prevailing sources of self-excitation of the cutting system and can change the topology of attracting sets of tool deformations relative to the workpiece, affecting the output characteristics of the cutting process. Unexpected is the proof of the propensity of the system to lose stability due to deformations in the direction of the cutting speed, as well as the formation in this case of super-low-frequency attracting sets of deformations of a complex time structure. It is also proved to increase the stability of the equilibrium if the spindle rotation frequency exceeds the natural frequencies of the oscillatory circuits formed by the tool subsystem. The results obtained open up a new direction of increasing cutting efficiency based on constructive changes in the elastic properties of the tool subsystem, its geometry and the coordination of the CNC program with the dynamic properties of the system.

References

1. Kudinov VA. Dynamics of Machines. Moscow: Mashinostroenie; 1967. 359 p. (in Russian).
2. Hahn RS. On the theory of regenerative chatter in precision-grinding operations. Transactions of American Society of Mechanical Engineers. 1954;76:593–597.
3. Tobias SA, Fishwick W. Theory of regenerative machine tool chatter. The Engineer. 1958;205(7):
199–203.
4. Tobias SA. Machine Tool Vibrations. London: Blackie; 1965. 351 p.
5. Tlustý J, Poláček M, Daněk O, Špaček L. Selbsterregte Schwingungen an Werkzeugmaschinen. Berlin: Verlag Technik; 1962. 431 s. (in German).
6. Tlustý J, Ismail F. Basic non-linearity in machining chatter. CIRP Annals. 1981;30(1):299–304. DOI: 10.1016/S0007-8506(07)60946-9.
7. Merritt HE. Theory of self-excited machine-tool chatter: Contribution to machine-tool chatter research. ASME Journal of Engineering. 1965;87(4):447–454. DOI: 10.1115/1.3670861.
8. Altintas Y, Budak E. Analytical prediction of stability lobes in milling. CIRP Annals. 1995;44(1):
357–362. DOI: 10.1016/S0007-8506(07)62342-7.
9. Eliasberg ME. Self-Oscillation of Machine Tools: Theory and Practice. Saint Petersburg: Osoboe KB Stankostroeniya; 1993. 180 p. (in Russian).
10. Zakovorotny VL, Fam DT, Bykador VS. Self-organization and bifurcations of dynamical metal cutting system. Izvestiya VUZ. Applied Nonlinear Dynamics. 2014;22(3):26–39 (in Russian). DOI: 10.18500/0869-6632-2014-22-3-26-39.
11. Zakovorotny VL, Gubanova AA, Lukyanov AD. Stability of shaping trajectories in milling: Synergetic concepts. Russian Engineering Research. 2016;36(11):956–964.
DOI: 10.3103/S1068798X16110216.
12. Zakovorotnyi VL, Gubanova AA, Luk'yanov AD. Parametric self-excitation of a dynamic end-milling machine. Russian Engineering Research. 2016;36(12):1033–1039.
DOI: 10.3103/S1068798X16120194.
13. Zakovorotny VL, Gvindzhiliya VE. Influence of spindle wobble in a lathe on the tool's deformational-displacement trajectory. Russian Engineering Research. 2018;38(8):623–631. DOI: 10.3103/S1068798X1808018X.
14. Zakovorotny VL, Gvindjiliya VE. Link between the self-organization of dynamic cutting system and tool wear. Izvestiya VUZ. Applied Nonlinear Dynamics. 2020;28(1):46–61 (in Russian). DOI: 10.18500/0869-6632-2020-28-1-46-61.
15. Veits VL, Vasilkov DV. Dynamics, modeling and quality assurance tasks in the machining of low-rigidity workpieces. Russian Engineering Research. 1999;(6):9–13 (in Russian).
16. Zakovorotny VL, Fleck MB. The Dynamics of the Cutting Process. Synergetic Approach. Rostov-on-Don: Terra; 2006. 876 p. (in Russian).
17. Pontryagin LS. Selected Works of L.S. Pontryagin. Moscow: MAKS Press; 2004. 551 p. (in Russian).
18. Tikhonov AN. Systems of differential equations with small parameters in senior derivatives. Mathematics of the USSR — Sbornik. 1952;31(3):575–586 (in Russian).
19. Lipski J, Litak G, Rusinek R, Szabelski K, Teter A, Warminski J, Zaleski K. Surface quality of a work material influence on vibrations in a cutting process. Journal of Sound and Vibration. 2002;252:729–737. DOI: 10.1006/jsvi.2001.3943.
20. Gorodetsky YI. Theory of nonlinear oscillations and machine tool dynamics. Vestnik of

Lobachevsky University of Nizhni Novgorod. Series: Mathematical Modeling and Optimal Control. 2001;(2):69–88 (in Russian).
21. Balachandran B. Nonlinear dynamics of milling processes. Phil. Trans. R. Soc. A. 2001;359(1781): 793–819. DOI: 10.1098/rsta.2000.0755.
22. Litak G, Rusinek R. Dynamics of a stainless steel turning process by statistical and recurrence analyses. Meccanica. 2012;47(6):1517–1526. DOI: 10.1007/s11012-011-9534-x.
23. Gouskov AM, Voronov SA, Paris H, Batzer SA. Nonlinear dynamics of a machining system with two interdependent delays. Communications in Nonlinear Science and Numerical Simulation. 2002;7(4):207–221. DOI: 10.1016/S1007-5704(02)00014-X.
24. Voronov SA, Ivanov II, Kiselev IA. Investigation of the milling process based on a reduced dynamic model of cutting tool. Journal of Machinery Manufacture and Reliability. 2015;44(1): 70–78. DOI: 10.3103/S1052618815010100.
25. Zakovorotnyi VL, Lukyanov AD, Gubanova AA, Khristoforova VV. Bifurcation of stationary manifolds formed in the neighborhood of the equilibrium in a dynamic system of cutting. Journal of Sound and Vibration. 2016;368:174–190. DOI: 10.1016/j.jsv.2016.01.020.
26. Litak G. Chaotic vibrations in a regenerative cutting process. Chaos, Solitons & Fractals. 2002;13(7):1531–1535. DOI: 10.1016/S0960-0779(01)00176-X.
27. Namachchivaya NS, Beddini R. Spindle speed variation for the suppression of regenerative chatter. Journal of Nonlinear Science. 2003;13(3):265–288. DOI: 10.1007/s00332-003-0518-4.
28. Wahi P, Chatterjee A. Self-interrupted regenerative metal cutting in turning. International Journal of Non-Linear Mechanics. 2008;43(2):111–123. DOI: 10.1016/j.ijnonlinmec.2007.10.010.
29. Warmiński J, Litak G, Lipski J, Wiercigroch M, Cartmell M. Vibrations in regenerative cutting process synthesis of nonlinear dynamical systems. Solid Mechanics and its Applications. 2000;73:275–283.
30. Stépán G, Szalai R, Insperger T. Nonlinear dynamics of high-speed milling subjected to regenerative effect. In: Radons G, Neugebauer R, editors. Nonlinear Dynamics of Production Systems. Hoboken, New Jersey: Wiley; 2004. P. 111–128. DOI: 10.1002/3527602585.ch7.
31. Stépán G, Insperger T, Szalai R. Delay, parametric excitation, and the nonlinear dynamics of cutting processes. International Journal of Bifurcation and Chaos. 2005;15(9):2783–2798. DOI: 10.1142/S0218127405013642.
32. Stépán G. Modelling nonlinear regenerative effects in metal cutting. Phil. Trans. R. Soc. A. 2001;359(1781):739–757. DOI: 10.1098/rsta.2000.0753.
33. Moradi H, Bakhtiari-Nejad F, Movahhedy MR, Ahmadian MT. Nonlinear behaviour of the regenerative chatter in turning process with a worn tool: Forced oscillation and stability analysis. Mechanism and Machine Theory. 2010;45(8):1050–1066. DOI: 10.1016/j.mechmachtheory.2010.03.014.
34. Gouskov AM, Guskov MA, Tung DD, Panovko GY. Modeling and investigation of the stability of a multicutter turning process by a trace. Journal of Machinery Manufacture and Reliability. 2018;47(4):317–323. DOI: 10.3103/S1052618818040052.
35. Lapshin VP. The influence of the cutting speed of metals on the regeneration of the vibratory oscillations of the tool in machines of the turning group. Metal Working and Material Science. 2020;22(1):65–79 (in Russian). DOI: 10.17212/1994-6309-2020-22.1-65-79.
36. Reith MJ, Bachrathy M, Stépán G. Improving the stability of multi-cutter turning with detuned dynamics. Machining Science and Technology. 2016;20(3):440–459. DOI: 10.1080/10910344.2016.1191029.
37. Brissaud D, Gouskov A, Guibert N, Rech J. Influence of the ploughing effect on the dynamic behaviour of the self-vibratory drilling head. CIRP Annals. 2008;57(1):385–388. DOI: 10.1016/j.cirp.2008.03.101.
38. Gouskov A, Gouskov M, Lorong P, Panovko G. Influence of flank face on the condition of

chatter self-excitation during turning. International Journal of Machining and Machinability of Materials. 2017;19(1):17–40. DOI: 10.1504/IJMMM.2017.081186.
39. Rusinek R, Wiercigroch M, Wahi P. Influence of tool flank forces on complex dynamics of cutting process. International Journal of Bifurcation and Chaos. 2014;24(9):1450115. DOI: 10.1142/S0218127414501156.
40. Rusinek R, Wiercigroch M, Wahi P. Modelling of frictional chatter in metal cutting. International Journal of Mechanical Sciences. 2014;89:167–176. DOI: 10.1016/j.ijmecsci.2014.08.020.
41. Grabec I. Chaos generated by the cutting process. Phys. Lett. A. 1986;117(8):384–386. DOI: 10.1016/0375-9601(86)90003-4.
42. Wiercigroch M, Budak E. Sources of nonlinearities, chatter generation and suppression in metal cutting. Phil. Trans. R. Soc. A. 2001;359(1781):663–693. DOI: 10.1098/rsta.2000.0750.
43. Wiercigroch M, Krivtsov AM. Frictional chatter in orthogonal metal cutting. Phil. Trans. R. Soc. A. 2001;359(1781):713–738. DOI: 10.1098/rsta.2000.0752.
44. Masoumi F, Pellicano F, Samani FS, Barbieri M. Symmetry breaking and chaos-induced imbalance in planetary gears. Nonlinear Dynamics. 2015;80(1–2):561–582. DOI: 10.1007/s11071-014-1890-3.
45. Zakovorotny VL, Gvindjiliya VE. The influence of fluctuation on the shape-generating trajectories stability with a turning. University News. North-Caucasian Region. Technical Sciences Series. 2017;(2(194)):52–61 (in Russian). DOI: 10.17213/0321-2653-2017-2-52-61.
46. Zakovorotny VL, Gvindjiliya VE. The influence of the vibration on the tool shape-generating trajectories when turning. Metal Working and Material Science. 2019;21(3):42–58 (in Russian). DOI: 10.17212/1994-6309-2019-21.3-42-58.
47. Zakovorotny VL, Gvindzhiliya VE. Synergetic concept of software control of machining processes on metal-cutting machines. BMSTU Journal of Mechanical Engineering. 2021;(5(734)):24–36 (in Russian). DOI: 10.18698/0536-1044-2021-5-24-36.
48. Lyapunov AM. The General Problem of the Stability of Motion. Moscow: Gostekhizdat; 1950. 472 p. (in Russian).
49. Zakovorotniy VL, Pham TH. Parametric self-excitation of cutting dynamic system. Advanced Engineering Research. 2013;13(5–6):97–103 (in Russian). DOI: 10.12737/1286.
50. Besekersky VA, Popov EP. Theory of Automatic Control Systems. Moscow: Nauka; 1975. 768 p. (in Russian).
51. Zakovorotniy V, Pham D, Nguyen X. Modeling of tool deformation offsetting to workpiece in turning. Advanced Engineering Research. 2010;10(7):1005–1015 (in Russian).
52. Ryzhkin AA. Synergetics of Wear of Tool Materials During Blade Processing. Rostov-on-Don: Don State Technical University Publishing; 2019. 289 p. (in Russian).
53. Push AV. Spindle Units: Quality and Reliability. Moscow: Mashinostroenie; 1992. 288 p. (in Russian).
54. Khusu AP, Vitenberg YR, Palmov VA. Roughness of Surfaces. Probabilistic Approach. Moscow: Nauka; 1975. 344 p. (in Russian).
55. Zakovorotny VL, Flek MB, Lukyanov AD, Voloshin DA. Tool wear modeling using integral operators. Russian Engineering Research. 2004;(3):9–14 (in Russian).
56. Zakovorotny VL, Gvindjiliya VE. Self-organization and evolution in dynamic friction systems. Journal of Vibroengineering. 2021;23(6):1418–1432. DOI: 10.21595/jve.2021.22033.
57. Altintas Y, Kersting P, Biermann D, Budak E, Denkena B, Lazoglu I. Virtual process systems for part machining operations. CIRP Annals. 2014;63(2):585–605. DOI: 10.1016/j.cirp.2014.05.007.
58. Kilic ZM, Altintas Y. Generalized mechanics and dynamics of metal cutting operations for unified simulations. International Journal of Machine Tools and Manufacture. 2016;104:1–13. DOI: 10.1016/j.ijmachtools.2016.01.006.