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Nonlinear elite generation change model

A. V. Kolesnikov[1], *G. G. Malinetskiy*[2], *A. V. Podlazov*[2], *S. N. Sirenko*[3]✉

[1]Institute of Philosophy of the National Academy of Sciences of Belarus, Minsk, Republic of Belarus
[2]Keldysh Institute of Applied Mathematics, Moscow, Russia
[3]Maksim Tank Belarusian State Pedagogical University, Minsk, Republic of Belarus
E-mail: kolesnikov@philosophy.by, gmalin@keldysh.ru, tiger@keldysh.ru, ✉sirenka@bspu.by
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Abstract. The *purpose* of the presented article was to build a concise conceptual mathematical model of the competitive dynamics of alternative types of social activity. The model was developed in the form of a discrete two-dimensional non-linear mapping. The proposed mapping is new and has not been previously studied either in the field of mathematical social dynamics (sociophysics), or in the section of discrete models of nonlinear dynamics. The approach we used corresponds to the ideas of the theory of social time put forward by F. Braudel. Nonlinear two-dimensional mapping, in a paradoxical way, given the general socio-economic ideas about the relationship between generations, as it turned out, has a Hamiltonian structure. The analysis showed that both formally and in terms of qualitative behavior it is close to the standard model describing a rotator under the action of impacts. It was found that, depending not only on the parameters of the problem, but also on the initial conditions, in this case, periodic, quasiperiodic, and chaotic dynamics are simultaneously possible. Within the framework of the model, this means a great variety of intergenerational relationships. Thus, the data in the system will not be "forgotten". The influence on the dynamics of the model of "dissipative additions" describing the degradation of the elite, the desire of society to "eliminate the best" is demonstrated. The dynamics of the system and its dependence on parameters become much simpler; nevertheless, cyclicity and multistability do not disappear in it. In this approximation, history turns out to be "local" — the details and peculiarities of society's behavior will be "forgotten" after several generations. The study of the constructed model opens up great prospects for the analysis of various types of cyclical processes in mathematical history.

Keywords: modeling of social processes, mathematical social dynamics, two-dimensional maps, Hamiltonian dynamics, generational conflict, dynamic chaos, space man, sensitivity to parameters and initial data, historical time.

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Introduction

> Трудные времена создают сильных людей. Сильные люди создают хорошие времена. Хорошие времена создают слабых людей. И слабые люди создают трудные времена.
>
> *Г. М. Хопер.* «Те, кто остаются»

Dynamic behavior of social systems is formed under the influence of competitive interactions of carriers of various own goals and value orientations. At the same time, the goals of social groups may be opposite, but intergroup relations are interdependent. A situation arises when the realization of the interests of one group of social agents creates conditions for the activation of their counterparties, and those, in turn, as their own goals are realized, begin to need the activity of agents of the first type again. This leads to more complex types of dynamic behavior. The purpose of this work is to construct as simple and concise a mathematical model of this type of competitive relationship as possible and to further study its dynamic behavior.

At one time, the representative of the Annals school, Fernand Braudel, put forward the idea of a quantitative description of historical processes. He showed that there are "slow variables"that change little from epoch to epoch. In addition, he identified three time scales in history.

The first level is geographical time, a "long time"associated with changes, trends, repetitions and cycles in the environment.

Second level — long-term social, cultural and economic history.

The third level is the level of events related to politics, reforms, rapid changes. That's what people usually pay attention to [1].

The development of nonlinear dynamics and the widespread use of computers made it possible to take the next step related to the mathematical modeling of historical events. In the 1990s, a research program was put forward related to the construction of *mathematical history* [2]. Within the framework of this program, a mathematical description of the multi-temporal social dynamics of historical processes was assumed based on the ideas of the theory of self-organization and the allocation of order parameters reflecting the most important cause-and-effect relationships [2]. The implementation of this program led to the construction of mathematical models, many of which describe oscillatory, cyclic processes [3, 4].

The use of mathematical and computer models as a tool for research and cognition of the mechanisms that shape the dynamics of society seems to be a promising means of understanding the philosophy of social processes, predicting their course and, most importantly, designing future progressive social development. Despite some progress in the field of modeling social processes and the dynamics of society's development, sudden crises and cycles occurring in social systems, containing periods of boom and bust, rapid development and regression, are difficult to describe mathematically, and even more so to predict. The model of social dynamics developed by us and presented in the article is quite concise, but at the same time it can be used to understand and in some sense explain the events of sociogenesis occurring in the past and present.

Apparently, the desire to identify periodically recurring states of the studied systems is characteristic of many developing disciplines. For example, within the framework of market economy research, the cycles of Kitchin (3-4 years), Zhuglyar (7-11 years), Kondratiev (40-50 years) are distinguished. The presence of cyclic repeatability allows you to make predictions. Wars, revolutions, crises, in accordance with the ideas of a number of researchers, are determined by Kondratiev cycles. For example, the forecast of the global economic crisis of 2008–2009 was presented five years before its onset [5].

Another example of rhythms and cycles is given by the analysis of the activity of the Sun, in which the Schwabe cycle (11 years), the Hale cycle (22 years), the Gleisberg cycle (70–100 years) are distinguished. Initially, the idea of the influence of solar activity cycles on historical events was expressed by A. L. Chizhevsky. Nevertheless, cyclical behavior in many cases is only a good approximation. A more detailed analysis allows us to identify a much more complex chaotic dynamics [6].

The emergence of oscillatory processes in complex systems is one of the important forms of self-organization. In such systems, order parameters can be formed that determine their behavior, and objects whose dynamics can be determined by fairly simple differential equations.

The model we propose allows for a broad interpretation in the framework of considering competitive interactions of alternatives in social dynamics. In this article we will consider the socio-

cultural and technological activity of different generations. Currently, the theory developed by W. Strauss and N. Howe is popular in sociology, describing patterns of behavior that are repeated in a number of generations. This theory, which became known after 1991, considers generations in Anglo-American history since 1433. to date, [7, 8].
Its authors define a generation as the totality of all people born in a time span of approximately 20 years, which corresponds to one phase of life - childhood, youth, middle age, old age. It is the personalities of the same type that are most popular in different epochs (generational archetypes). He is a Prophet, A Wanderer, A Hero, An Artist.

The constructiveness of this approach is manifested in the fact that it has become popular not only among historians and sociologists, but also among marketers, advertising specialists, HR specialists, "headhunters". They usually consider the generations of our country, singling out the greatest generation (1903–1923), the silent generation (1924–1943), baby boomers (1944–1963), generation X (1964–1984), generation Y (1985–2003), generation Z (2004–2024).

Let's pay attention to a simple sociological pattern — if you want to know the mood and prospects of society, look 20 years ago and you will see about the same. Folk wisdom says that grandchildren understand much better and take more from their grandparents than from their parents.

The mentioned generational theories are both too complex and too simple. They are difficult because we have to take into account the rich, contradictory historical context, which largely depends on the views of the researcher. In addition, it is difficult to compare quantitative characteristics of entities considered in theory. At the same time, it is quite obvious that the dynamics of society depends very significantly on the initial data — on the state and resources that it has.

Some fundamental foundations of the mechanism of the evolution of ethnic groups are revealed by the theory of ethnogenesis of L.,N.Gumilyov [9]. Within the framework of this theory, the share of passionaries in society plays a fundamental role. This is the term he calls active, energetic people who are ready to give a lot up to their lives for the embodiment of their meanings, values, ideals, projects. Along with them, there are subpassionaries in society, for whom the personal is much more important than the public and who are happy to lead a parasitic lifestyle, if there is an opportunity for this. The middle position is occupied by people who harmoniously combine personal and public interests (harmonics).

Numerous historical trajectories of ethnic groups show the traditional fate of these social structures, which have been developing for 1000–1200 years. Passionaries who believe that the world is bad and it should be changed give rise to a new ethnic group. The share of these people is growing, their imperatives determine the state of society. At this stage, unexpected horizons are opening up, new technologies are being created and actively used. In order to keep what has been won and effectively manage it, harmoniously developed people become necessary. After breaking the ball, the subpassionaries begin to rule, the ethnos goes to sunset [9]. It can be said that the dynamics of ethnogenesis described by L. Gumilev deals with one leading, slow variable, which he called the passionate tension of society. Arnold Toynbee's theory of civilizations gives about the same picture.

However, it is natural to assume that in the fast times of 20-100 years, we will also see fluctuations in the share of passionaries in society. The presence of these rapid oscillations is consistent with the theory of N., D.Kondratiev. At different phases of the economic wave, which takes 40-50 years, people with different socio-psychological characteristics are in demand. This is confirmed by the observations of the classic of economic thought J. Schumpeter, who saw innovation as the most important factor of production [10]. He found out that no more than 10% of entrepreneurs are interested in the development, in the promotion of new technologies, in changing the existing system. It is their strategy that is close to a passionate way of acting.

At the same time, 90% are ready to invest efforts in ensuring stability, preserved relationships, abandoning the new. Their course of action is naturally compared with that which is characteristic of harmonics and subpassionaries. And these shares vary significantly depending on the phase of economic development.

Attention should be paid to the fundamental difference between the considered approach and the models of social dynamics and mathematical history described earlier [3, 4]. In these models, the most important parameters are economic and demographic, as well as the amount of resource consumed by the ruling stratum. In some cases, these models allow us to describe the instabilities that took place in the historical dynamics. As a rule, they refer to "slow variables". In our case, we are talking rather about "fast variables" and about the difference in the contribution to social development of different generations, essentially about social philosophy.

1. Building a model

Понять — значит упростить.
Стругацкие, «Волны гасят ветер» (1984)

Following the works of [11–13], as well as following the spirit of the philosophy of Russian cosmism, we will simplify the situation. Passionaries who create new opportunities, discover and master new things, offer original life-saving technologies, we will call *space people*. It was thanks to their activities that humanity expanded its habitat and became a technological civilization, which fundamentally distinguished our species in the course of evolution. He can be contrasted with *molecular man*, who seeks to use the existing opportunities and does not want to go beyond them. In fact, we simplify the Gumilev scheme by not considering "harmonics" and distinguishing only two types of people in society.

We introduce the concept of a cumulative resource, including material and intangible resources produced by the social system, R_n, which reflects the change in what the social system reproduces and consumes during the generational change (approximately 20 years) at the time n. Under n we will understand the "number" of the generation. At the same time, we are talking not only about the economy and material resources, but also about industrial and humanitarian technologies.

This moment is very important. We will deal not with continuous time $0 \leqslant t < \infty$ and differential equations, but with discrete time $n = 1, 2, 3...$ and models of the form

$$\vec{z}_{n+1} = \vec{G}\left(\vec{z}_n, \vec{\lambda}\right).$$

We assume that the function $\vec{G}$ is continuous and unambiguous. This allows us to talk about a continuous dependence on the initial data, the existence and uniqueness characteristic of differential equations. There are two reasons for the transition from continuous to discrete time. We will be interested in qualitative features of historical dynamics, not a detailed description of a particular generation. In addition, the discrete mapping can be considered as a system describing the dynamics on the Poincare section. This makes it possible to get rid of the errors introduced during the numerical integration of differential equations that could describe the same situation. On the other hand, it simplifies the picture — the limit cycle of the differential equation corresponds to several points in the mapping, quasi-periodic motion (invariant torus) — line, etc.

Note that values of this sort are traditionally introduced in historical and world dynamics [14]. In fact, the Forrester model considers such quantities as world resources, global product, pollution level, quality of life. In historical dynamics, in many models, the share of the product assigned by the elite and such values as passionarity or asabia appear.

Let the proportion of the passionate ("cosmic") population in the population at time n is q_n. This allows you to write out a simple recurrence relation

$$R_{n+1} = k_l q_n R_n + k_{ml} (1 - q_n) R_n, \quad R_0 = 1, \quad 0 \leqslant q \leqslant 1, \tag{1}$$

where $k_l > 1$, $k_{ml} < 1$ — coefficients of reproduction of the total resource by representatives of different parts of society. The total resource at time $n+1$ (or in the next cycle of reproduction) consists of the resource reproduced by passionaries and molecular consumers. We consider the simplest linear relationship between the parts of the resource produced by different parts of society. By its nature, the passionate sociotype is focused on incrementing the total resource of the system. Therefore, the reproduction coefficient k_l is greater than one. If $k_l = k_{ml}$, then we have a convergent geometric progression with the denominator k_{ml}, so that $R_n \to 0$ for $n \to \infty$. Society is dying.

Passionaries, as historians show, are a big burden for society, — they do not allow society to live in peace, offering big projects, conquests or large-scale reforms. The Crusades and the exploration of America provide illustrative examples of the “sending” of a large number of passionaries to distant frontiers.

With the decline in the security of society, the need for passionaries grows, in “well-fed” periods it falls. This makes it possible to supplement the equation (1) with a ratio for the dynamics of the proportion of passionaries and obtain a system (2)

$$\begin{aligned} R_{n+1} &= k_l q_n R_n + k_{ml} (1 - q_n) R_n, \quad R_0 = 1, \quad 0 \leqslant q \leqslant 1, \\ q_{n+1} &= q_n^{R_{n+1}}. \end{aligned} \tag{2}$$

The properties of the power curve in the section [0; 1] (namely, the fraction q makes sense in this section) are such that at high values of the exponent R, the values of q will approach zero. At low values of R, the values of q will be close to one. This is quite consistent with the statement of Sheikh Rashid ibn Said Al Maktoum: “My grandfather rode a camel, my father rode a camel, I drove a Mercedes, my son drove a Land Rover and my grandson drives a Land Rover, but my great-grandson will ride a camel,” which metaphorically describes cyclical processes in society.

The principle is that in the equation $q_{n+1} = q_n^{R_{n+1}}$ of the system (2) there is an advance. uses R_{n+1} as the exponent, not R_n. This reflects the fact that the “social order” for the share of passionaries is formed at the end of the current cycle of reproduction, based on the amount of resource that was produced when the share of passionaries was q_n. A similar logic is used in modeling the innovative development of the economy. It turns out that the key innovations of this technological cycle were developed during the development of the previous one, but they were not yet in demand in it. In other words, the researchers and engineers of the previous cycle were ahead of their time, which will be used only in decades.

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The mathematical model of social competition developed by us is a two-dimensional discrete mapping containing nonlinearity. The mapping (2) was not considered earlier in the section of nonlinear dynamics studying discrete models. It is being investigated for the first time.

We don't consider controls in our model. Nevertheless, the conclusion from the relation (2) is quite obvious. Planning and forecasting for the generation cycle (20 years) is very important for society. It is important to imagine how many "passionaries" will be needed for the upcoming changes [13]. Based on this, *advanced education* is of fundamental importance — setting tasks for schoolchildren and students not today, but tomorrow [15]. is very useful to tell them about what they have to master in the future, and formulate problems that the country or humanity is solving today. Research by psychologists shows that it is very useful to ask questions to those who are studying, even if we do not give answers to them.

2. Model research

Мой дед ездил на верблюде, мой отец ездил на верблюде, я на Мерседесе, сын на Ленд Ровере, и мой внук водит Ленд Ровер, но правнук будет ездить на верблюде.

Рашид ибн Саид Аль Мактуси

Let's rewrite the mapping (2) as

$$\begin{aligned} R_{n+1} &= R_n(aq_n + b), \\ q_{n+1} &= q_n^{R_{n+1}}, \end{aligned} \tag{3}$$

где $a = k_l - k_{ml} > 0$ и $b = k_{ml}$.

Let's introduce new variables

$$x_n = \ln R_n, \quad y_n = \ln\left(-\ln q_n\right). \tag{4}$$

In these variables , the mapping will be written as follows

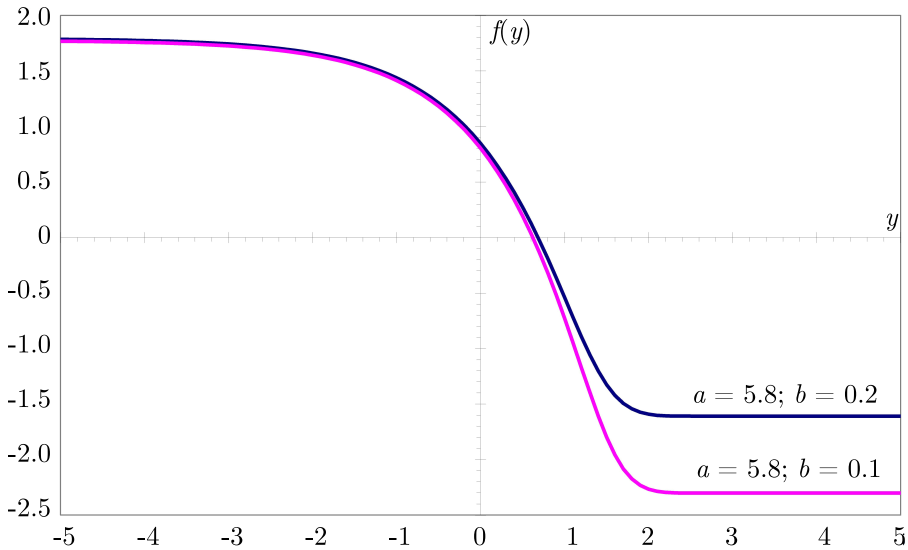

Fig. 1. Вид функции отображения. Функция $f(y)$ является монотонно убывающей между двумя асимптотическими пределами

Fig. 1. View of the mapping function. The function $f(y)$ is monotonically decreasing between two asymptotic limits

$$
\begin{aligned}
x_{n+1} &= x_n + f\left(y_n\right), \\
y_{n+1} &= x_{n+1} + y_n,
\end{aligned}
\tag{5}
$$

where $f(y) = \ln\left(ae^{-e^y} + b\right)$. The form of the function f for several parameter values is shown in Fig. 1. It can be seen that these graphs of functions connect two sections close to the constants $f\left(y\right) \to \ln b$ при $y \to \infty$ и $f\left(y\right) \to \ln\left(a+b\right)$ при $y \to -\infty$.

Let's pay attention to the special case when $y \to \infty$, $f\left(y\right) \to \mathrm{const}$. We see that if $f\left(y\right) = c$, then $\{x_n\}$ form an arithmetic progression $x_n = x_0 + cn$, a $y_n = y_0 + nx_0 + cn\left(n+1\right)/2 \to \infty$ при $n \to \infty$.

From the second equality in (5) follows

$$
y_n = y_0 + S_n, \qquad \text{где} \quad S_n = \sum_{k=1}^{n} x_k. \tag{6}
$$

Therefore

$$
x_{n+1} = x_n + f\left(y_0 + S_n\right). \tag{7}
$$

This equality is very interesting — it shows that the logarithm of the resource at the moment is determined by a function of the sum of the logarithms of the resource taken with the same weight. It turns out that everything is essential, that the system does not "forget" anything.

Note that if the whole society is passionate ($q_n = 1$), then the sequence $\{R_n\}$ defines a geometric progression with the denominator $a + b$. The resource received by society grows indefinitely over time.

It is very interesting that the mapping (7) is very close to the one that is classical in nonlinear dynamics. In this area, we consider standard mappings describing a rotator under the action of periodic shocks, with a Hamiltonian in the variables action – angle

$$
H = \frac{1}{2}I^2 - K\cos\Theta \sum_{n=-\infty}^{\infty} \delta\left(t/T - n\right).
$$

This model results in the mapping

$$
\begin{aligned}
p_{n+1} &= p_n + K\sin x_n \,(\mathrm{mod}\ 2\pi), \\
x_{n+1} &= x_n + p_{n+1} \,(\mathrm{mod}\ 2\pi).
\end{aligned}
$$

It is also called the standard mapping or the Chirikov mapping– of Taylor. This mapping arises in plasma theory. It is widely used in the analysis of chaos in Hamiltonian systems. It has trajectories describing chaotic dynamics belonging to the so-called *stochastic sea*. They are adjacent to quasi-periodic trajectories on the so-called *islands* [16].

In the class of periodic functions with a period of 2π, we have the mapping

$$
p_{n+1} = p_n + K\sin\left(x_0 + \sum_{k=1}^{n} p_k\right). \tag{8}
$$

As you can see, we have the same functional dependence of the equations (7) and (8). Moreover, if we consider an equation for a nonlinear oscillator solved by a semi-implicit Euler scheme, then we get an analogue of the relation (7).

Despite the complex dynamics, Hamiltonian systems with perturbation were considered in detail. In fact, the motion of the planets of the Solar System can be considered as quasi-periodic over a $2p$-dimensional torus in space $(\Theta_1, \dot{\Theta}_1, \ldots\Theta_p, \dot{\Theta}_p)$, where Θ_i characterizes the angular coordinate of the ith planet, p — the number of planets. There is a question that worried Newton, — how stable is the Solar system, whether the quasi-periodic solution of this Hamiltonian system will be preserved with small perturbations. After three centuries of efforts by outstanding mathematicians in Kolmogorov's theory–Arnold–It was established by Moser that for sufficiently small perturbations in the vicinity of a given invariant (relative to the dynamical system under consideration) torus (quasi-periodic motion), there is an invariant torus close to it [19]. It should be noted that in the absence of perturbation in the classical problem there is an integrable limit. It is not in our task. Nevertheless, the analogy seems quite obvious — we also have a set of stochastic layers and ordered islands in phase space.

These results were obtained using perturbation theory, but the possibilities of this approach are rather limited — in many interesting cases, perturbations are not small: "Such complexity, as well as a variety of types of trajectories, cannot be obtained by any known method of perturbation theory. Narrow zones with chaotic trajectories are called stochastic layers. Inside these stochastic layers there are islands with nested curves, sub-islands and even smaller stochastic layers" [18, p. 28]. Therefore, the main tool for analyzing such systems remains a computer experiment.

Due to the similarity of the (7) and (8) mappings, both for the model under study and for the system under consideration, we can expect the presence of rather complex dynamics. Therefore, it is worth formulating the issues under consideration.

Does the (3) model describe generational change and cycles of reproduction of the aggregate resource? Do we really have the simplest cyclical dynamics with a small periodicity? Are we always dealing with a periodic process or is chaotic dynamics possible? Does the model always describe limited trajectories? How do these processes behave in the presence of dissipative effects?

3. Dissipative version of the model

Отсутствие лучших (по крайней мере их дефицит) имело глубокое отрицательное влияние на отечественную историю, окончательно воспрепятствовав нашему превращению в более или менее нормальную страну.

Х. Ортего-и-Гассет

The theory of self-organization is based on the analysis of dissipative processes that allow us to consider asymptotics and a set of attractors. The details of the initial data are "forgotten". From this point of view, accounting for dissipative processes does not complicate, but simplifies the phenomenon under study. Let's consider what in our case would be given by taking into account the processes of dissipation, taking into account that society often seeks to reduce the number of passionate "space people". This is consistent with the description of a number of epochs by historians. Taking this into account gives a two-dimensional mapping instead of (3)

$$\begin{aligned} R_{n+1} &= R_n(aq_n + b), \\ q_{n+1} &= q_n^{R_{n+1}}(1-\alpha). \end{aligned} \tag{9}$$

The value of $\alpha \ll 1$ characterizes the proportion of "space people" that society "weeds out" in the next generation. A characteristic view of the phase plane is shown in Fig. 2. It is clear that cycles with different periods take place here.

We see two fundamental features.

The first is — *multistability*. Depending on the initial data (x_0, y_0), the modes established at $n \to \infty$ fundamentally change.

In this parameter area there is a large area in which $R_n \to R^*$, $q_n \to q^*$ at $n \to \infty$. In other words, there is a stationary state in which neither the quantity of the product being created nor the share of passionaries in society changes. This is similar to the idea of a stable “internal state” put forward by a number of sociologists and political scientists. According to their ideas, it is this state that determines the key decisions made in society.

However, away from this state, we observed cycles of length l: $(R_{n+l}, q_{n+l}) \to (R_l^*, q_l^*)$. In other words, the system returns to its original state after l generations. At the same time, the value of l itself may be different for different values of abb. This is cyclical dynamics — exactly the pattern for which the model was proposed to explain.

In Fig. 2 the elements of each cycle are connected by straight lines so that they can be distinguished from each other. It can be seen that there is a fixed point here, and cycle-5, and cycle-8, and cycle-11. This shows that “generational cycles” of different lengths can exist in the same society. There are opportunities to change the rhythm of elite reproduction, and with it the state of society in the “middle” (in Braudel’s terminology) times. A significant crisis associated with the removal or repression of a part of the elite, or a significant deterioration in the financial situation can change the historical trajectory ("...the wrong people came").

In addition, it should be noted that a naive approach to historical dynamics (for example, to "rejuvenate the elite"), the desire to do “as best as possible” without understanding the areas of attraction of different attractors may not improve, but worsen the situation. The fact is that the regions of attraction of various cycles are intricately intertwined, since we are not dealing with differential equations, but with a mapping capable of making “jumps” along the phase plane.

Let us explain what has been said. Due to the continuity of the display, the segment turns into a segment. However, the set of points converging to a given attractor may have a complex structure. A classic example is the mapping $x_{n+1} = 1 - 2\,|x_n|$, in which each rational x_0 leads to a cycle, each irrational — to a chaotic trajectory. And in the mapping $x_{n+1} =$

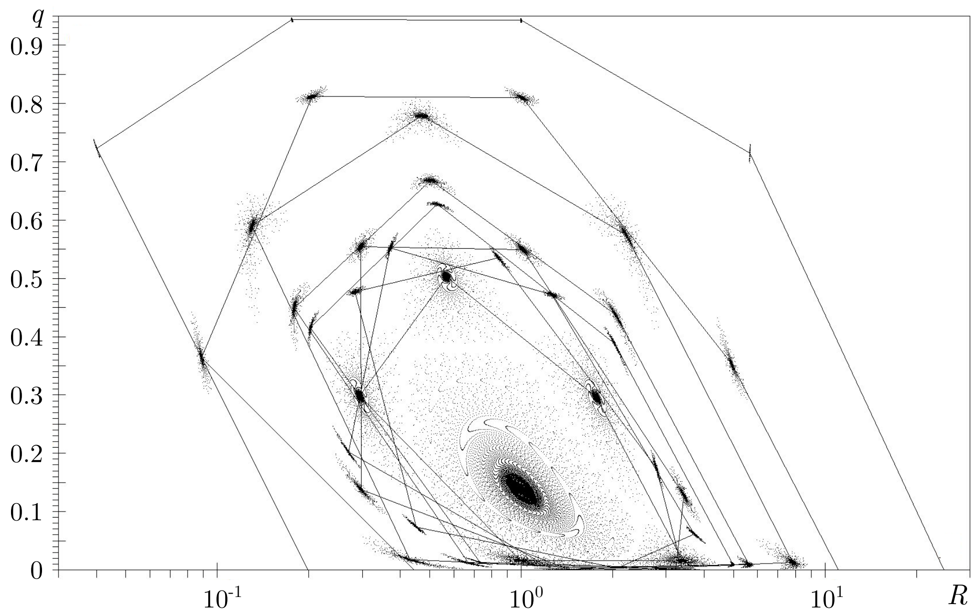

Fig. 2. Циклы диссипативной системы (9) в координатах (R, q) при $a = 5.8$, $b = 0.2$ и $\alpha = 10^{-3}$. Некоторое количество точек, попадающих в область притяжения каждого цикла, показано вблизи каждой его точки. При различных начальных данных происходит переход на разные циклы, что позволяет говорить о мультистабильности в системе

Fig. 2. Cycles of a dissipative system and their mapping areas

$= \lambda x_n (1 - x_n)$ there is a stable cycle near each "chaos". In this case, we found sensitivity with respect to the parameters — small changes in the initial data led to cycles of different periods. Unfortunately, it is not possible to simply display this amazing dynamics in black and white. Therefore, the study of the question of the detailed arrangement of the regions of attraction of cycles is not carried out in this work.

4. Detailed study of the model

Общество не консервативно, когда оно не видит, что сохранять. Дайте ему что-то положительное и увидите, что оно будет консервативно.

Ф. М. Достоевский

The dynamics under study turns out to be quite complex, so it is natural to turn to the simplest mathematical models that demonstrate similar dynamics. In fact, this is the main way to understand the behavior under study.

The results of the calculations give a complex picture that repeats itself on smaller and smaller scales. Therefore, it is worth paying attention to two simple models that give an understanding and logic of studying the constructed model. Complex dynamics is provided by the interaction of two fairly simple effects.

The first one is related to the intersection of separatrix and stochastic layer. Consider the simplest physical pendulum with a small periodic perturbation in time. Its phase space is $(\varphi, \dot{\varphi})$. Singular points (0,0), (0,π), (0,$-\pi$). If we identify the points φ and $\varphi + 2\pi$, then the phase space will be a cylinder $(-\pi \leqslant \varphi \leqslant \pi,\ -\infty \leqslant \dot{\varphi} \leqslant \infty)$. When the energy is $E = (1/2)\ dot\varphi^2 - \omega_0^2 \cos\varphi$ is small, then the system makes small fluctuations. When it is very large, the pendulum "scrolls", rotating in one direction or the other depending on the direction of the initial velocity. Near the origin, the angle φ returns to the starting point for the period. When the energy is high, φ increases (or decreases) by 2π per period. There can be no continuous transition from the first type of behavior to the second. This transition is associated with a separatrix leading at a certain energy value E_{crit} from one saddle $(-\pi,\ 0)$ to another $(\pi,\ 0)$. Movement along this trajectory (separatrix) takes an infinitely long time. No matter how small the disturbances are during this time, they can have a very significant impact on the system. The role of these perturbations is to make part of the "oscillatory" trajectories (in which φ does not change during the period) "flyby"(in which φ changes by 2π or by -2π during the period). And, on the contrary, a small part of the "flight" trajectories can become "oscillatory". The smaller the magnitude of the disturbance ε, the narrower the layer in the phase space in which this occurs.

For example, such dynamics is observed in a system with a Hamiltonian

$$H = \frac{1}{2}\dot{\varphi}^2 - \omega_0^2 \cos\varphi + \varepsilon\omega_0^2 (\omega t)\,.$$

The Poincare section can be taken by considering the trajectory for the period $= 2\pi/\nu$. This excitation, which depends on both time and the angle φ, creates uncertainty. The well-known physicist D.,S.Chernavsky compared such systems with "Chinese billiards", in which, as in a roulette game, small changes in the initial velocity of the ball can change the hole into which it falls [20].

The second effect is associated with a huge number of "islands" and a complex dependence on the parameters of the problem, in which a small change in parameters can qualitatively change the observed picture. At the same time, this is based on the realization of a set of resonances and a well-known phenomenon in the theory of oscillations — frequency capture. This phenomenon occurs with an auto-oscillatory system with a periodic external signal with a given frequency and amplitude. The simplest system illustrating what is happening is the representation of a circle

$$\varphi_{n+1} = \varphi_n + \alpha + \varepsilon \sin(2\pi\varphi_n).$$

When $\varepsilon = 0$, this mapping is a rotation of the angle α. For this mapping, we can consider the rotation number $\rho(\alpha,\varepsilon)$, showing by what amount the angle φ changes on average per iteration. The set of points at which this function takes rational values $\rho(\alpha,\varepsilon) = p/q$ are called languages Arnold.

For $\varepsilon = 0$ we have periodic orbits starting at rational points $\varphi_0 = p/q$ and a set of quasi-periodic orbits starting with other initial data.

When $\varepsilon > 0$, a “language” begins to grow near each value of p/q, which is the wider the larger ε. At the same time, inside the language we have a rotation not with frequencies α (which was at $\varepsilon = 0$), but with a frequency p/q, which "captures"frequencies in its speed.

At the same time, the structure of the parameter space turns out to be very complex.

If α is a "good"irrational number

$$\left|\alpha - \frac{m}{s}\right| > \frac{k(t)}{s^{2.5}}, \quad k(t) \to 0 \text{ при } \varepsilon \to 0,$$

then an invariant torus is stable for a sufficiently small ε.

Otherwise, the frequency is “captured”. The interval near α, where this happens, is estimated at $k(\varepsilon)/s^{2.5}$. The total length of these intervals

$$L < \sum_{s=1}^{\infty} \frac{k(\varepsilon)}{s^{2.5}} = k(\varepsilon) \sum_{s=1}^{\infty} s^{-1.5}.$$

At the same time, the number of rotation as a function of the parameter α is a kind of cantor ladder. Unlike the usual Cantor ladder, each “step” corresponds to its p/q (where the frequency is captured), but there is an interval between them corresponding to the irrational p/q and invariant tori. At the same time, the Lebesgue measure of both is positive. In other words, “throwing at random” the initial data φ_0, we will get into a cycle with a non-zero probability and with a non-zero — into chaos [21,22].

Linearizing the mapping (5) near an arbitrary point (x, y)

$$\begin{aligned} \Delta x_{n+1} &= \Delta x_n + p\Delta y_n, \\ \Delta y_{n+1} &= \Delta x_n + (1+p)\Delta y_n, \end{aligned} \tag{10}$$

where

$$p = f'(y) = -\frac{e^y}{1 + \frac{b}{a}e^{e^y}} < 0. \tag{11}$$

Matrix eigenvalues

$$\begin{vmatrix} 1 & p \\ 1 & 1+p \end{vmatrix} \tag{12}$$

are found as the roots of the quadratic equation $\lambda^2 - (p+2)\lambda + 1 = 0$. Its discriminant is $D = p^2 + 4p < 0$ at $-4 < p < 0$. In this case, the roots λ_1 and λ_2 are complex conjugate, and since, according to Vieta’s theorem, their product $\lambda_1\lambda_2 = 1$, then $|\lambda_{1,2}| = 1$, that is, the mapping only rotates the points around an arbitrarily selected decomposition point, without bringing them closer to it or removing them from it. In other words, the system is conservative. Neither the real $p/2+1$, nor the imaginary $\pm\sqrt{-(p/2)^2-p}$ the parts of the roots modulo do not exceed 1, that is, they are equal to the cosine and sine of the rotation angle. The case $D>0$ for the considered function f is of no interest, since it can be shown that the necessary values of $p<-4$ are achievable only with fantastic parameter ratios $a/b>4e^5\approx 594$.

The determinant of the matrix (12) is always 1. This means that the system is conservative, the area of the phase space is preserved, the details of the initial data are not forgotten. We have a property often called in synergetics *sensitivity to parameters* [17].

Due to the complexity of the models of the form (6) and (7), it is natural to start with the analysis of their simplest, linear variant. Using linearity, we will look for a solution in a complex form

$$x_n = e^{i(n\omega+\varphi)}, \tag{13}$$

in order to take into account only the real part of this entry at the end $x_n=\cos(n\omega+\varphi)$. For values (13) is easily calculated as an increment

$$x_n - x_{n-1} = e^{i(n\omega+\varphi)}\left(1-e^{-i\omega}\right),$$

so is the amount

$$S_{n-1}=\sum_{k=0}^{n-1}x_k = e^{i\varphi}\frac{e^{in\omega}-1}{e^{i\omega}-1}.$$

These two expressions are obviously connected by a linear relation

$$x_n = x_{n-1} + AS_{n-1} + B, \tag{14}$$

in which the free term B is also complex, that is, only the real part of it will also need to be preserved. Substitute the increment and the sum into this ratio

$$e^{i(n\omega+\varphi)}\left(1-e^{-i\omega}\right) = Ae^{i\varphi}\frac{e^{in\omega}-1}{e^{i\omega}-1}+B$$

and, accordingly, equating the coefficients for terms that depend on and do not depend on n, we find the parameters

$$A=\left(1-e^{-i\omega}\right)\left(e^{i\omega}-1\right)=2\left(\cos\omega-1\right)\leqslant 0,$$
$$B=\frac{Ae^{i\varphi}}{e^{i\omega}-1}=e^{i\varphi}\left(1-e^{-i\omega}\right)\big|_{Re}=\cos\varphi-\cos\left(\varphi-\omega\right).$$

Thus, for specific values of A and B, we have a mapping of the form (14). For him, the set of images of any point forms an ellipse in the coordinates (x,y) with an irrational relation ω/π.

Let us now turn to the phase portraits (Fig. 3 for displaying (3) and Fig. 4–7 to display (5)). They show a large structure in which the trajectories of the points are close to elliptical.

If we had a dynamical system for which the map under study would be a Poincare section, then this type of ordering inside regular islands would correspond to invariant tori. In addition to the change of elites, this means that the change of the type of elite "differing both in the number of passionaries and in the volume of the product produced which the theory of elites says, does not occur.

However, with other initial data (R,q), the situation turns out to be fundamentally different, exactly what the theory of elites says. In this case, about "l-cycles" in the dissipative system, "l-tori" arise. Each of these l islands can be mapped to its own type of elite. In the next generation, there is a transition to a different type. With the appropriate parameter values, the model describes exactly this picture, for which it was created. It should be noted that by changing the parameters, we can change the number of "islands"that determine the dynamics of the elite. In addition, we do not consider management tasks. However, the results of the calculation show that by reducing the number of "passionaries"or the product of society (and in some cases increasing it), it is possible to change the temporal dynamics of the evolution of elites, meaning either stability or diversity, allowing for more effective consideration of the changes taking place.

However, the conservative model is significantly richer than the dissipative one. It has two more important features. The first one is connected with the fact that there are non-periodic, chaotic trajectories in it, whereas the authors failed to find it in the dissipative model of chaos. An example is shown in Fig. 3 for $a = 5.8$, $b = 0.2$.

It should be noted that in the model of elite change under consideration, there are several disjoint "regions of chaos" located in different parts of the phase space, which may differ significantly in their characteristics.

A remarkable feature of the observed stochastic sea is its fractal character. In Fig. 3 it can be seen that it turns out to be riddled with islands of orderliness of ever smaller size and a large period. Within the framework of the model, this means that, depending on the initial data, there are more or less likely variants of quasi-periodic elite dynamics.

The dynamics and representation of the islands become much clearer in the coordinates (x, y) (Fig. 4). Here you can see the presence of a system of islands that characterize different cycles of elite change. A very important parameter is the value of a, which characterizes the

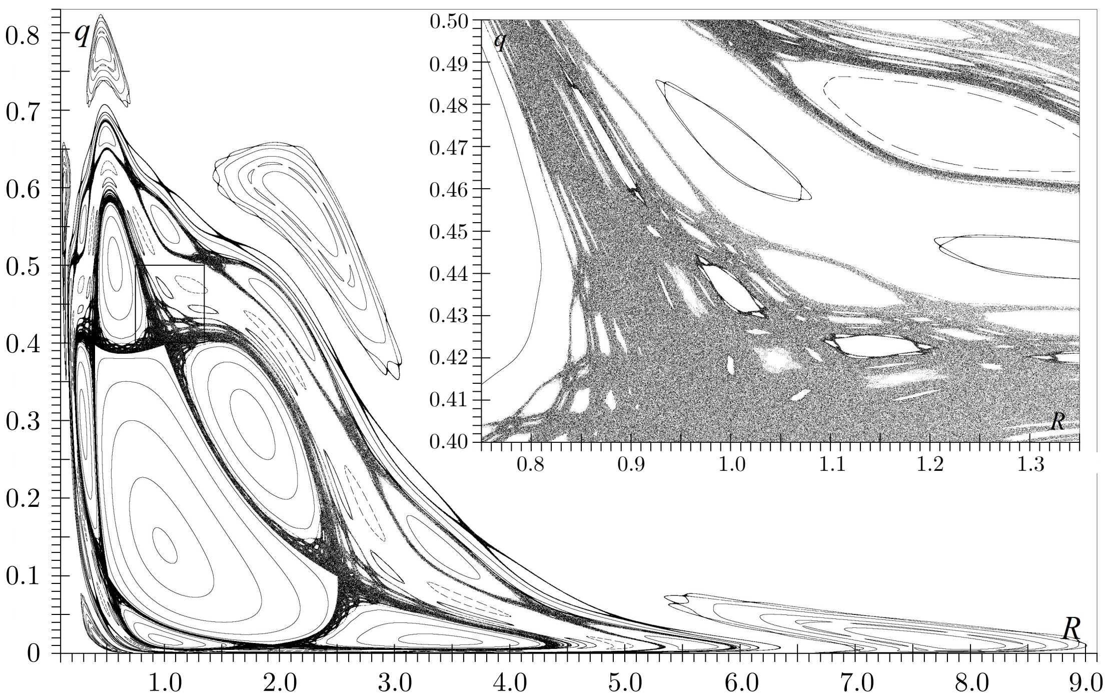

Fig. 3. Траектории различных начальных точек отображения (3) при $a = 5.8$, $b = 0.2$ в координатах (R, q). Одни начальные данные рождают квазипериодическое движение вокруг неподвижных точек (ему соответствуют замкнутые линии), другие — квазипериодическое движение вокруг циклов (ему соответствуют наборы замкнутых линий), а третьи — хаотическое движение (ему соответствуют шлейфы точек). На врезке увеличен выделенный фрагмент хаотической области (при этом число итераций увеличено с 500 тыс. до 5 млн). Обратим внимание, что находящиеся рядом с ним траектории оказались при увеличении не замкнутыми линиями, а тоже хаотическими областями, что следует из самопересечения

Fig. 3. Trajectories of different initial points of mapping (3) for $a = 5.8$, $b = 0.2$ in coordinates (R, q). sets of closed lines), and the third — chaotic movement (it corresponds to trails of points). The inset shows a zoomed-in fragment of a chaotic region. Note that the trajectories next to it turned out to be not closed lines when magnified, but also chaotic regions, which follows from self-intersection

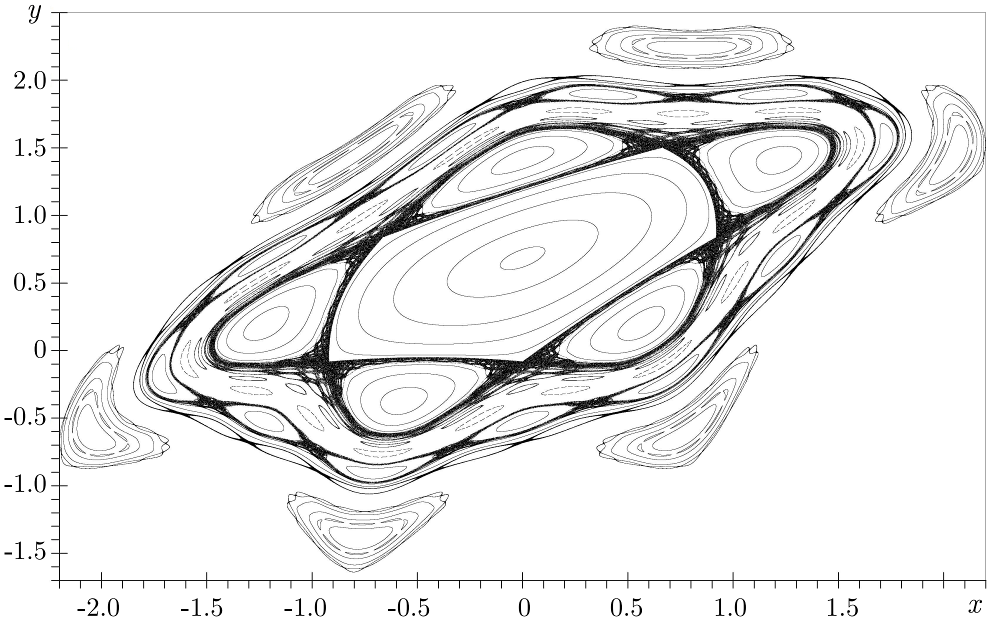

Fig. 4. Траектории с рис. 3 в координатах (x, y). Обратим внимание на наличие основной большой области, где не происходит эволюции поколений, на большой цикл порядка 5, где такие перемены происходят, а также на многообразие малых циклов самых разных порядков

Fig. 4. Trajectories from fig. 3 in coordinates (x, y). Let us pay attention to the presence of the main large area where there is no evolution of generations, to the large cycle of order 5, where such changes occur, as well as to the variety of small cycles of various orders

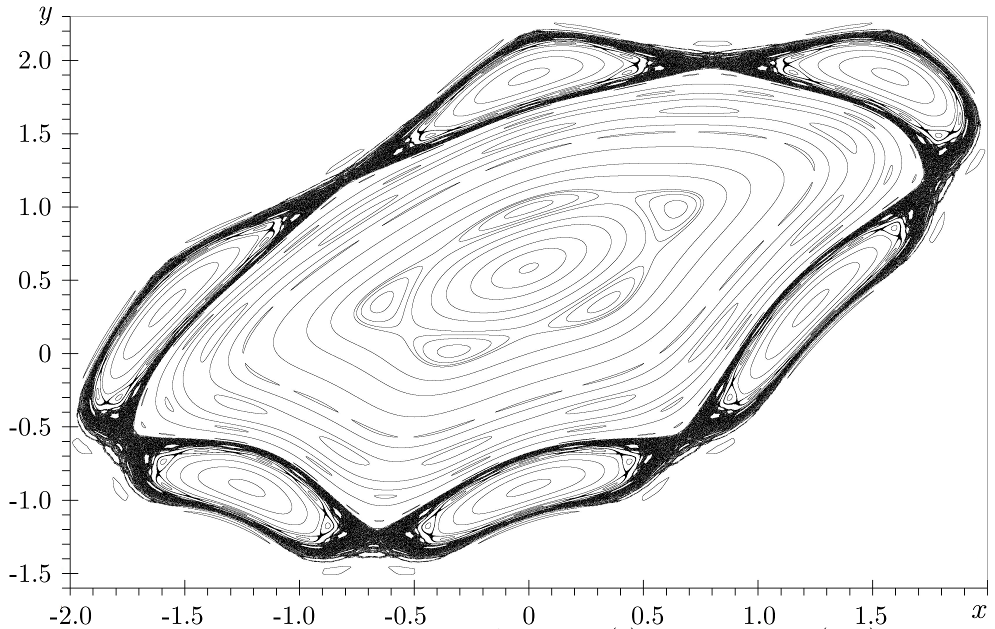

Fig. 5. Траектории различных начальных точек отображения (3) в координатах $(x,\ y)$ для $a = 4.8$, $b = 0.2$. Этот случай отображает ситуацию, в которой активность пассионарного слоя снижена. Размер областей циклов изменился, но в целом структура осталась той же

Fig. 5. Trajectories of different starting points of the mapping (3), for $a = 4.8$, $b = 0.2$. This case reflects a situation in which the activity of the passionary layer is reduced. The size of the loop areas has changed, but the overall structure remains the same

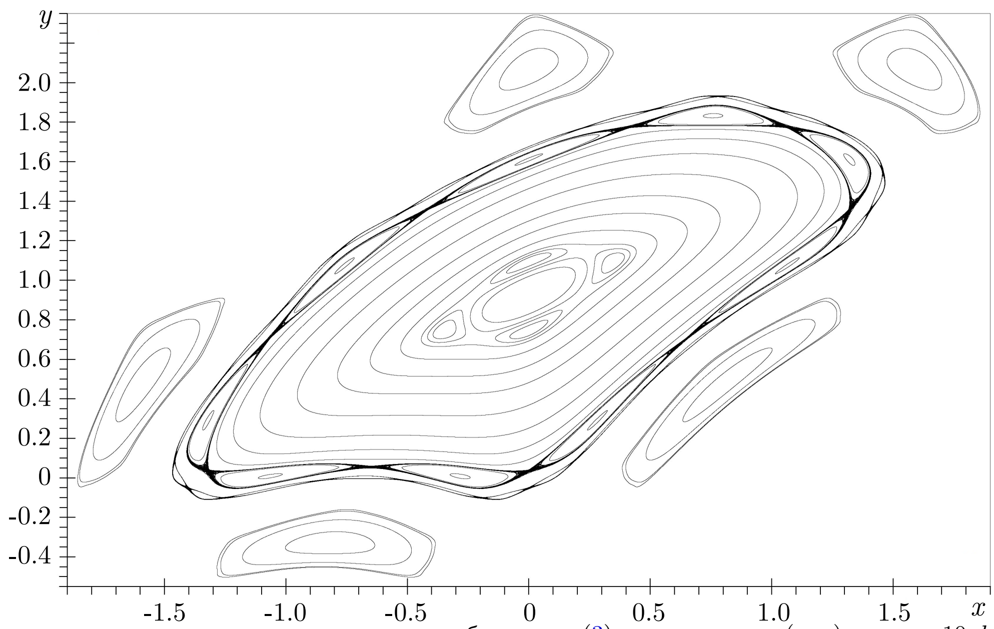

Fig. 6. Траектории различных начальных точек отображения (3) в координатах (x, y) для $a = 10$, $b = 0.2$, что соответствует высокой активности пассионарного класса. Области, в которых возможны квазипериодические циклы смены элит или квазипериодический режим, занимают много меньше места в фазовом пространстве

Fig. 6. Trajectories of different starting points of the mapping (3), for $a = 10$, $b = 0.2$, which corresponds to the high activity of the passionary class. Areas in which quasi-periodic cycles of elite change or a quasi-periodic regime are possible occupy much less space in the phase space

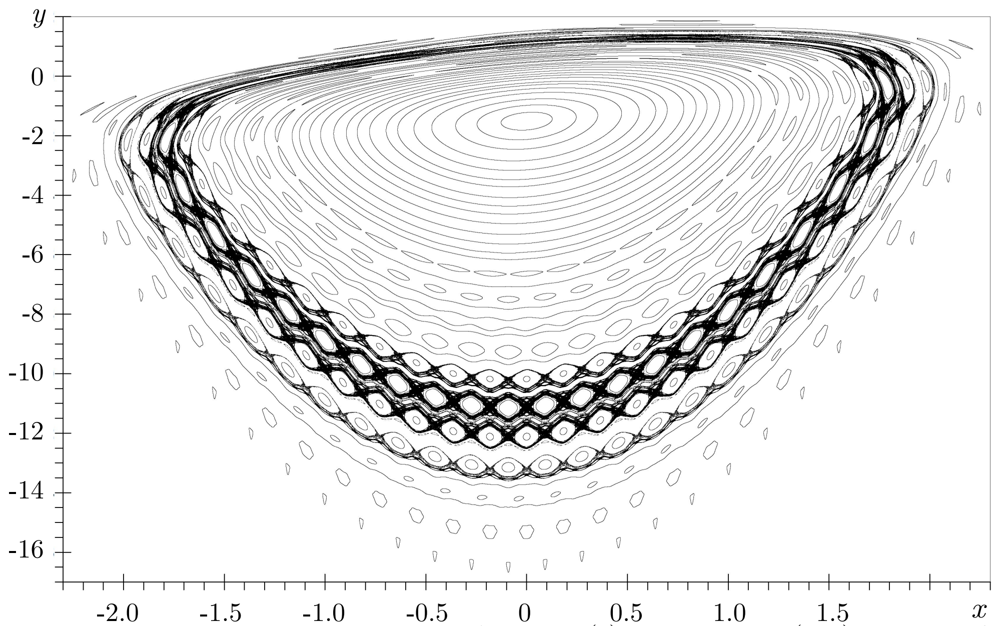

Fig. 7. Траектории различных начальных точек отображения (3) в координатах (x, y) для $a = 1$, $b = 0.2$, что соответствует низкой активности креативного класса. Виден качественный эффект — наличие большого количества траекторий, соответствующих большим циклам, а также — изменение величины q практически от 0 до 1

Fig. 7. Trajectories of different starting points of the mapping (3), for $a = 1$, $b = 0.2$, which corresponds to a low activity of the creative class. A qualitative effect is seen — the presence of a large number of trajectories corresponding to large cycles, as well as — a change in the value of q practically from 0 to 1

activity of the passion class.

The phase trajectories for different values of a are shown in Fig. 4–7. The higher this activity, the larger the area of the simplest (close to that in the linear system) regime of elite change. The smaller the a, the more complex and longer the cycles of elite change.

The second interesting feature is associated with the presence of a model area where its behavior is unstable (рис. 8).

Let, for example, $R \gg 1$ and $q \ll 1$, that is, $x > 0$ and $y > 0$. In this case, x decreases until $f(y) < 0$ is executed, and y increases. At some step, x becomes negative and y also begins to decrease. This leads to the fact that it becomes $f(y) > 0$, which leads to a further increase in x, after which y begins to increase after 0. And everything repeats. It is important that the increments that these variables receive are large numbers, which is why x jumps past 0 in a small number of steps (most often in 1), which leads to switching between the cases of $q \ll 1$ and $1 - q \gg 1$ (see Fig. 7). Similarly, y quickly jumps through the sign change point $f(y)$, which switches the cases of $R \gg 1$ and $R \ll 1$. In other words, the value of R in this model can increase indefinitely.

Such dynamics has its own interpretation within the framework of the theory of elites. She describes a revolutionary situation, which is reflected by the proverb: "There was not a penny, but suddenly altyn." With a low level of social product and an almost complete absence of the elite, significant successes lead to the fact that "almost everyone becomes passionate." Obviously, limiting factors should be taken into account here, — the limits of the use of the applied technologies (1) or the appearance of a large number of "harmonics" in terms of L. N. Gumilev, striving to combine personal and public interests. The model in question does not describe them.

Thus, the study of the conservative model shows that it has multistability, — depending on the initial data (R_0, q_0) it can describe both stable (without qualitative changes), quasi-periodic,

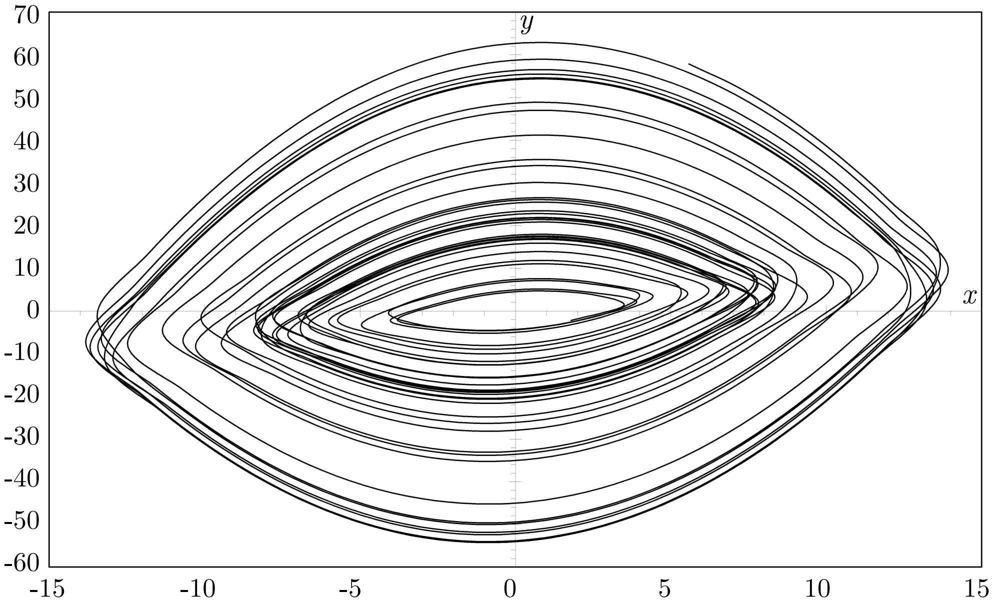

Fig. 8. Динамика траектории, уходящей на бесконечность. Здесь $a = 5.8$, $b = 0.2$, $R_0 = 6$, $q_0 = 0.9$. Такая циклическая динамика соответствует очень быстрым переходам q между значениями очень близкими то к 0, то к 1, а R — между очень большими и малыми значениями. Движение происходит в положительном направлении (против часовой стрелки)

Fig. 8. Dynamics of a trajectory going to infinity. Here $a = 5.8$, $b = 0.2$, $R_0 = 6$, $q_0 = 0.9$. Such cyclic dynamics corresponds to fluctuations q between values very close to 0, then to 1, and R — between very large and small values. The movement is in the positive direction (counterclockwise)

and chaotic regimes of changing elites of the society in question.

The model is sensitive to parameters. The presence of islands of order in a chaotic sea means the possibility of transition from chaotic to quasi-periodic regimes of elite change with an arbitrarily small change in parameters. The latter can be considered as one of the disadvantages of the investigated model.

At the same time, such dynamics perfectly correspond to the ideas of many researchers and philosophers about the uniqueness, uniqueness of many historical trajectories and events taking place.

5. System with noise

The observability of the predicted effects is fundamental from the point of view of modeling. Non-rough systems that have a sensitive dependence on incoming parameters, as a rule, do not have this property. Small changes in parameters can lead to a qualitative change in the trajectory. Nevertheless, the theory of generations has a number of serious confirmations. What are they related to?

Social systems are characterized by the presence of many random factors during the transition to the third "everyday" time scale introduced by F. Braudel [1]. It is natural to take them into account by introducing a small random noise into the equation describing the social product

$$\begin{aligned} R_{n+1} &= R_n(aq_n + b) + \varepsilon_n, \\ q_{n+1} &= q_n^{R_{n+1}}. \end{aligned}$$

As a parameter for simplicity, you can choose ε_n evenly distributed over the interval $[-\varepsilon, +\varepsilon]$. An example of the behavior of a system with noise is shown in Fig. 9.

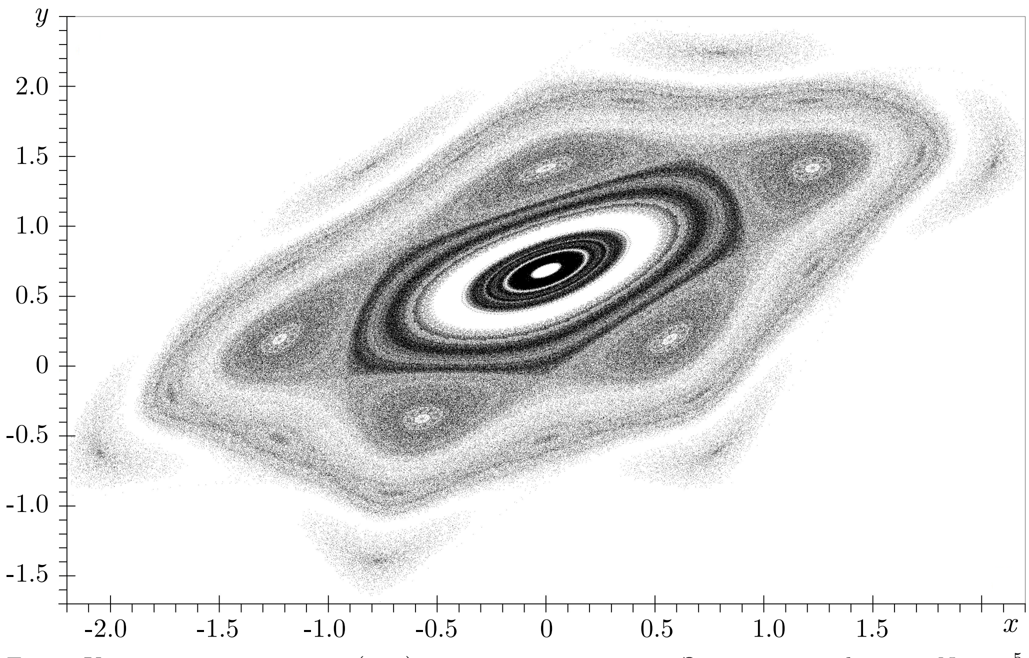

Fig. 9. Картина в координатах $(x,\ y)$ для системы с шумом. Здесь $a = 5.8$, $b = 0.2$, $N = 10^5$, $\varepsilon = 10^{-3}$

Fig. 9. Picture in coordinates (x, y) for a system with noise. Here $a = 5.8$, $b = 0.2$, $N = 10^5$, $\varepsilon = 10^{-3}$

However, there is another qualitative effect here. Not only the initial data and the noise level are essential, but also N — the sample length on which we analyze the model, and a specific sequence of pseudorandom numbers defining ε_n. In Fig. 9 it can be seen that for a long time the sequence (x_n, y_n) can be close to cyclic.

At large characteristic times, it may turn out to be close to a chaotic trajectory. On even larger ones — unlimited solutions may arise. As a rule, the higher the noise level, the larger the initial data area, the trajectory goes to infinity. In other words, this model has a kind of forecast horizon — at short times, the change of elites can be predicted, despite the noise. On the b*o*lshe — the sequence of accidents that society faces becomes decisive.

6. Discussion and connection with the concepts of history and social dynamics

Революции готовят гении, делают романтики, а её плодами пользуются проходимцы.

О. Бисмарк

History and social dynamics are formed as a result of the interaction of many actors, participants in local interactions, endowed with psyche, consciousness, having their own local goals and interests. These numerous contacts make up the evolutionary event historical flow of society as a whole, and of each of its social subsystems separately. Society is self-organizing. Various institutional and ideological structures are constantly emerging and collapsing in it. These macrosocial phenomena O. G.Bakhtiyarov designated by the term Large Processes [24]. He gave an analogy with the metamorphosis of a caterpillar, when a caterpillar that has fulfilled its functions pupates and again turns into an amorphous cell mass, from which a new creature differentiates. At the same time, individual cells are rebuilt under the influence of local causes and do not realize their involvement in the big process as a whole. Elements of social systems — people, although endowed with psyche, mind and consciousness, are also involved in local interactions and for the most part do not realize that they are cells forming a large historical process at the moment of its completion. Some minimally holistic analytics, as a rule, appears later than a separate phase of a large process that has occurred. Nevertheless, by scientific means, one can try to make some breakthrough to the essence of the macro-social outbursts and cycles that are taking place. One of such attempts is the presented work. The use of mathematical apparatus in this case cannot be aimed at predicting the course of a large process for the simple reason that it, being a temporal cooperative self-organizing phenomenon, is being accomplished, and its specific scenario is extremely sensitive to the initial and current values of variables and parameters. Its specific course is not predetermined and, thus, in relation to these processes, the very concept of quantitative forecasting loses its meaning. A much more important purpose of mathematical modeling in this case is to understand the mechanism of the phenomenon and determine the points of impact on a large process.

The dissipative and conservative models, in fact, reflect two fundamentally different views on the description of the historical process. In physics, the concepts of short-range and long-range action have been struggling for many years. This process is now taking place in mathematical history and sociodynamics.

The dissipative model assumes that the details of the initial data will be forgotten. In the theory of elites also tend to think within three generations. Grandchildren can correct the mistakes that grandfathers made. This point of view is shared by many historians, as well as researchers seeking to use the ideas of proximity in this area [1–4]. From their point of view, it is useless and unnecessary to complain about Peter's reforms or Vladimir's choice of religion, considering today's geopolitical and geo-economic problems. These events should be interpreted as initial data for a system describing a historical trajectory.

The actions of the current and several next generations can change the situation. Difficulties can be considered both as an insurmountable barrier and as an opportunity for the desired changes.

"Long-range action" in history is associated with a number of religious cults, with astrology, with the theories of a number of historians, for example, highlighting 100-year, 400-year and even longer cycles of historical development. An overview of such views is given in the book [17]. This approach repeats in many ways the view of modern theoretical physics, which is based on the idea of symmetry, and therefore on the laws of conservation. In this context, it is impossible to ignore the activities of Peter and Vladimir, explaining today's events. The actions of these people have determined the current realities. An alternative story largely proceeds from this.

The problem with this approach arises in the fact that, unlike physics, symmetries and conservation laws are not presented for such a description of history.

The study of a dissipative system with "weeding out part of the elite" with noise coming from F. Braudel's ideas about the hierarchy of historical times shows that the judgment about the rivalry of "molecular" and "cosmic" man in the development of society, the ideas about the cyclical change of elites considered by sociological theories, as well as the possibility of "historical chaos" are well described the investigated model. A remarkable feature of the considered model is that the effect of the "change of elites" on certain initial data is described in both dissipative and Hamiltonian versions.

An important and interesting difference from most of the studied Hamiltonian systems is the presence of unlimited oscillatory solutions. In the theory of modes with exacerbation, it is shown that such trajectories can be a reflection, an intermediate asymptotic of many processes in systems with strong positive feedback [25]. Here, probably, a part of the trajectory describes the socio-economic instability known as the "Singapore miracle" [26]. The rigorous selection and appointment of the best to key positions, the evaluation of not the process, but the result of the work led to a great economic effect in this country. A similar dynamic is observed on part of the trajectory in this model, but then a crisis occurs.

In other words, the study of the model has shown that it is significantly more complex and richer than was assumed in a number of humanitarian theories describing the dynamics of large processes involving changes of elites.

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