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# Synchronization of excitation waves in a two-layer network of FitzHugh–Nagumo neurons with noise modulation of interlayer coupling parameters

*I. R. Ramazanov, I. A. Korneev*✉*, A. V. Slepnev, T. E. Vadivasova*

Saratov State University, Russia
E-mail: ibadulla.ramazanov@yandex.ru, ✉ivankorneew@yandex.ru,
a.v.slepnev@gmail.com, vadivasovate@yandex.ru
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***Abstract.*** The *purpose* of this work is to study the possibility of synchronization of wave processes in distributed excitable systems by means of noise modulation of the coupling strength between them. *Methods.* A simple model of a neural network, which consists of two coupled layers of excitable FitzHugh–Nagumo oscillators with a ring topology, is studied by numerical simulation methods. The connection between the layers has a random component, which is set for each pair of coupled oscillators by independent sources of colored Gaussian noise. *Results.* The possibility to obtain a regime close to full (in-phase) synchronization of traveling waves in the case of identical interacting layers and a regime of synchronization of wave propagation velocities in the case of non-identical layers differing in the values of the coefficients of intra-layer coupling is shown for certain values of parameters of coupling noise (intensity and correlation time). *Conclusion.* It is shown that the effects of synchronization of phases and propagation velocities of excitation waves in ensembles of neurons can be controlled using random processes of interaction of excitable oscillators set by statistically independent noise sources. In this case, both the noise intensity and its correlation time can serve as control parameters. The results obtained on a simple model can be quite general.

***Keywords***: networks of oscillators, nonlinear systems, FitzHugh–Nagumo model, nonlinear coupling, colored noise, noise modulation, synchronization.

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## Introduction

Sources of internal and external noise have an impact on all objects and systems of wildlife and technology. They can significantly change the behavior of the system [1–9]. This should be taken into account when modeling and predicting various possible effects. The effect of noise can be very diverse depending on the noise characteristics and on the dynamics of the system itself. In conditions of strong nonlinearity and complex dynamics, theoretical research methods may not always be applicable. In this case, numerical modeling plays an important role.

Not all effects associated with impact of noise on nonlinear systems are currently sufficiently studied. This is especially true for multicomponent ensembles and distributed systems. A number of works are devoted to the study of the influence of noise on the dynamics of ensembles and distributed systems [10–15]. They consider noise-induced phase transitions and the effects of stochastic and coherent resonances in complex multicomponent systems. One of the important issues is the effect of noise on the synchronization of oscillations. The fundamental phenomenon of synchronization plays an important role in the behavior of various nonlinear systems in systems of different nature and underlies the formation of spatial structures. The presence of noise sources in the self-oscillator leads to the fact that the phase locking is not strict and is observed at a finite time – the so-called «effective synchronization» [16]. The effect of general noise on non-interacting or weakly interacting self-oscillating and excitable systems leads to their frequency-phase synchronization [17–22]. It has been shown that stochastic oscillations arising in bistable and excitable oscillators under the influence of noise can also be synchronized [23–29]. In ensembles of excitable systems, the synchronization of stochastic oscillations at certain noise parameters leads to the emergence of complex cluster structures (coherent resonance chimera) [30]. Parametric noise modulating the coupling parameter can cause partial synchronization of complex spatio-temporal dynamics in the connected layers of chaotic maps [31, 32] and allows you to control the behavior of the ensemble demonstrating chimera states [33].

Noise plays a fundamental role in neurodynamic models, since neurons are excitable oscillators and their behavior is largely determined by the level and parameters of noise in the system [34–42]. The question may also be raised about the possible influence of noise present in the links chains on the dynamics of neural ensembles. In this paper, we consider the simplest model of a neural system, which is a two-layer network of excitable FitzHugh-Nagumo oscillators with a ring topology. Within the limits of one layer (ring), the oscillators are connected by local dissipative coupling. Thus, a traveling excitation wave mode can be implemented in each of the layers. At the same time, noise sources inside the layers are not considered. The connection between the layers is also local and dissipative. However, the interaction strength of the oscillators is random and is determined by independent noise sources. This model is not a realistic model of any neural system. However, it allows us to identify a number of effects caused by noise modulation of the coupling coefficients, which can be quite general. The paper shows how the characteristics of the random coupling component between the layers can affect the synchronization mode of the excitation waves observed in them.

## 1. The model under study and methods of numerical analysis

A model of an excitable neural system is studied in the paper, which is an ensemble of two layers (rings) of FitzHugh-Nagumo (FHN) oscillators [43, 44] with local interaction and interlayer coupling coefficient having a noise component. The equations of the system can be written as

$$\begin{cases} \dot{x}_{j,1} = \dfrac{1}{\varepsilon}\left(x_{j,1} - y_{j,1} - \alpha x_{j,1}^3\right) + \sigma_1\left(x_{j-1,1} + x_{j+1,1} - 2x_{j,1}\right) + (k_0 + kz_j)\left[x_{j,2} - x_{j,1}\right], \\ \dot{y}_{j,1} = \gamma x_{j,1} - y_{j,1} + \beta, \\ \dot{x}_{j,2} = \dfrac{1}{\varepsilon}\left(x_{j,2} - y_{j,2} - \alpha x_{j,2}^3\right) + \sigma_2\left(x_{j-1,2} + x_{j+1,2} - 2x_{j,2}\right) + (k_0 + kz_j)\left[x_{j,1} - x_{j,2}\right], \\ \dot{y}_{j,2} = \gamma x_{j,2} - y_{j,2} + \beta, \\ \dot{z}_j = -\mu z_j + \sqrt{2\mu}n_j(t), \quad j = 1, 2, \ldots, N; \\ \text{boundary conditions:} \quad x_{j\pm N,i}(t) = x_{j,i}(t),\ y_{j\pm N,i}(t) = y_{j,i}(t),\ i = 1, 2. \end{cases} \tag{1}$$

Here $j$ is the element number in the layer, $i$ is the layer number. The layers consist of identical FHN oscillators with dissipative local coupling inside the rings. The coefficients of the internal coupling $\sigma_1$ and $\sigma_2$ for two rings may differ. All other parameters are the same. Oscillators with the same numbers $j$ belonging to different layers are locally connected. The strength of the interlayer coupling is characterized by the same constant value of the coupling coefficient $k_0$, to which random components $kz_j(t)$ are added. In the studies carried out, it is assumed that

$k_0 = 0$, i.e. there is no constant component of the coupling. Random variables $z_j(t)$ are described by one-dimensional Ornstein-Uhlenbeck processes− with the same statistical characteristics generated by independent sources of Gaussian white noise $n_j(t)$. Independent processes $z_j(t)$ have a Gaussian distribution with a stationary mean values $\langle z_j(t)\rangle \equiv 0$. Thus, the coupling coefficient determined by noise changes sign over time, becoming either positive (attractive coupling) or negative (repulsive coupling). The variance of the processes $z_j(t)$ in stationary mode is equal to one: $D(z_j) \equiv 1$, and the correlation function is described by a decreasing exponent:

$$\Psi_{z_j}(\tau) = e^{-\mu|\tau|}, \quad \tau = t_2 - t_1. \tag{2}$$

Accordingly, the power spectral density of these processes has the Lorentzian form

$$W_{z_j}(\omega) = \frac{4\mu}{\mu^2 + \omega^2}, \quad \omega > 0. \tag{3}$$

The correlation time and the width of the spectrum at the half-power level are determined by the parameter $\mu$: $\tau_{\rm cor} = \mu^{-1}$; $\Delta\omega_{1/2} = \mu$. It is important to note the following feature of the considered color noise model. The variance of all normalized sources is equal to one, and taking into account the multiplier $k$, it is equal to $k^2$. Since the variance is proportional to the integral of the spectral density, a fixed value of the variance means that the integral power of noise is constant regardless of the width of the power spectrum. With an increase in the spectrum width (with an increase in the parameter $\mu$), the integral power of noise sources is distributed over an increasingly wide frequency range. The spectral density at the maximum point at zero and at other frequencies decreases. At $\mu \to \infty$, the spectral density tends to zero. This does not allow us to consider the limiting transition to white noise with a finite spectral density.

During numerical simulation, the following parameters of the system (1) were fixed: $N = 100$ (the number of elements in the ensemble layer); $\alpha = 1/3$, $\beta = 0.2$, $\gamma = 0.8$, $\varepsilon = 0.01$ (oscillator parameters corresponding to excitable mode); $\sigma_1 = 4.5$ (coupling coefficient in the first layer); $\sigma_2 = 4.5$ or $\sigma_2 = 5.5$ (coupling coefficient in the second layer); $k_0 = 0$ (constant component of the interlayer coupling coefficient). The intensity of the noise component of the interlayer coupling $k$ and the parameter $\mu$, which controls the width of the spectrum of noise sources, are considered as controlling and change in the course of research.

To establish the initial modes in two non-interacting rings , the initial conditions were set as follows:

$$\begin{aligned} &x_{j,1}(0) = 2\sin(2\pi j/N), && y_{j,1}(0) = 2\cos(2\pi j/N), \\ &x_{j,2}(0) = 2\sin(2\pi j/N + \varphi), && y_{j,2}(0) = 2\cos(2\pi j/N + \varphi), \quad z_j(0) = 0. \end{aligned} \tag{4}$$

The initial conditions (4) provide a traveling wave mode in both rings with a wavelength equal to the length of the system (the main wave mode). The parameter $\varphi$ defines the phase shift of the two waves. When obtaining a steady-state mode of traveling waves in two non-interacting rings, the parameter $k$ is assumed to be zero. At the same time, the noise variables $z_j$ do not affect the wave processes. However, integration at the setting time of wave modes also provides the establishment of statistical characteristics of noise sources. The instantaneous states of oscillators and noise variables in steady-state mode in the absence of a connection between the rings were stored and then used as initial conditions for studying the interaction of the rings.

The paper investigates the effect of phase synchronization of excitation waves in two identical layers, as well as effective frequency-phase synchronization when the parameters of the intra-layer coupling are different. To diagnose the synchronization of waves in identical layers, a value is calculated that characterizes the degree of identity (in-phase) of spatial structures, which we will call the error of in-phase synchronization. The instantaneous value of the synchronization error is defined as

$$\delta_t = \frac{1}{N}\sum_{j=1}^{N}\Big([x_{j,2}(t) - x_{j,1}(t)]^2 + [y_{j,2}(t) - y_{j,1}(t)]^2\Big). \tag{5}$$

Since $\delta_t$ can change over time, its average value was calculated

$$\delta = \langle \delta_t \rangle \,, \tag{6}$$

where the brackets $\langle \ldots \rangle$ mean time averaging.

To diagnose the synchronization of oscillation frequencies (phase velocities), the average periods (inter-spike intervals) of oscillations of the elements in the two layers are calculated (they are the same for all oscillators in one layer). The instantaneous period $\tau_n$ is the time between two consecutive intersections in the same direction by the value of the variable $x$ of the level taken as the firing threshold (when calculating, the value $x_{\text{th}} = 1.5$ was chosen). The average period is

$$T = \lim_{M\to\infty}\frac{1}{M}\sum_{n=1}^{M}\tau_n, \tag{7}$$

where $M$ is the number of consecutive intersections in one direction of the level $x_{\text{th}}$ at the observation time. The average oscillation frequency $f$ is the inverse of the average period. Even if the fluctuations are periodic, the values of $\tau_n$ may differ for different $n$, so averaging is applied. To diagnose the synchronization of mean periods and frequencies, the following ratio was calculated

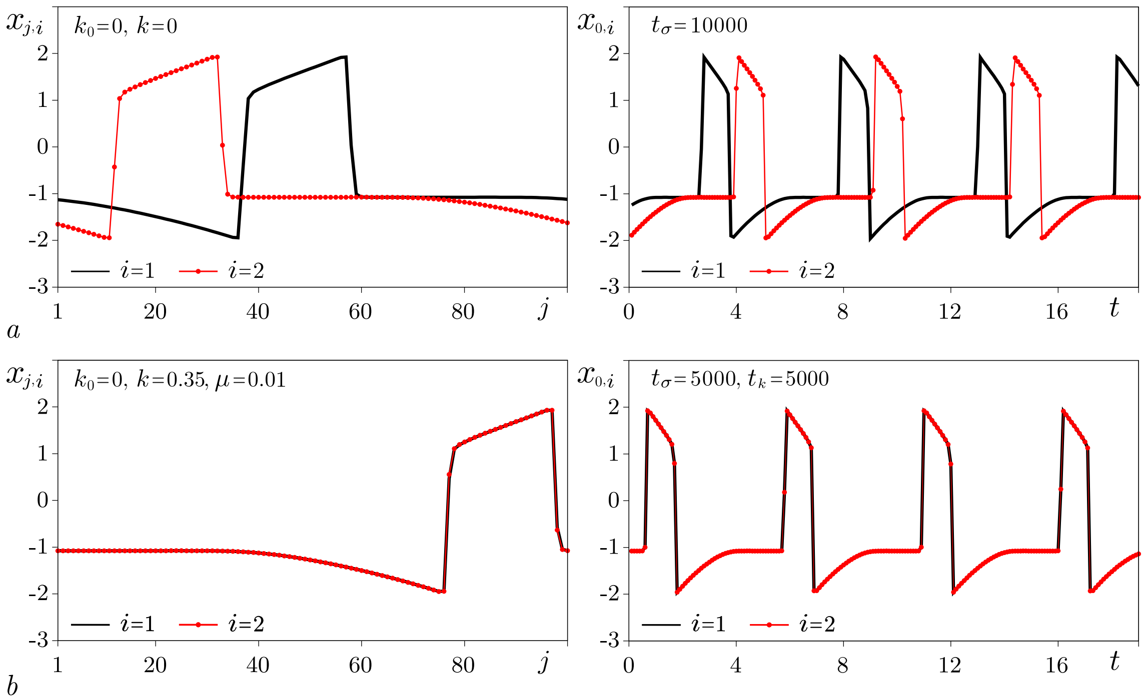
$$\theta = \frac{T_2}{T_1} = \frac{f_1}{f_2}, \tag{8}$$

where the indexes indicate the layer number.

## 2. Synchronization of phases of traveling waves in two identical layers

Traveling wave modes were set in two non-interacting layers ($k = k_0 = 0$) with selected parameter values and initial conditions (4). The wave propagation velocities and oscillation frequencies were exactly the same, but there was a constant phase shift between the waves. The steady states of two non-interacting layers were fixed and used as initial conditions for further research. When studying the interlayer interaction, the system of equations (1) was first integrated with the selected initial conditions (at $k = k_0 = 0$) at the time $t_\sigma$. Then a connection was introduced between the layers, which was completely random (noisy) in our studies, i.e. it was assumed that $k_0 = 0$, $k \neq 0$. The system was integrated in the presence of coupling at the setting time $t_k$. Then the average value of the synchronization error $\delta$ was calculated at the time $t_\delta$. The transient times of $t_\sigma$ and $t_k$ were at least 5000 units of the dimensionless time of the system (i.e. $5 \cdot 10^6$ integration steps). The averaging time was chosen to be equal to 4000 dimensionless units.

Instantaneous wave profiles and oscillations of units with the same number $j = 0$ in rings in steady-state mode in the absence of interaction are shown in Fig. 1, *a*. The phase shift of the oscillations and wave profiles is clearly visible. With the introduction of interlayer noisy coupling with intensity $k = 0.35$ and parameter $\mu = 0.01$, after the setting time $t_k$, synchronization of instantaneous wave profiles and oscillation phases of all oscillators with the same numbers in two layers is observed. The corresponding regime is illustrated in Fig. 1, *b*.




*a*

*b*

Fig. 1. Snapshots of traveling waves (left column) and oscillations $x_{0,i}$ (right column) in two identical rings $\sigma_1 = \sigma_2 = 4.5$: *a* — in the absence of interlayer coupling $k = 0$; *b* — in case of random coupling $k = 0.35$, $\mu = 0.01$

A more detailed study of the behavior of two layers with noisy coupling shows that periodic oscillations with a constant phase shift between the layers are not established. The observed synchronization effect strongly depends on the time of observation and the realizations of random processes that provide interlayer coupling. A significant role is played by the value of the parameter $\mu$, which determines the spectral-correlation properties of noise sources. These properties of noise induced synchronization of identical layers are illustrated in Fig. 2.

In Fig. 2, *a*, *b* the dependences of the instantaneous value of the synchronization error (the value of $\delta_t$) on time are given, obtained at $k = 0.35$, $\mu = 0.01$ for two different noise realizations as noise 1 and noise 2. When studying stochastic differential equations (1), the increments of the states of all random noise sources $n_j(t)$ at the integration step are set using a sequence of random numbers generated by a special program - a generator of uncorrelated random numbers with a standard Gaussian distribution. One or another realization of noise (more precisely, a set of realizations of all sources $n_j(t)$) is determined by some input parameter of this program (initializing variable). Noise sources at $\mu = 0.01$ are narrow-band low-frequency and the processes of interaction between the elements of the two layers will be «slow».

These graphs indicate that a constant phase shift between the waves in the two layers is not established at the considered observation time. The dependencies of $\delta_t$ on time have the form of an intermittency process. This indicates the complex nature of oscillations of the elements in two layers in two layers, which is caused by noise. At the same time, long intervals of completely synchronous behavior can be observed. They are replaced by intervals at which $\delta_t$ takes large values. The specific sequence of states depends on the realizations of noise. Due to the duration of intervals with different behavior, it is not possible to obtain a steady-state average value of the synchronization error at times acceptable for numerical study.

The corresponding graphs of the dependence of $\delta$ on the averaging time are shown in Fig. 2, *e* (curves 1, 2). The averaging time varied from zero to 5000 dimensionless units. To

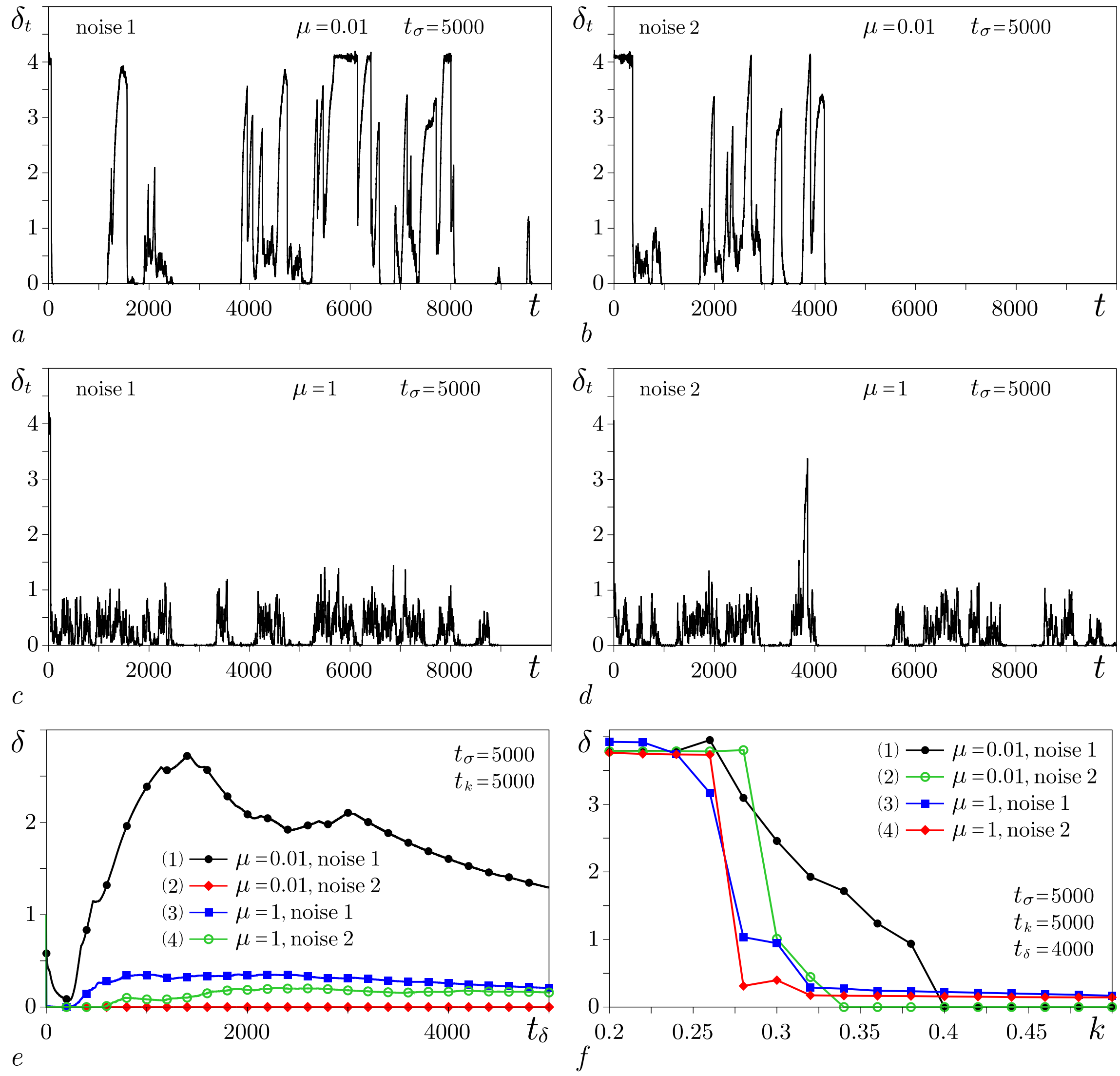



Fig. 2. Behavior of the synchronization error with different choice of noise realizations: *a*, *b* — dependences of the instantaneous value of the synchronization error $\delta_t$ on time at $k = 0.35$, $\mu = 0.01$; *c*, *d* — time dependences of $\delta_t$ at $k = 0.35$, $\mu = 1.0$; *e* — dependences of the average synchronization error $\delta$ on the averaging time at $\mu = 0.01$, $\mu = 1.0$ and a fixed coupling parameter $k = 0.35$; *f* — dependences of $\delta$ on the coupling parameter $k$ obtained for two noise realizations at $\mu = 0.01$, $\mu = 1.0$ and fixed settling and averaging times (shown on the graphs)

implement noise 2, any averaging interval in this range after the establishment time ($t_k = 5000$) does not include non-zero values of the mgn synchronization error ( Fig. 2, *b*) and the average error is strictly zero. However, the conclusion about perfect synchronization may be premature. It is possible that with an increase in the integration time, the synchronization failure will appear again.

Similar dependences of $\delta_t$ on time with the same value of the coupling parameter $k = 0.35$, the same noise realizations, but with more broadband noise with the parameter $\mu = 1.0$ are shown in Fig. 2, *c*, *d*. These dependences also show the absence of a steady-state regime with a constant phase shift of the waves. Unlike narrowband noise, long intervals of complete synchronization of layers are not observed at $\mu = 1.0$. However, the values of $\delta_t$ themselves become significantly smaller. Averaging the synchronization error at times of the order of 4000...5000 dimensionless units gives an almost steady and close value of $\delta$ for both realizations of noise (curves 3, 4 in

Fig. 2, *e*).

In Fig. 2, *f* the dependences of the values of the average error δ on the intensity of the noise interaction $k$ obtained for two noise realizations at $\mu = 0.01$ and $\mu = 1.0$ are compared. The time of averaging of synchronization error is chosen to be $t_\delta = 4000$ dimensionless units. According to the graphs, one can approximately find the synchronization threshold. It corresponds to the value of the parameter $k$, at which there is a sharp decrease in the value of δ. The quantity δ can take zero values (full synchronization) or reach the level of sufficiently small values (partial synchronization). For noise with the parameter $\mu = 1.0$, the dependencies $\delta(k)$ obtained for the two noise realization behave similarly and the threshold values of $k$ are approximately the same (curves 3, 4). For narrowband low-frequency noise with $\mu = 0.01$, the dependences $\delta(k)$ obtained for the two noise realizations are very different (curves 1, 2). In the case of noise 1 realization, the synchronization error decreases gradually with the growth of $k$ (curve 1) and there is a difficulty with determining the synchronization threshold. If we consider full synchronization and determine the threshold by the condition $\delta_m = 0$, then at $\mu = 1.0$ full synchronization is not observed, and at $\mu = 0.01$ the threshold significantly depends on the realization of noise.

Problems in determining the synchronization threshold are associated with an insufficiently long averaging time of $t_\delta$. However, the slow nature of the processes (especially with low-frequency noise) and the large time spent on integrating the equations (1) do not allow averaging over a time sufficient to establish the value of δ. Nevertheless, the obtained graphs $\delta(k)$ show the presence of the effect of layer synchronization (at least partially) and the dependence of the synchronization threshold on the spectral-correlation properties of color noise, which determines the interlayer connection.

In Fig. 3, *a* the dependences of the value δ on the intensity of the noise modulation of the coupling coefficient $k$ are given. The same realizations of noise sources $z_j(t)$ (noise 1), times of regime setting and times of averaging the synchronization error are used for calculations. With an increase in the width of the noise spectrum (parameter μ), at first there is a slight decrease in the synchronization threshold. Then, with a very wide range of noise, the threshold increases. At $\mu = 100$, synchronization ceases to be observed within the change of the parameter $k \leqslant 1$ (the behavior at $0.5 \leqslant k \leqslant 1$ is not reflected in the graphs).

The reason for the dependence of the synchronization threshold on the width of the noise spectrum is explained in Fig. 3, *b*. It shows graphs of the spectral density of noise sources $z_j(t)$

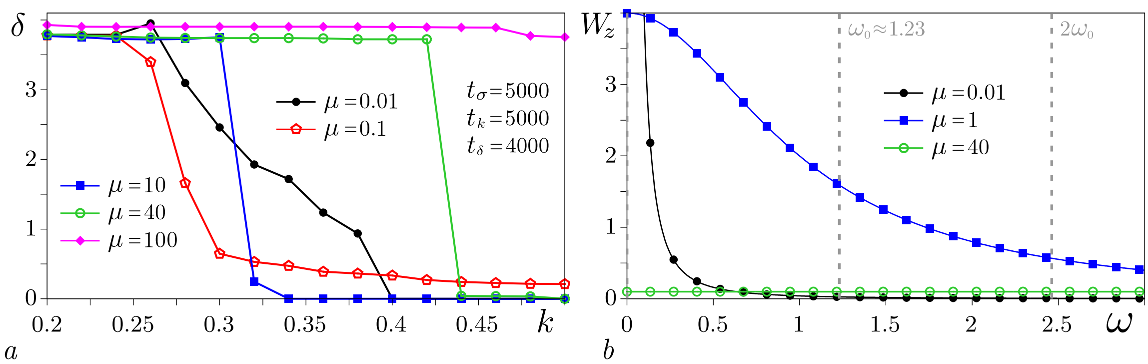
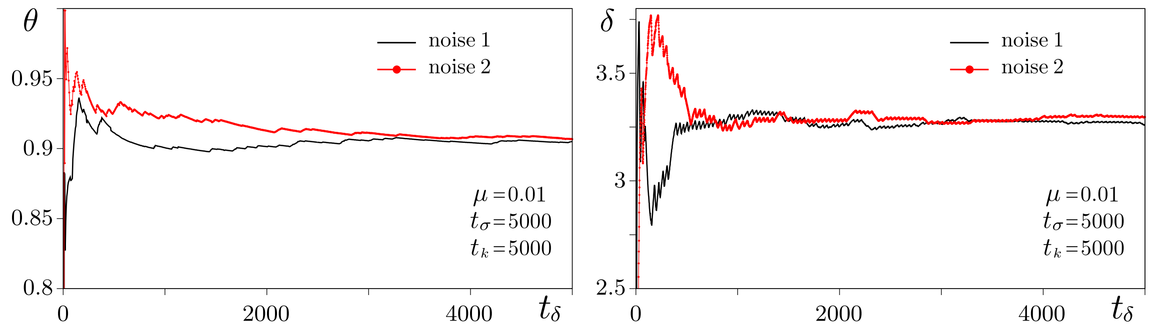



Fig. 3. The effect of synchronization at different values of the spectrum width of the noise coupling: *a* — dependencies of the synchronization error δ of travelling waves on the noise coupling intensity $k$ in two identical rings $\sigma_1 = \sigma_2 = 4.5$ at different values of noise parameter μ; *b* — graphs of noise spectral density for different values of parameter μ calculated by the formula (3). When calculating all dependences $\delta(k)$, the noise 1 realization was used. The settling and averaging times are shown in the figure. The vertical dotted lines in panel (*b*) mark the spectral lines in the spectrum of oscillations of oscillators in the regime illustrated in fig. 1 at $k = 0$

calculated using the formula (3) for various values of the parameter μ. Vertical dotted lines show the spectral components in the power spectrum of oscillators in layers without coupling. Since these oscillations are periodic (Fig. 1, *a*), then their spectrum is discrete and the spectral density consists of δ peaks at the fundamental frequency $\omega_0 = 1.23 \pm 0.001$ and its harmonics, as well as at zero frequency, since the average value of the oscillations is different from zero. At $\mu = 0.01$, the noise spectrum is concentrated at low frequencies, and in the region of the main oscillation frequency, the spectral noise density is very small. With the growth of the parameter μ, the noise spectrum becomes wider and the spectral density at the frequency $\omega_0$ increases somewhat. This leads to a more effective influence of noise coupling on the behavior of layers. With a further increase in the parameter μ, the spectral noise density at the harmonics of the main frequency $\omega_0$ increases, but decreases at the main frequency and at lower frequencies. This leads to an increase in the synchronization threshold, and further, with large values of μ, synchronization does not occur.

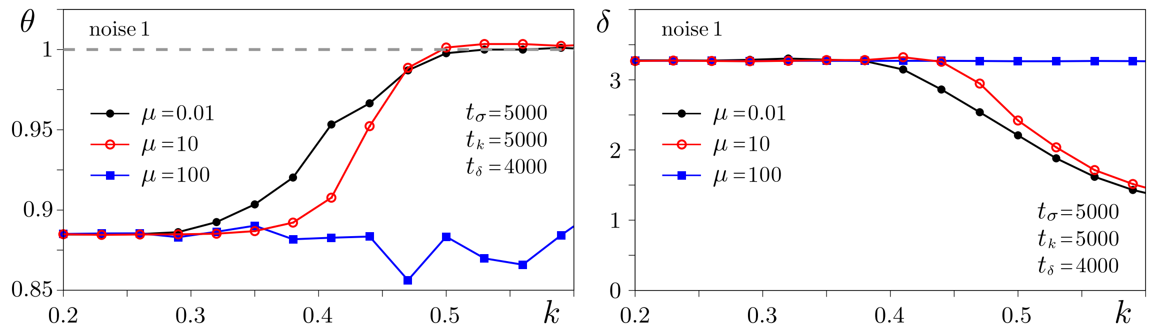
## 3. Synchronization of traveling wave speeds in two rings with different values of coupling coefficients of elements in the two rings

The speed of the traveling excitation wave in the ring of the FitzHugh-Nagumo oscillators significantly depends on the coupling coefficient of the ring elements. Thus, by setting the value $\sigma_2 \neq \sigma_1$ in the second layer, it is possible to obtain traveling waves in each of the layers (in the




*a* *b*

Fig. 4. The process of establishing the average synchronization characteristics in non-identical layers with intralayer coupling coefficients $\sigma_1 = 4.5$, $\sigma_2 = 5.5$ for interlayer coupling parameters $k = 0.35$, $\mu = 0.01$ and two different noise realizations: *a* — dependence on averaging time $t_\delta$ for the ratio of the average periods $\theta = T_2/T_1$ and *b* — the average synchronization error δ. Settling times are indicated on the graphs




*a* *b*

Fig. 5. Synchronization characteristics in two non-identical rings $\sigma_1 = 4.5$, $\sigma_2 = 5.5$ at different values of noise parameter $\mu = 0.01, 10, 100$: *a* — dependencies of the ratio of average oscillation periods $T_2/T_1$ and *b* — synchronization errors δ on the interlayer coupling coefficient $k$. In the calculations, one realization of noise (noise 1) was used. The settling and averaging times are shown in the figure

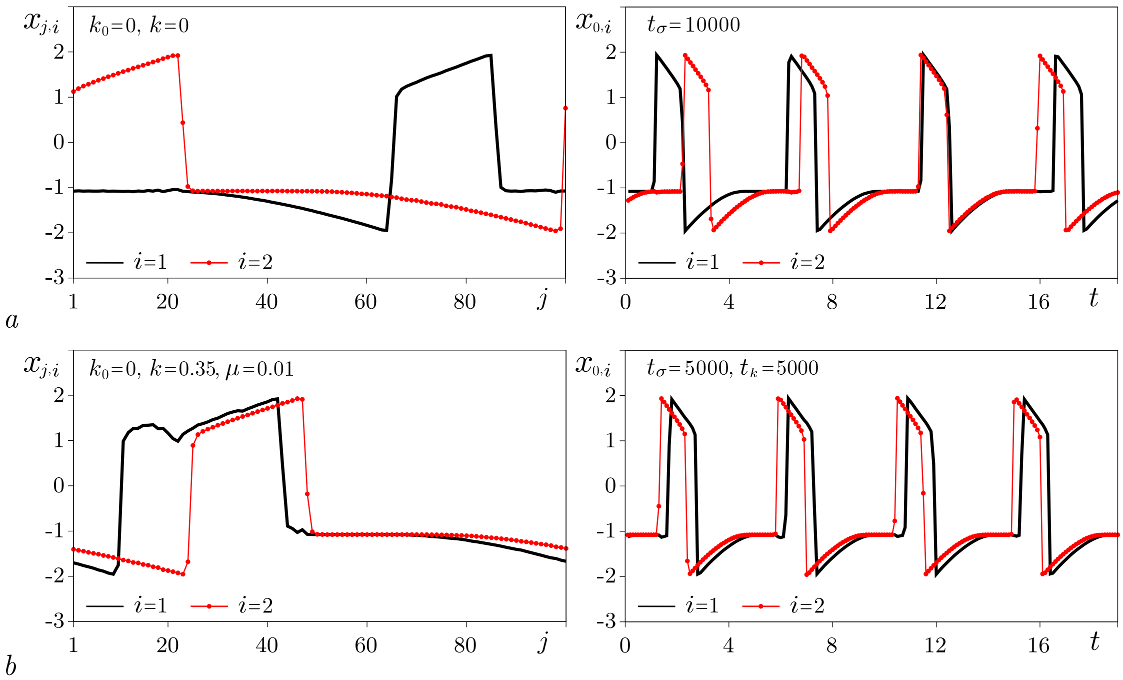



Fig. 6. Snapshots of traveling waves (left column) and oscillations $x_{0,i}$ (right column) in two non-identical rings $\sigma_1 = 4.5$, $\sigma_2 = 5.5$: *a* — in the absence of interlayer coupling $k = 0$; *b* — in case of noisy coupling $k = 0.35$, $\mu = 0.01$

absence of interaction between them), differing not only in initial phases, but also in velocities. Accordingly, the oscillators in the two layers will oscillate at different frequencies. In our studies, the values of intralayers coupling coefficients are: $\sigma_1 = 4.5$ or $\sigma_2 = 5.5$. This ensured the difference in the velocities of wave processes in the absence of coupling between the layers. With the introduction of random coupling, the capture of the average interspike intervals and the average elements oscillation frequencies was observed. The behavior of average interspike intervals (average oscillation periods) in two layers layers, their relations $\theta = T_2/T_1$ and synchronization errors $\delta_m$ were studied. Coupling was carried out only through random variables ($k_0 = 0$).

In Fig. 4 examples of dependencies of the ratio of the average oscillation periods $\theta = T_2/T_1$ and the synchronization error $\delta$ on the averaging time $t_\delta$ for two noise realizations with fixed parameters $k = 0.35$ and $\mu = 0.01$ are given. In this case, the establishment of average values even with low-frequency coupling noise is faster than in identical layers. At $t_\delta$, the averaged characteristics of $\theta$ and $\delta$ can be considered practically steady. Moreover, their stationary values within the accuracy of calculations do not depend on the realization of noise.

The dependences of the ratio of the average oscillation periods in two layers $\theta = T_2/T_1$ and the synchronization error $\delta$ on the intensity of the coupling noise $k$ obtained for three values of the parameter $\mu$ are shown in Fig. 5. At a certain (threshold) value of the random coupling intensity $k$, the ratio of the average periods becomes close to unity (Fig. 5, *a*). In general, calculations show that the synchronization of wave speeds, average periods/ average frequencies of oscillations is observed in a fairly wide range of values of $\mu$. The synchronization thresholds at $\mu = 0.01$ and $\mu = 10$ are practically the same. However, if the noise becomes very broadband ($\mu = 100$), then synchronization is not observed.

The disappearance of the frequency synchronization effect in the case of broadband noise, as well as the disappearance of the synchronization of wave phases in identical layers, is explained by a decrease in the spectral density of noise with an expansion of the frequency range and a fixed variance value. The dependences of the synchronization error δ on the intensity of the coupling noise $k$ show that in the case of frequency synchronization, the value of δ decreases markedly, but remains far from zero (Fig. 5, $b$). Thus, in the studied range of values of the parameter $k \leqslant 0.5$, in-phase or close to it synchronization is not observed in the frequency synchronization region. However, it occurs (at least partially) with a higher intensity of noisy coupling.

In Fig. 6, $a$ instantaneous profiles of waves and oscillations of elements with the same number $j = 0$ in two layers in steady-state regime in the absence of interaction are given. The out-of-sync oscillations of elements in two layers is clearly visible on the time realizations of the variables $x_{0,1}(t)$ and $x_{0,2}(t)$ (right column). With the introduction of noise coupling between the layers, synchronization of wave speeds and oscillation frequencies is observed (Fig. 6, $b$). Oscillations cease to be strictly periodic. The instantaneous interspike intervals in the two rings do not completely coincide. This can be seen in the time realizations shown on the right. Synchronization of wave velocities, frequencies and interspike oscillation intervals is observed only on average and does not depend on the choice of the setting time of noise sources or initializing variables that determine the realization of noise.

## Conclusion

The carried out studies allow us to conclude that there are effects of frequency and phase (full and partial) synchronization of layers in the network of excitable FitzHugh-Nagumo oscillators in the mode of traveling waves with a random nature of the intensities of interlayer interactions of oscillators set by independent sources of colored Gaussian noise. The average value of the coupling noise for each pair of interacting oscillators in steady-state regime was chosen to be zero, i.e. there was no constant coupling component. Nevertheless, the effects of layer synchronization were observed both in the case of identical layers and in the presence of differences in the parameters of the layers. With a fixed integral power of noise sources, the change in the spectral-correlation properties of noise significantly affects the effects of synchronization, which ceases to be observed with an increase in the width of the noise spectrum. This effect is explained by a decrease in the spectral density of noise with the expansion of the spectrum. Probably, in the case of modulation of the interlayer coupling by white noise with a certain spectral density (intensity), synchronization will also be observed.

The process of setting the synchronization mode in identical and non-identical layers was studied. It is shown that this process can be very long and have the character of random intermittency. This is especially noticeable in the case of narrow-band low-frequency coupling noise. The constant phase difference of traveling waves is not established even in identical layers. The stationary value of the average synchronization error is not achieved even with very long averaging times. This behavior of the system prevents the determination of the synchronization threshold, which turns out to depend on the time of establishment and averaging, as well as on the choice of noise realization. However, these effects decrease with an increase in the width of the noise spectrum and are weakly manifested in the case of non-identity of the interacting layers.

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