Изв. вузов «ПНД», т.12, № 4, 2004

COUPLED ROTATORS APPROACH TO THE DYNAMICS OF INTERACTING MAGNETIC LAYERS*

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The dynamical properties of two anisotropic rotators modelling magnetic layers coupled by bilinear and biquadratic interactions terms are considered. The easy plane case is considered in detail and it is shown that the presence of the biquadratic coupling produces a sequence of bifurcations in the rotational phase spaces. In a first bifurcation a new ground state with rotators located in the easy plane and including a finite angle is generated. Hyperbolic points appear by increasing the biquadratic coupling in a second bifurcation. These hyperbolic points correspond to a nonparallel rotator configuration with rotators splitted off from the easy plane and symmetrically displaced along the transversal direction. The corresponding energy is located between the energies of the ground state and the parallel rotator configuration. Phase space portraits displaying how the stationary states are embedded in the rotational flow are shown.

80th anniversary of Professor
Yu.L.Klimontovich is devoted

Introduction

Magnetic materials display a wealth of nonlinear phenomena [1] among which in the last decade the biquadratic coupling mechanism between magnetization vectors in layer systems has attracted particular attention (see e. g. [2-4]). In such systems the coupling between the magnetization vectors of different layers can be influenced to a large extent by selecting specific spacers through which the magnetic layers interact. Then by changing the spacer material and layer configurations the magnetic properties of the layer systems can be varied to a great extent and desired properties for applications produced, for the calculations of the interlayer exchange coupling from microscopic quantum models for different systems we refer to the recent work [5]. In the discussion of the coupling between the magnetic layers the relation between the bilinear and biquadratic coupling types has turned out to be of particular importance [2]. Systematic studies of the coupling coefficients for different magnetic layer systems reveal that the magnitude of the biquadratic coupling coefficient can be well above the bilinear

* Данная работа выполнена на кафедре физики низких температур и сверхпроводимости МГУ, на которой долгие годы работал профессор Ю.Л. Климонтович, и на кафедре статической физики и нелинейной динамики Гумбольдтского университета Берлина, возглавляемой в течение многих лет профессором В. Эбелингом.

coefficient for some parameter regions (e. g. for some values of the spacer layer thickness [6]). So far the corresponding calculations were static in the sense that they focussed on the energies of the energetically lowest states, which can display transitions when the bilinear and biquadratic coupling coefficients are changed, as was pointed in [2,3].

The aim of this paper is to consider the implications of the presence of both the bilinear and biquadratic coupling mechanism on the dynamical level for coupled magnetization vectors. By using a classical approach familiar from the Landau-Lifshitz equations we investigate the dynamics of the magnetization vectors in a model of coupled rotators with both the bilinear and biquadratic interaction terms present. Starting from the equations of motion for the coupled rotators we will show that a sequence of bifurcations is possible in the rotational phase space of the rotators. In order to restrict the number of possible relations between the parameters in the equations of motion we will consider this bifurcation sequence for the easy plane case of the intralayer part. Analysing the bifurcation sequence we find all the stationary rotator configurations including the ground state and the relations between these configurations from the energetic side. In particular we will show that besides the bifurcation to a new ground state there is another bifurcation in the excited state region of the rotator configurations creating hyperbolic points. The full dynamic analysis presented here is a necessary condition for a further understanding of dynamical phenomena in coupled magnetic layer systems such as switching between different stationary states and relaxation.

The paper is organized as follows. In the section 1 the model for the coupled rotators is formulated and the basic equations are derived. Stationary states and their stability are treated in the sections 2 and 3, respectively. Based on the results of the sections 2 and 3 in the 4 section the bifurcation sequence is discussed and representative phase space portraits are shown. Finally in the last section we summarize our conclusions.

1. Model

We consider a system of two magnetic layers with coupled magnetization vectors. The magnetization vectors are represented by rotators and described by a classical Hamilton function

$$H = H_0 + H_{\text{int}}, \tag{1}$$

where H_0 and H_{int} are the Hamiltonians of the isolated rotators and their interaction, respectively. The Hamiltonian of the isolated rotators H_0 is taken as to characterize an anisotropic system with different in plane and out of plane constants $\alpha>0$ and $\beta>0$, i.e.

$$H_0 = \Sigma_{i=1}^{2} \alpha/2(M_{ix}^2+M_{iy}^2) + \beta/2 M_{iz}^2. \tag{2}$$

In (2) the z-axis is chosen transversal to the plane, M_{ix}, M_{iy} and M_{iz} are the components of the angular momentum vectors of the rotators $\mathbf{M}_i$, i=1,2. The interaction is taken as an expansion in powers of the scalar products of these vectors $(\mathbf{M}_1,\mathbf{M}_2)$

$$H_{\text{int}} = \Sigma_k \mathrm{A}_k(\mathbf{M}_1,\mathbf{M}_2)^k, \tag{3}$$

where A_k, k=1,2,.. are the corresponding interaction constants. We will consider the case of two anisotropic rotators coupled by both the bilinear and biquadratic interaction terms, i.e. we will keep the interaction terms up to the second order k=2. For the sign of A_1 we will take the case $A_1=-|A_1|<0$. Then in the absence of biquadratic coupling the minimum energy corresponds to a parallel orientation (ferromagnetic case) of the magnetization vectors of the layer system. For the sign of the biquadratic interaction constant A_2 we assume for the sake of definiteness that $A_2>0$, which is observed in different systems relevant for applications [2]. A schematic representation of the model is shown in the fig. 1.

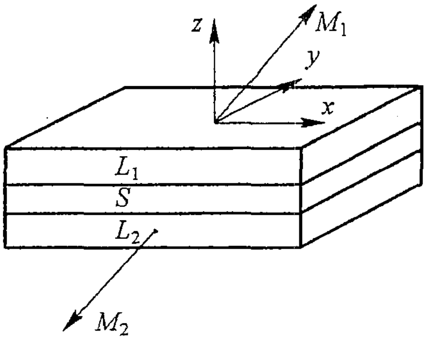

Fig. 1. Schematic representation of the magnetic layer system. Two magnetic layers L_1 and L_2 are shown with the magnetization vectors $\mathbf{M}_1$ and $\mathbf{M}_2$, respectively. The anisotropy axis of both layers is directed transversal to the layer planes and is parallel to the z-axis. The magnetizations $\mathbf{M}_1$ and $\mathbf{M}_2$ interact through the spacer S by both bilinear and biquadratic interaction terms, see eq. (3). By changing the spacer material and/or thinkness the interaction strength is changed.

The equations of motion for the coupled rotators are obtained from

$$d\mathbf{M}_i/dt = \{\mathbf{M}_i, H\}, \tag{4}$$

where {.,.} represent the classical Poisson brackets. The r.h.s. of (4) is calculated by using the explicit forms of H_0, H_{int} and the standard angular momentum brackets for M_{ix}, M_{iy} and M_{iz}. In particular for the interaction part one finds the relation

$$\{\mathbf{M}_i, (\mathbf{M}_1\mathbf{M}_2)^k\} = (-1)^i k(\mathbf{M}_1\mathbf{M}_2)^{k-1}[\mathbf{M}_1\times\mathbf{M}_2], \quad i=1,2. \tag{5}$$

Collecting all terms one obtains the system of coupled rotators

$$d\mathbf{M}_1/dt = (\beta - \alpha)(\mathbf{n}\mathbf{M}_1)[\mathbf{n}\times\mathbf{M}_1] + (A_1+2A_2(\mathbf{M}_1,\mathbf{M}_2))[\mathbf{M}_2\times\mathbf{M}_1], \tag{6}$$

$$d\mathbf{M}_2/dt = (\beta - \alpha)(\mathbf{n}\mathbf{M}_2)[\mathbf{n}\times\mathbf{M}_2] + (A_1+2A_2(\mathbf{M}_1,\mathbf{M}_2))[\mathbf{M}_1\times\mathbf{M}_2]. \tag{7}$$

Here $\mathbf{n}$ denotes a unit vector directed transversal to the (x, y)-plane along the z-axis. The equations of motion (6), (7) conserve the system energy $E=H$, the z-projection of the total momentum $M_z=M_{1z}+M_{2z}$ and the square of each of the vectors $\mathbf{M}_i^2$, $i=1,2$, as is easily verified by calculating the corresponding Poisson brackets. We will assume conditions for which $\mathbf{M}^2=\mathbf{M}_1^2=\mathbf{M}_2^2$, i. e. the moduli of the angular momenta of rotators are equal and will be denoted by M below. This represents the situation when the interacting layers are prepared from the same magnetic material. The easy plane case, i. e. $\beta>\alpha$, will be considered. As will be shown below, in this case by remaining in the plane the magnetization vectors can pass from the parallel into a nonparallel ground state configuration, if the biquadratic coupling becomes strong enough.

2. Stationary states and energies

In view of the conservation of $\mathbf{M}_i^2$ the phase space of each of the vectors $\mathbf{M}_i$, is located on a sphere (rotational phase space). On this sphere according to (2) an equator and poles formed by the intersections of the (x,y)-plane and the z-axis with the sphere are distinguished (the interaction part (3) is isotropic and does not introduce particular directions). The z-axis transversal to the equator plane determines a reflection symmetry with respect to the two half spaces $z>0$ and $z<0$. This symmetry is present in the locations of the stationary states outside the equator plane and in the structure of the phase space portraits.

From the zeros of the r.h.s. of the system (6) and (7) we now find the following stationary configurations of the coupled rotators.

Parallel equator plane configuration. From the r.h.s. of the equations of motion (6) and (7) one obviously finds a stationary state when both vectors $\mathbf{M}_1$ and $\mathbf{M}_2$ are parallel and located in the equator plane. According to (2) and (3) the energy of this stationary state is given by

$$E_\alpha = \alpha M^2 - |A_1|M^2 + A_2M^4. \tag{8}$$

Nonparallel equator plane configuration. For $\mathbf{M}_1$ and $\mathbf{M}_2$ located in the equator plane the r.h.s. of the equations (6) and (7) admit another solution for a stationary state if

$$A_1 + 2A_2(\mathbf{M}_1,\mathbf{M}_2) = 0 \tag{9}$$

holds. This condition determines an angle θ between the vectors $\mathbf{M}_1$ and $\mathbf{M}_2$, for which the vectors are at rest. The angle is given by

$$\cos\theta = |A_1|/(2A_2M^2). \tag{10}$$

The solution (10) exists for parameters A_1, A_2 and moduli of angular momenta M, for which the condition

$$|A_1|/(2A_2M^2) < 1 \tag{11}$$

is fulfilled, i.e. the stationary state (10) exists, if the biquadratic coupling is strong enough. The stationary configuration with both vectors $\mathbf{M}_1$ and $\mathbf{M}_2$ located in the equator plane and constituting the angle θ, eq. (10), is degenerate with respect to a total rotation of both vectors in the equator plane. This configuration was also found from an experimentally analysis of the domain structure in magnetic multilayer systems modelled by an energy function with a biquadratic coupling term and called the canted configuration in the literature on coupled magnetic layers [3]. We will call this stationary state the nonparallel equator plane configuration in order to distinguish it from another nonparallel configuration in which the vectors $\mathbf{M}_1$ and $\mathbf{M}_2$ are located outside the equator in the meridian plane. The nonparallel meridian plane configuration will be considered below. Inserting (10) into (2), (3) one obtains for the energy E_θ of the nonparallel equator configuration

$$E_\theta = E_\alpha - A_2M^4(1-\cos\theta)^2 \tag{12}$$

where E_α is the energy of the parallel equator configuration given by (8). As is evident from (12) the energy E_θ is below E_α.

Nonparallel meridian plane configuration. These stationary states are found by representing the system (6), (7) in the equivalent form

$$d\mathbf{M}_1/dt = [\mathbf{K}_{21}\times\mathbf{M}_1], \tag{13}$$

$$d\mathbf{M}_2/dt = [\mathbf{K}_{12}\times\mathbf{M}_2], \tag{14}$$

where the vectors $\mathbf{K}_{21}$ and $\mathbf{K}_{12}$

$$\mathbf{K}_{21} = (\beta - \alpha)(\mathbf{n}\mathbf{M}_1)\mathbf{n} + (A_1 + 2A_2(\mathbf{M}_1,\mathbf{M}_2))\mathbf{M}_2 \tag{15}$$

and

$$\mathbf{K}_{12} = (\beta - \alpha)(\mathbf{n}\mathbf{M}_2)\mathbf{n} + (A_1 + 2A_2(\mathbf{M}_1,\mathbf{M}_2))\mathbf{M}_1 \tag{16}$$

were introduced. Zeros of the r.h.s. of the equations (13) and (14), corresponding to stationary states are obtained if the vector $\mathbf{K}_{21}$ is parallel to $\mathbf{M}_1$ and the vector $\mathbf{K}_{12}$ parallel to $\mathbf{M}_2$, simultaneously. This occurs for the special configuration when the vectors $\mathbf{M}_1$ and $\mathbf{M}_2$ are located in the meridian plane transversal to the equator plane and have opposite projections on the z-axis. In this plane they form a configuration symmetric with respect to a reflection in the equator plane and include the angle ϕ

$$\cos\phi = [{}^1/{}_2(\beta - \alpha) + |A_1|]/(2A_2M^2). \tag{17}$$

The solution (17) exists for parameters A_1, A_2 and angular momenta M, for which the condition

$$[{}^1/_2(\beta - \alpha) + |A_1|]/(2A_2M^2) < 1 \tag{18}$$

is fulfilled. The solution (17) can be directly checked by noting that for this particular value of ϕ

$$A_1 + 2A_2(\mathbf{M}_1,\mathbf{M}_2) = {}^1/_2(\beta - \alpha). \tag{19}$$

Inserting (19) into (15), (16) and using for $\mathbf{M}_1$, $\mathbf{M}_2$ the decompositions $\mathbf{M}_1=\mathbf{M}_e+\mathbf{M}_z$, $\mathbf{M}_2=\mathbf{M}_e-\mathbf{M}_z$, where the vectors $\mathbf{M}_e$ and $\mathbf{M}_z$ denote the parts located in the equator plane and directed along the z-axis, respectively, one obtains $\mathbf{K}_{21}=1/2(\beta-\alpha)\mathbf{M}_1$ parallel to $\mathbf{M}_1$ and $\mathbf{K}_{12}=1/2(\beta-\alpha)\mathbf{M}_2$ parallel to $\mathbf{M}_2$ as required.

Using (17) in (2), (3) one finds for the energy E_ϕ of the nonparallel meridian plane configuration

$$E_\phi = E_\alpha - A_2M^4(1\text{-}\cos\phi)^2 \tag{20}$$

which similar to (12) is lower than the energy of the parallel equator configuration. The relation between the energies E_θ and E_ϕ is established by comparing (10) with (17), which gives $\cos\theta \leq \cos\phi$, $0 \leq \phi \leq \theta \leq \pi/2$ and consequently $E_\theta \leq E_\phi$, if both the solution branches E_θ and E_ϕ coexist. We will consider the implications for the phase space flow of the latter situation when the discussing the bifurcation sequence below.

3. Stability

We now turn to the results for the stability and local phase space structure around each of the stationary points found in the previous section.

3.1. Parallel equator plane configuration. We start the stability analysis with small deviations from the parallel equator configuration when the vectors $\mathbf{M}_1$ and $\mathbf{M}_2$ are almost parallel and deviate by a small vector $\mathbf{m}$, i. e.

$$\mathbf{M}_1 = \mathbf{M} + \mathbf{m}, \quad \mathbf{M}_2 = \mathbf{M} - \mathbf{m} \tag{21}$$

and

$$\mathbf{M} = {}^1/_2\,(\mathbf{M}_1+\mathbf{M}_2), \quad \mathbf{m} = {}^1/_2\,(\mathbf{M}_1-\mathbf{M}_2). \tag{22}$$

The vector $\mathbf{M}$ is located in the equator plane, i. e. $(\mathbf{Mn})=0$, and $|\mathbf{m}| << |\mathbf{M}|$. With this assumption one obtains from the r.h.s. of (6), (7) to first order in $|\mathbf{m}|$

$$d\mathbf{M}_1/dt = (\beta - \alpha)(\mathbf{nm})[\mathbf{n}\times\mathbf{M}] + 2(A_1+2A_2M^2)[\mathbf{M}\times\mathbf{m}], \tag{23}$$

$$d\mathbf{M}_2/dt = -(\beta - \alpha)(\mathbf{nm})[\mathbf{n}\times\mathbf{M}] - 2(A_1+2A_2M^2)[\mathbf{M}\times\mathbf{m}]. \tag{24}$$

Adding both equations one finds $d\mathbf{M}/dt=0$, i. e. the central vector $\mathbf{M}$ is constant and remains in the equator plane. Subtracting the equations one obtains for $\mathbf{m}$

$$d\mathbf{m}/dt = (\beta\text{-}\alpha)(\mathbf{nm})[\mathbf{n}\times\mathbf{M}] + 2(A_1+2A_2M^2)[\mathbf{M}\times\mathbf{m}]. \tag{25}$$

We now choose the x-axis parallel to $\mathbf{M}$. Then for the components of $\mathbf{m}$ one has

$$dm_x/dt = 0, \tag{26}$$

$$dm_y/dt = (\beta\text{-}\alpha)Mm_z - 2(A_1+2A_2M^2)Mm_z, \tag{27}$$

$$dm_z/dt = 2(A_1+2A_2M^2)Mm_y. \tag{28}$$

The transition to the tangential space with m_x=const is a result of the linearization. The phase space structure of the remaining system of two coupled variables m_y and m_z is of elliptic or hyperbolic type depending on the sign of the roots λ of the characteristic equation

$$\lambda^2 = 4M^2\{(\beta - \alpha)/2-(A_1+2A_2M^2)\}(A_1+2A_2M^2). \tag{29}$$

For $\lambda^2<0$ ($\lambda^2>0$) the point $\mathbf{m}=0$ is elliptic (hyperbolic), i. e. the parallel equator configuration is stable (unstable).

3.2. Nonparallel equator plane configuration. In this case the linearization of the equations of motion (6) and (7) has to be performed around the configuration of vectors $\mathbf{M}_1$ and $\mathbf{M}_2$ located in the equator plane and including the fixed angle θ, eq. (10). We denote the particular vectors, which satisfy (10), as $\mathbf{M}_1=\mathbf{P}$ and $\mathbf{M}_2=\mathbf{Q}$, respectively. Introducing now the small vectors $\mathbf{m}_1$ and $\mathbf{m}_2$ for the deviations as

$$\mathbf{M}_1 = \mathbf{P}+\mathbf{m}_1 \tag{30}$$

and

$$\mathbf{M}_2 = \mathbf{Q}+\mathbf{m}_2 \tag{31}$$

one obtains by keeping the linear terms for the components of $\mathbf{m}_1$ and $\mathbf{m}_2$ in the equator plane

$$d\mathbf{m}_{1;x,y}/dt = (\beta - \alpha)(\mathbf{n}\mathbf{m}_1)[\mathbf{n}\times\mathbf{P}], \tag{32}$$

$$d\mathbf{m}_{2;x,y}/dt = (\beta - \alpha)(\mathbf{n}\mathbf{m}_2)[\mathbf{n}\times\mathbf{Q}] \tag{33}$$

and for the components transversal to the equator plane

$$d\mathbf{m}_{1;z}/dt = -2A_2[\mathbf{P}\times\mathbf{Q}]((\mathbf{m}_1\mathbf{Q})+(\mathbf{m}_2\mathbf{P})), \tag{34}$$

and

$$d\mathbf{m}_{2;z}/dt = +2A_2[\mathbf{P}\times\mathbf{Q}]((\mathbf{m}_1\mathbf{Q})+(\mathbf{m}_2\mathbf{P})). \tag{35}$$

Now calculating the roots λ of the characteristic equation for the system (32)-(35) one finds

$$\lambda^2 = -4A_2(\beta - \alpha)|[\mathbf{P}\times\mathbf{Q}]|^2, \tag{36}$$

i.e. the nonparallel equator plane configuration is elliptic for the easy plane case $\beta>\alpha$.

3.3. Nonparallel meridian plane configuration. In this case we proceed in analogy to the nonparallel equator plane configuration and denote the vectors $\mathbf{M}_1$ and $\mathbf{M}_2$ satisfying the condition (17) as $\mathbf{M}_1=\mathbf{P}$ and $\mathbf{M}_2=\mathbf{Q}$, respectively. Introducing the small vectors $\mathbf{m}_1$ and $\mathbf{m}_2$ for the deviations from $\mathbf{P}$ and $\mathbf{Q}$ as

$$\mathbf{M}_1 = \mathbf{P} + \mathbf{m}_1, \tag{37}$$

and

$$\mathbf{M}_2 = \mathbf{Q} + \mathbf{m}_2, \tag{38}$$

and performing the linearizations of the equations of motion (6) and (7) one obtains

$$\begin{aligned} d\mathbf{m}_1/dt = 1/2\ (\beta - \alpha)\{2(\mathbf{n}\mathbf{m}_1)[\mathbf{n}\times\mathbf{P}]+2(\mathbf{n}\mathbf{P})[\mathbf{n}\times\mathbf{m}_1] + \\ + [\mathbf{m}_2\times\mathbf{P}]+[\mathbf{Q}\times\mathbf{m}_1]\}+2A_2\{(\mathbf{Q}\mathbf{m}_1)+(\mathbf{P}\mathbf{m}_2)\}[\mathbf{Q}\times\mathbf{P}] \end{aligned} \tag{39}$$

and

$$d\mathbf{m}_2/dt = 1/2\ (\beta - \alpha)\{2(\mathbf{n}\mathbf{m}_2)[\mathbf{n}\times\mathbf{Q}]+2(\mathbf{n}\mathbf{Q})[\mathbf{n}\times\mathbf{m}_2]+[\mathbf{m}_1\times\mathbf{Q}] + [\mathbf{P}\times\mathbf{m}_2]\} + \tag{40}$$

$$+ 2A_2\{(\mathbf{Qm}_1)+(\mathbf{Pm}_2)\}[\mathbf{P}\times\mathbf{Q}].$$

Proceeding as with the nonparallel equator plane configuration and representing the system (39), (40) by the components of the vectors one obtains for the roots λ of the characteristic equation

$$\lambda^2 = 2A_2(\beta - \alpha)|[\mathbf{P}\times\mathbf{Q}]|^2, \tag{41}$$

i. e. the nonparallel meridian plane configuration is hyperbolic in the easy plane case $\beta>\alpha$.

4. Bifurcations and phase space portraits

We now turn to the sequence of bifurcations and the corresponding changes in the rotational phase spaces of the biquadratic coupled rotators. To be systematic in the exposition we will fix the value of A_1 and increase the value of the biquadratic coupling A_2. As mentioned above we consider the case $A_1<0$ and to avoid confusion in all equations below the absolute value of A_1, i. e. $|A_1|$ is used. We note that in the easy plane case, which is considered in this paper, $\beta>\alpha$ and correspondingly $|A_1|<|A_1| + {}^1/_2\,(\beta-\alpha)$. Then the condition $|A_1|<2A_2M^2$ is fulfilled before $|A_1| + {}^1/_2\,(\beta-\alpha)<2A_2M^2$ when increasing the biquadratic coupling A_2. Correspondingly according to (11), (18) the bifurcation to the nonparallel equator plane configuration appears before the bifurcation to the nonparallel meridian plane configuration when A_2 is increased.

Phase space portraits were obtained by a direct numerical integration of the equations of motion (6) and (7) by a standard 4th-order Runge-Kutta method and by projecting the dynamics of the vectors M_1 and M_2 on tangential spaces. A time integration interval from $t=0$ to $t_{max}=15$ with a time step equal to 0.001 was used. Accuracy of integration was checked by calculating the integrals of motion.

Increasing the biquadratic coupling constant A_2 one obtains next cases.

Low biquadratic coupling, $2A_2M^2<|A_1|$: In this case the parallel equator configuration of the rotators is a stable elliptic state and represents the lowest energy E_α. Deviations of the vectors $\mathbf{M}_1$ and $\mathbf{M}_2$ from the parallel orientation result in rotations of $\mathbf{M}_1$ and $\mathbf{M}_2$ around the parallel configuration (we do not display the phase space portrait for this trivial case).

Intermediate biquadratic coupling, $|\mathbf{A}_1|<2\mathbf{A}_2\mathbf{M}^2<|\mathbf{A}_1|+1/2(\beta-\alpha)$: According to (10) and (11) in a first bifurcation a new stationary state appears. This is a pitchfork bifurcation which converts the parallel rotator configuration into an unstable hyperbolic state and creates a new stable elliptic configuration with nonparallel rotators including the angle θ, eq. (10), in the equator plane. According to (12) the energy of this new stable stationary state E_θ is below the energy of the unstable parallel configuration E_α from which it has splitted off. For a representative set of parameters the phase space portrait is shown in the Fig. 2.

Strong biquadratic coupling, $|\mathbf{A}_1|+1/2(\beta-\alpha)<2\mathbf{A}_2\mathbf{M}^2$: A second bifurcation occurs which according to (29), (41) converts the parallel configuration from an unstable hyperbolic point into a stable elliptic point with the creation of two new hyperbolic states arranged symmetrical in the z-direction. In this configuration the rotators include the angle ϕ. The corresponding phase space portrait is shown in the Fig. 3. According to (20) the energy E_ϕ of the newly created hyperbolic configuration is below E_α. In view of $0\leq\phi\leq\theta\leq\pi/2$, the lowest energy remains associated with the states of the stable nonparallel equator configuration E_θ, i.e. the relation between the energies of the three stationary solution branches is $E_\theta\leq E_\phi\leq E_\alpha$.

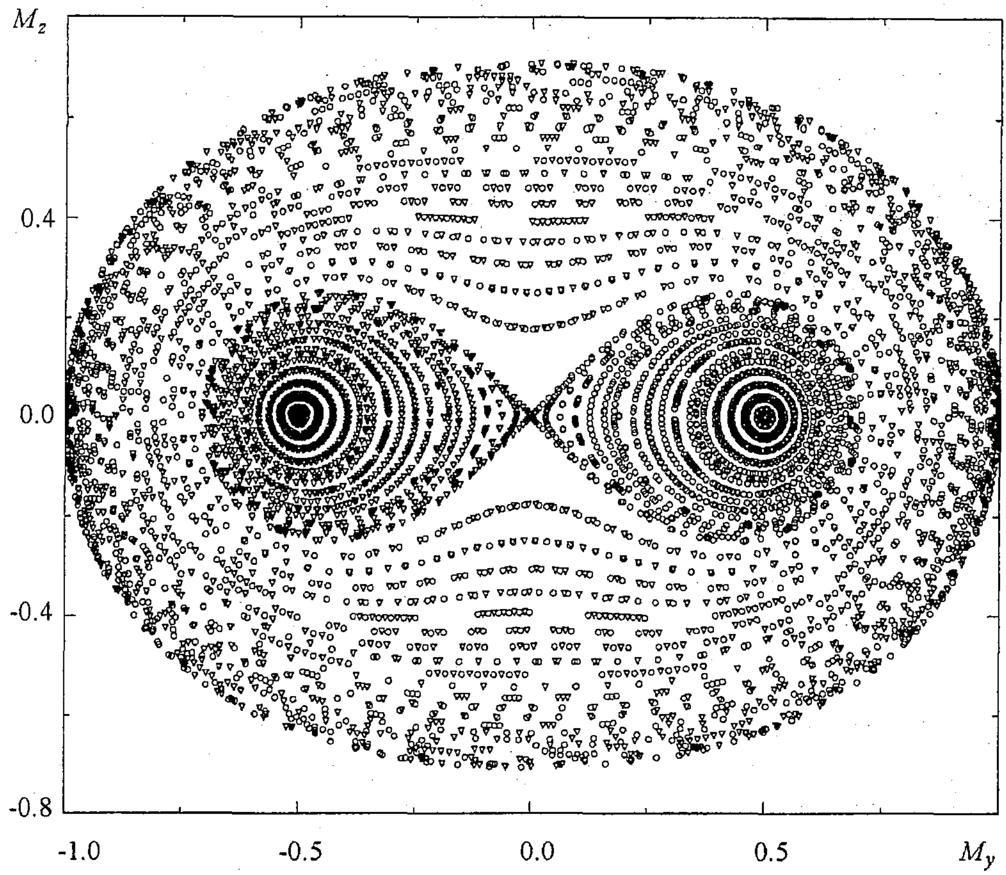

Fig. 2. Phase space portraits of two coupled rotators for an intermediate biquadratic coupling representative for the first bifurcation creating a new ground state (see text): A_1=-1, A_2=1, β-α=4 and $\mathbf{M}_1^2=\mathbf{M}_2^2=\mathbf{M}^2=1$. Initial conditions in the equator plane: $M_{1z}=M_{2z}=0$, $M_{1x}=M_{2x}=M\cos\vartheta$ and $M_{1y}=-M_{2y}=M\sin\vartheta$, where ϑ scans the equator plane from ϑ=0 to $\vartheta_{max}=\pi/2$, scan step $d\vartheta$=0.01. Shown are the trajectories of the components M_{1y}, M_{1z} (triangels) and M_{2y}, M_{2z} (circles) corresponding to the projections of the vectors $\mathbf{M}_1$ and $\mathbf{M}_2$ on the (M_y,M_z) - plane tangential to the sphere $\mathbf{M}^2$=1 with the origin (0,0) of this plane located in the centre of the initial condition scan at ϑ=0

Conclusions

Summarizing we note that a system of two bilinear and biquadratic coupled anisotropic rotators displays a sequence of bifurcations from low to intermediate and intermediate to strong values of the biquadratic coupling constant. The first bifurcation is a pitchfork bifurcation in which a new ground state with a nonparallel alignment of the rotators in the easy plane appears. The new ground state energy E_θ of this nonparallel alignment of the rotators or canted configuration of the corresponding magnetization vectors is located below the energy E_α of the parallel configuration of rotators. For a strong biquadratic coupling in a second bifurcation hyperbolic points with the energy E_ϕ in the excited states region of the rotator dynamics appear. Furthermore in this second bifurcation the parallel rotator configuration passes from the hyperbolic state back to an elliptic state. The energy of the unstable branch E_ϕ generated in the second bifurcation is located between the energy of the nonparallel ground state configuration E_θ and the energy of the parallel alignment E_α of rotators in the easy plane. As possible candidates for the observation of the bifurcation sequences as described above we point to multilayer systems such as Fe/Mn/Fe [7] or Gd/Cr/Co [6], where the bilinear coupling can be small and the biquadratic coupling can dominate. Then controlled changes of the layer

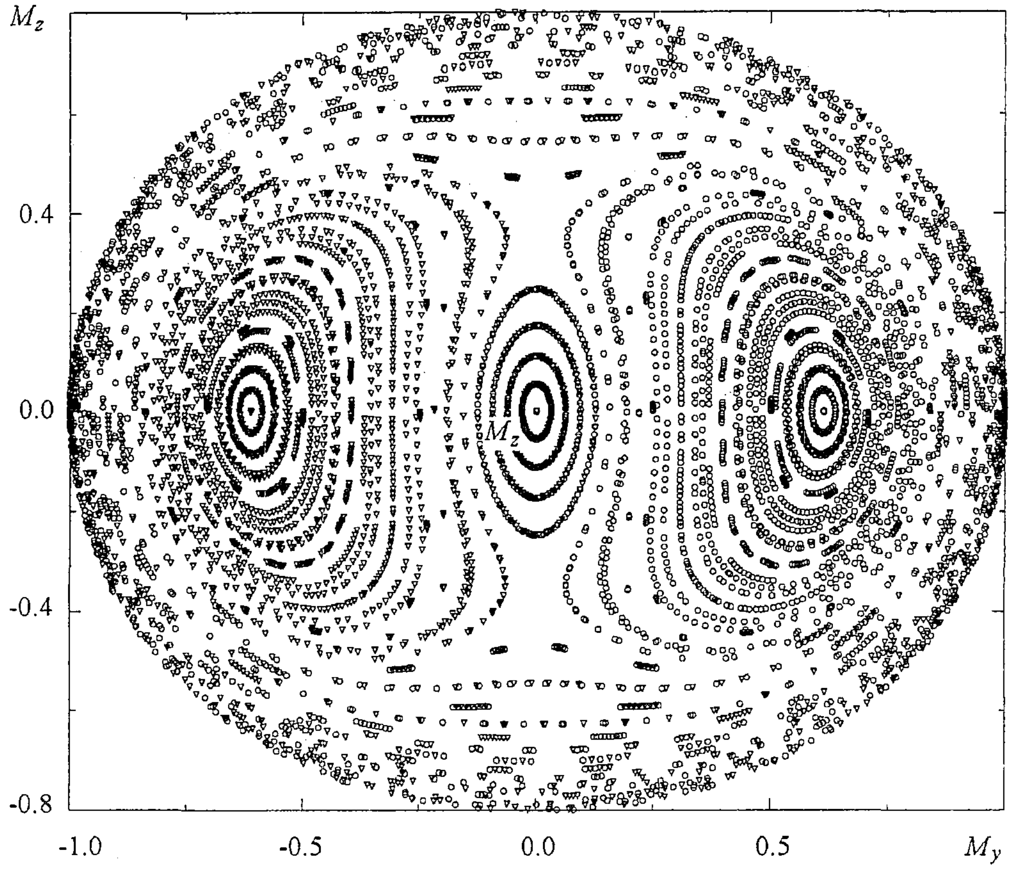

Fig. 3. Phase space portrait for a strong biquadratic coupling A_2=2 representative for the second bifurcation creating hyperbolic points in the excited states region, other parameters and initial conditions as in Fig. 2

configuration and spacer thickness can change the strength of the biquadratic coupling such as to make the bifurcation sequence described above observable. We also point to the important implications of the presence of the unstable hyperbolic rotator configuration in the excited states region for switching and relaxation in such multilayer systems.

The authors would like to thank the Humboldt-Universität Berlin and the Moscow State University for hospitality during exchange visits and the Deutscher Akademischer Austauschdienst (DAAD) for financial support.

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Received 27.02.2004

УДК 517.9; 538.221

МОДЕЛЬ СВЯЗАННЫХ РОТАТОРОВ В ИССЛЕДОВАНИИ ДИНАМИКИ ВЗАИМОДЕЙСТВУЮЩИХ МАГНИТНЫХ СЛОЕВ

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Исследована динамика нелинейных взаимодействий в слоистых магнетиках на модели двух биквадратично взаимодействующих ротаторов. Рассмотрены стационарные состояния и проведен анализ их устойчивости в изотропном и анизотропном случаях. Получены бифуркационные переходы в зависимости от величины биквадратного взаимодействия. Оказывается, что наблюдаемые экспериментально параллельные и «скрещенные» состояния векторов намагниченности слоев являются предельными случаями общего бифуркационного поведения системы. В частности, показано, что энергия основного состояния понижается при увеличении бифуркационного угла, который определяется параметрами системы. Приводятся фазовые портреты для рассматриваемой модели на основе численных расчетов.

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