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Introduction to the statistical theory of differential communication based on chaotic signals

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Abstract. The *purpose* of this paper is to analyse the statistical characteristics of a Direct Chaotic Differentially Coherent communication scheme based on chaotic radio pulses in a communication channel with additive white Gaussian noise, where the chaotic signal is given by different instantaneous distributions. *Methods.* To achieve this goal, numerical modelling of the noise immunity of Direct Chaotic Differentially Coherent communication is conducted and compared with the results of analytical research. *Results.* The regularities associated with the use of chaotic signals with various statistical distributions of instantaneous values were studied. The minimum values of energy per bit to white Gaussian noise power spectral density ratio were obtained, providing the required error probabilities. *Conclusion.* It is shown that the proposed system works efficiently at high values of processing gain, and as the processing gain increases, the dependence of noise immunity on the specific statistical distribution of the chaotic signal is levelled out.

Keywords: chaotic radio pulses, differential communication scheme, numerical simulation, statistic characteristics, bit-error probability.

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Introduction

The phenomenon of dynamic chaos has been discovered and actively investigated since the mid-60s of the twentieth century. In the first 20 years, this phenomenon has been studied in detail with examples in various fields of natural science. Many of the properties of DC turned out to be amazing, they certainly include the possibility of synchronizing two or more systems with dynamic chaos. It was this property that caused the initial interest in chaos as a potential information carrier. And, although the first information transmission schemes proposed on the basis of chaotic synchronization turned out to be insufficiently effective in terms of noise immunity

[1–12], the beginning of using dynamic chaos for transmitting and processing information was laid.

It should be noted that the task of transmitting information depends very much on the physical conditions in which it is carried out. In reality, there are now a significant number of examples of the use of dynamic chaos for transmitting information in the radio range of electromagnetic waves (via cable and through free space) [7–9, 13–15], optical communication systems (via fiber optic cables) [10, 11, 16–18], in an aqueous environment (ultrasonic chaos) [12, 13, 19, 20], etc.

Although the results presented in this article may be relevant to any of the mentioned areas of application of dynamic chaos to the transmission and processing of information, it is primarily devoted to the wireless transmission of information using high-frequency and ultra-high-frequency electromagnetic signals (radio waves). These results can also be used to create means for observing the environment using radio light lamps - miniature sources of broadband incoherent microwave radiation based on dynamic chaos [21, 22].

The specifics of using chaotic signals for wireless communications are determined by two factors: 1) it is necessary to ensure a certain ratio of signal energy to spectral noise density at the required transmission range, which, under conditions of restrictions on the spectral density of radiated power, is regulated by signal processing (base); 2) it is necessary to take measures to protect against multipath interference, for example, through the use of pulse modes and protective intervals.

Although attempts to use dynamic chaos as information-carrying signals began primarily with ideas related to synchronization, in fact it has a number of other properties that are attractive from the point of view of data transmission. Therefore, after the relative failures of using chaotic synchronization, attention was drawn to other possibilities.

Among them are relative transmission methods and the use of energy reception. Here we will talk about relative transmission: a set of transmission methods in which the comparison signal is transmitted together with the information signal via the communication channel [14–16, 23, 24]. However, before proceeding to the analysis of these systems, let us indicate in which situations it is of interest to use chaotic signals to transmit information.

These situations are direct consequences of the following two properties of chaotic signals themselves.

Firstly, a fragment of a chaotic signal is used to transmit a symbol, which is long enough to be characterized as a segment of a noise-like process.

Secondly, a chaotic signal, as a rule, is characterized by a fairly wide spectrum.

The combination of these two properties leads to the fact that the use of chaos for information transmission is made by broadband signals with a base significantly exceeding one, or, in other words, signals with a significant expansion of the spectrum.

1. Signals that extend the spectrum

Unlike a narrowband signal, whose power is concentrated inside a band numerically approximately equal to the data transfer rate, an extended spectrum signal "smears" its power over a much larger frequency band [17, 25] and, with equal power with a narrowband signal, has a lower spectral power density. As a result, in the area of confident reception, the spectral power density of the received signal may be close to the spectral power density of background noise and may even be significantly lower than it. However, without prior (a priori) knowledge of the structure and parameters of the communication system, it is not so easy to fix even the very presence of a signal. But even if the presence of a signal is detected, it is difficult to extract the message without the appropriate additional information.

The basic requirements for spectrum extension communication systems can be summarized as follows.

1. The transmitted signal occupies a frequency band much wider than the information transfer rate.
2. The frequency band of the transmitted signal does not depend on the data transfer rate.
3. Demodulation can be carried out, in particular, by correlating the received signal with a replica (copy) of the signal used in the transmitter to expand the data.

Initially, spectrum expansion technologies were created around two main schemes: direct sequence (Direct Sequence — DS) and frequency hopping (Frequency Hopping — FH) [1–3,14–17].

Later, chirp technology was added to these two spectrum expansion technologies - pulse frequency modulation, in which the carrier swipes the frequency over a certain wide band in a given pulse interval. It is used, for example, in low-consumption long-range networks such as LoRaWAN [26].

The bandwidth of the carrier signal itself does not characterize the signal in terms of spectrum expansion. For example, an ultra-wide-band ultrashort pulse as a data carrier has a processing (or base) signal of the order of one, that is, it is ultra-wide-band, but it is not a signal with a spectrum extension, since the width of the spectrum coincides with the transmission rate. But a bundle of N ultrashort pulses of pseudo-random polarity, with which one bit of information is transmitted, has N processing, since for this signal $K = \Delta T \Delta F = N$, and represents a signal with a spectrum extension.

The idea of spectrum expansion technology follows from the K. Shannon ratio for channel bandwidth

$$C = W \cdot \log_2 \left(1 + \frac{S}{N}\right), \tag{1}$$

where C is the bandwidth of the channel in bits per second, W is the frequency band in hertz, N is the noise power, S is the signal power. The equation (1) shows the relationship between the ability of a channel to transmit information without errors, depending on the signal-to-noise ratio occurring in the channel and the frequency band used to transmit information.

So, let C be the desired information transfer rate, then, moving from the binary logarithm to the natural one, we get

$$\frac{C}{W} = 1.44 \cdot \log_e \left(1 + \frac{S}{N}\right) \tag{2}$$

and for small values of the ratio $\frac{S}{N}$, say, less than 0.1 (since we want to use them in a noise-tolerant system), decomposing the logarithm into a series and using the first term of the decomposition, we have

$$\frac{C}{W} = 1.44 \cdot \frac{S}{N}\,. \tag{3}$$

Which means that

$$W = \frac{C \cdot N}{1.44 \cdot S}\,. \tag{4}$$

That is, for any given noise-to-signal ratio, it is possible to ensure a low flow of information errors by increasing the bandwidth used to transmit information: for example, if we want to have a connection in which interference noise (noise exposure) is 100 times (20 dB) greater than the signal, and the flow is 10 kbit/s, then 10 kbit of information should be transmitted in a channel with a frequency band

$$W = \frac{10 \cdot 10^3 \cdot 10^2}{1.44} = 6.7 \cdot 10^5 \text{ Hz}. \tag{5}$$

In practice, the information itself can be introduced into an extended spectrum signal by several methods. The most common is to add information to an extended spectrum by using an expansion modulation. This technology is applicable to any spectrum-extending system that uses a code sequence to determine the radio frequency band (both forward-extending sequence systems and frequency-hopping systems are good candidates).

2. Communication based on chaotic signals

Chaotic signals with their inherent broadband are natural candidates for expanding the frequency spectrum of the original information signal. Since, when using chaotic signals to encode information, the resulting signals are signals with an expanded spectrum, having a wider band and lower spectral density compared to the original information signals, they have all the advantages of signals with an expanded spectrum, such as the complexity of detection without additional information about the system, resistance to multipath fading, interference, and etc. Moreover, as a result of the highly sensitive dependence on initial conditions and parameter variations, a large number of different expansion signals (wave forms) can easily be reproduced. Therefore, dynamic chaos is an inexpensive and versatile solution for spectrum extension communication systems.

Since the early 1990s, a number of modulation and demodulation schemes with spectrum expansion based on dynamic chaos have been proposed. At the same time, those that do not use chaotic synchronization are of practical importance in the first place.

Among them are DCC — Direct Chaotic Communications [5, 10, 13–15, 19, 20], as well as an ideologically similar scheme to DCC to COOK — Chaos on-off Keying [6, 11] they belong to the first type (energy reception), andthey belong to the first type (energy reception), and DCSK — Differential Chaotic Shift Keying [6, 11, 23, 24, 27, 28] and CDSK — Correlation Delay Shift Keying [6, 11, 25, 29] — to the relative method

According to their statistical characteristics, the mentioned systems were close to classical narrowband communication systems. But experimentally, up to today, only the DCC scheme has been implemented. As for the relative chaotic communication systems, even the best of them at that time — DCSK — had problems with practical implementation.

Formally, both DCC and DCSK belong to spectrum-broadening communication systems, however, due to the lack of copies of the waveform on the receiving side, they use a noisy waveform transmitted over the air, which leads to a deterioration in their statistical characteristics compared to "truly" coherent reception. This is especially noticeable when using signals with large and very large processing coefficients. At the same time, the effect of signal accumulation in them still works, which allows you to extract a useful signal even at a signal/noise level less than zero.

2.1. Signal structure and modulation method. In this paper, we consider a scheme of direct chaotic relative information transmission (DC^2 — Direct Chaotic Differentially Coherent), where chaotic radio pulses [30–33] are used as information carriers. The relative transmission of information based on chaotic radio pulses DC^2 differs from the DCSK [11, 27, 28] from the point of view of practical implementation in that the delays in it have a significantly shorter duration.

In the mentioned works [30–33], the operability of the DC^2 communication scheme and its effectiveness at medium and large signal bases were shown, analytical studies of the influence of the presence of white noise in the channel on the probability of errors during digital data transmission were carried out.

In this paper, the statistical characteristics of the DC^2 system are studied by numerical modeling methods for various distributions of instantaneous values of a chaotic signal, which, together with previously obtained analytical estimates, make it possible to formulate the main theoretical provisions of the statistical theory of the scheme of direct chaotic relative information transmission DC^2.

3. Relative transmission of information based on chaotic radio pulses

Just as in the direct chaotic communication system with energy reception [5,7–10,13–15,19, 20], information carrier in DC^2 chaotic radio pulses are in the communication system. The chaotic signal has a noise-like implementation and a rapidly decreasing autocorrelation function. These key properties are used in the considered scheme of relative information transfer. The frequency band of a chaotic radio pulse is determined by the frequency band of the initial chaotic signal and does not depend on the pulse duration over a wide range of pulse length changes.

If the duration of the chaotic radio pulse is $\Delta T \gg 1/(2\Delta F)$, then the power spectrum of the stream of chaotic radio pulses will practically not differ from the power spectrum of the initial chaotic signal. Since the value $K = \Delta T \Delta F$ represents the signal processing coefficient, an increase in the length of a chaotic radio pulse leads to an increase in its processing coefficient.

For the DC^2 circuit, an important characteristic is the autocorrelation time of the chaotic radio pulse, which is inversely proportional to the frequency band of the chaotic signal $\Delta\tau{\sim}1/\Delta F$. If the chaotic radio pulse is shifted by a time greater than the autocorrelation time, then these two radio pulses can be considered uncorrelated. This feature is the basis of the DC^2 circuit for modulation and data transmission.

The considered data transmission scheme refers to relative coherent reception, where, unlike the classical coherent reception scheme, a copy of the transmitted signal is not stored in the receiver, but is sent over the radio channel.

To transmit data to DC^2, a modulated chaotic radio pulse and its unmodulated copy are transmitted to the channel with a delay between them greater than the autocorrelation time. During reception, a correlation (coherent reception) is performed between the modulated radio pulse and its delayed, unmodulated copy. When modulating to transmit a logical unit, the chaotic radio pulse is transmitted unchanged, and to transmit a logical zero, the chaotic radio pulse is multiplied by -1. Thus, after relative coherent reception, pulses with positive and negative values occur in the receiver.

Let us consider in more detail the functional structures of the transmitter and receiver of the proposed DC^2 circuit. The transmitter of the system (Fig. 1, *a*) consists of a source of chaotic radio pulses; a divider; a modulator controlled by an external information signal; delays for a time τ exceeding the time of autocorrelation of the signal; an adder and a transmitting antenna. The source of chaotic radio pulses generates pulses with a duration of T_p; the intervals between the pulses — protective intervals — have a duration of T_{gi}. The total duration of the pulse and the protection interval is the duration of the transmitted bit T_b. Each pulse enters the divider, after which it enters two channels. In the first channel, it is modulated by an information signal by multiplying by 1, and in the second channel, it is delayed for a time τ. Multiplication by $+1$ corresponds to the transfer "1", multiplication by -1 — transfer "0". After that, the signals are summed up, the total signal is amplified, it enters the antenna and is emitted. In this case, the length of the radiated total pulse is equal to $T_r = T_p + \tau$.

The receiver of the system (Fig. 1, *b*) consists of an antenna, a low-noise amplifier, a divider, a time delay τ, a multiplier, an integrator and a threshold device. The signal received by the antenna is amplified to the desired level in a low-noise amplifier, divided in half and fed into two channels. In the first channel, no actions are performed with the signal, and it goes to

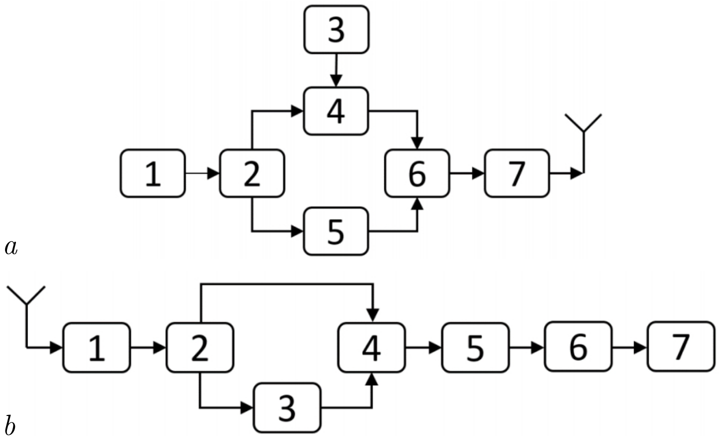


Fig. 1. *a* — Transmitter structure: 1 — source of chaotic pulses, 2 — divider, 3 — information data, 4 — modulator, 5 — time delay, 6 — combiner, 7 — amplifier; *b* — receiver structure: 1 — low-noise amplifier, 2 — divider, 3 — time delay, 4 — multiplier, 5 — integrator, 6 — threshold device, 7 — information data

the multiplier. In the second channel, the signal is delayed for a time τ, after which it also goes to the multiplier. Note that the pulse duration obtained by multiplying the pulses received by the multiplier is equal to T_p. The pulse received from the output of the multiplier is integrated during the time T_p. The signal is then sent to a threshold device with a zero threshold. If the received signal is greater than zero, then the threshold device fixes the reception of "1", if it is less than zero, then the reception is fixed "0".

3.1. Signal demodulation. When studying the process of receiving the DC^2 circuit, the following model of its functioning is considered [30–33].

Let $S_k(t)$ is the k-th chaotic pulse in the stream formed by the source of chaotic radio pulses (they must be indexed, because, due to randomness, they are all different); $\alpha_k \in \{-1, 1\}$ — the value of the information modulating signal. When transmitting the k-th binary information symbol, the signal at the output of the transmitter will look like this:

$$Y_k(t) = (\alpha_k S_k(t) + S_k(t-\tau))/2, \tag{6}$$

In the receiver, in the absence of noise, the pulse at the output of the multiplication block corresponding to the k-th information symbol goes to the integrator, after which it takes the form:

$$Z_k(t) = \left[\int\limits_{\tau}^{T_p+\tau} \alpha_k S_k(t-\tau) S_k(t-\tau)dt\right]/4 + \theta_k(t), \tag{7}$$

where

$$\theta_k(t) = \left[\int\limits_{\tau}^{T_p+\tau} \alpha_k S_k(t) S_k(t-\tau)dt + \right.$$
$$\left. + \int\limits_{\tau}^{T_p+\tau} \alpha_k S_k(t) S_k(t-2\tau)dt + \int\limits_{\tau}^{T_p+\tau} \alpha_k S_k(t-\tau) S_k(t-2\tau)dt\right]/4. \tag{8}$$

The $\theta_k(t)$ component of the signal (8) is noise generated by the circuit itself. Since the delay time of τ exceeds the autocorrelation time, all components of $\theta_k(t)$ will be significantly smaller compared to the first term in the expression (7), which is a useful signal. Thus, the sign α_k ("+" or "−") also determines the sign $Z_k(t)$. The signal from the integrator output goes to the decision block, where it is compared with the zero threshold. The sign determines the value of the output binary information symbol.

4. Analytical estimates of noise immunity in a white noise channel

Analytical estimates of noise immunity for the relative DC^2 transmission scheme were obtained in [30] under the assumption that fluctuating noise with a Gaussian distribution of instantaneous values and a constant spectral density is added to the k-th signal at the receiver input:

$$V_k(t) = Y_k(t) + \eta_k(t). \tag{9}$$

The probability of an error when receiving each bit message can be represented as:

$$P_{\text{err}} = f\left(N_c \Delta F T_p / \sqrt{N_c N_0 \Delta F T_p \left(2 + \frac{N_0}{2N_c} + \frac{5N_c}{2N_0}\right)}\right), \tag{10}$$

where

$$f(x) = \left[1 - \frac{2}{\sqrt{2\pi}} \int_0^x e^{-\frac{t^2}{2}} dt\right]. \tag{11}$$

Here N_c is the spectral density of the chaotic signal, ΔF is the band of the chaotic signal, N_0 is the spectral density of noise.

When analyzing the analytical form for the probability of error, it was found out that it has an interesting asymptotic property, namely, when the signal-to-noise ratio tends to infinity, the probability of error tends not to zero, but to a certain limit:

$$P_{\text{err lim}} = f\left(\sqrt{\frac{2\Delta F T_p}{5}}\right). \tag{12}$$

Analytical estimates of noise immunity provide a reliable basis for practical calculations in the design of ultra-wideband relative communications based on chaotic radio pulses. However, they have certain limitations that can be overcome by statistical modeling in cases where it is necessary to clarify, for example, the limits of applicability of analytical estimates.

Such cases include the influence of statistical characteristics of a chaotic signal on noise immunity. The fact is that the above analytical estimates are made for the case when the probability distribution of instantaneous values of a chaotic signal is Gaussian. That is, in principle, there are outliers with arbitrarily large amplitude for such a signal. Real chaotic signals are limited in amplitude. The effect of the limited values of the signal in comparison with signals having a Gaussian distribution is conveniently considered by direct statistical modeling using signals with corresponding probability distributions of instantaneous values.

5. Statistical modeling of noise immunity

Below, a discrete time signal model is used to numerically analyze the noise immunity of relative information transmission based on chaotic radio pulses. A time-discrete chaotic signal is formed by pseudo-random samples corresponding to three different distributions (normal, uniform and telegraphic), covering typical cases of distribution of instantaneous signal values.

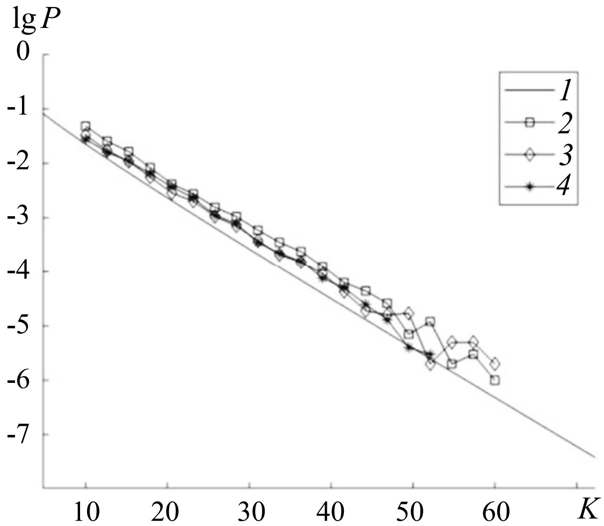


Fig. 2. Error probability versus processing coefficient graph without fluctuation noise. Line *1* corresponds to analytical estimate, *2* — to computer modeling with gaussian distribution, *3* — to uniform distribution, and *4* — to telegraph distribution

In other words, to analyze the noise immunity of the DC^2 communication circuit, the signal is sampled in time, as a result of which the signals $S_k(t)$ and $Y_k(t)$ are replaced by the signals $S_k(i)$ and $Y_k(i)$, and the noise $\eta(t)$— for noise samples $\eta(i)$, where i — the number of the sample, $S_k(i) = S_k(iT/B)$, $Y_k(i) = Y_K(iT/B)$, $\eta(i) = \eta(iT/B)$, where B is the base of the signal, $B = 2K$.

As in the continuous case, if the signal at the output of the integrating device (in this case, the adder) is negative, then it is decided that the symbol "0" is accepted, if positive, then — «1».

The number of samples of a random signal simulating a chaotic signal is $KS+1$, where K is the processing coefficient, S is the duty cycle. To calculate the probability of error, the initial information signal and the signal at the detector output are compared at different values of the ratio of energy per bit to the spectral density of noise.

Knowing the value of the bit energy ratio to the spectral noise density, the processing coefficient and measuring the power level at the transmitter output, it is possible to calculate the required value of the noise signal power level for various cases

$$\frac{P_S}{P_N}\ \text{dB}\ + 10\log_{10} K = \frac{E_\text{b}}{N_0}\ \text{dB}\ , \tag{13}$$

where P_S is the power level of the signal at the output of the transmitter, P_N is the power level of the noise signal at the input of the receiver, K is the processing coefficient.

White Gaussian noise in the communication channel was modeled by samples of pseudo-random numbers with a normal distribution generated using the standard library function "randi()" in the package MATLAB.

The band of the noise signal is consistent with the band of the chaotic signal due to the fact that in the process of mathematical modeling, both signals are formed in a discrete manner with the same sampling frequency.

5.1. Modeling for a channel without noise. The DC^2 communication circuit was modeled according to the model described above. As a result, the dependence of the probability of error per bit on the values of the processing coefficient was obtained (Fig. 2). According to this

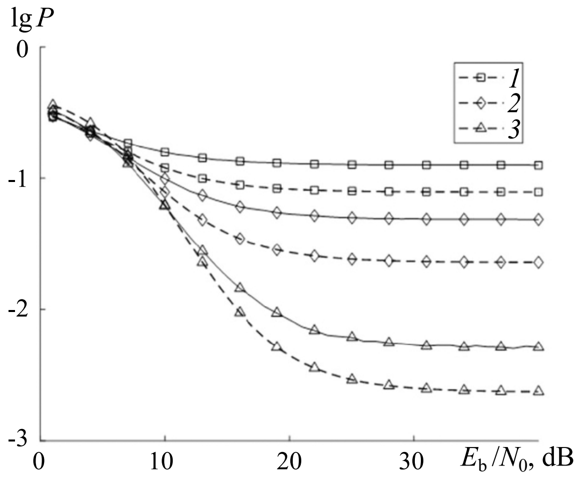


Fig. 3. Bit error probability as a function of E_b/N_0 obtained for the low values of the processing coefficient K (gaussian distribution). Dashed series *1*, *2*, *3* correspond to the simulation results for $K = 5; 10; 20$ and solid curves *1*, *2*, *3* — to the analytical estimates for $K = 5; 10; 20$ respectively

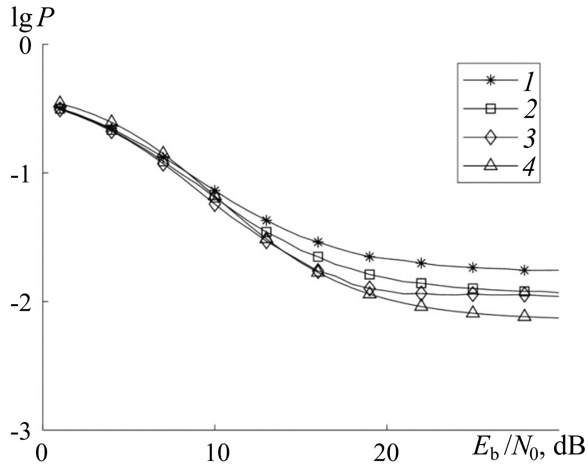


Fig. 4. Bit error probability as a function of E_b/N_0 obtained for $K = 15$. Line *1* corresponds to computer modeling with gaussian distribution, *2* — to uniform distribution, *3* — to telegraph distribution, and *4* — to analytical estimate

graph, it can be seen that the communication system starts working at error levels of $P = 10^{-3}$ with high values of the processing coefficient (starting from $K \sim 25$), and with its increase, you can count on lower values of the error probability per bit.

The analytical estimate for this dependence according to the formula (10) and the simulation results are in good agreement.

5.2. Modeling for a channel with Gaussian noise. Let's move on to statistical modeling of the DC^2 communication circuit in the presence of noise in the communication channel and compare the results obtained with analytical estimates using the formulas (8)–(10) from the works [30–33].

Statistical numerical modeling, with the help of which the noise immunity of the relative information transmission scheme DC^2 was calculated, was carried out for the following values of the processing coefficient: $K = 5, 10, 15, 20, 50, 100, 200, 300, 500, 10000$. First, calculations were performed for the case of a chaotic signal model in the form of a stream of values with a Gaussian distribution. Its results for small values of the processing coefficient $K = 5, 10$ and 20 are shown in Fig. 3. It can be seen that for $K = 5, 10$ and 20 it is impossible to achieve the probability of error $P < 10^{-3}$ for any ratio of the energy of the chaotic radio pulse E_b to the spectral density of Gaussian bandpass noise N_0.

Experiments conducted with other types of distributions of chaotic signal models, namely uniform and telegraphic distributions, show that even with these signal distributions, the probability of error P less than 10^{-3} is not achieved for small values of K. At the same time, it should be noted that with low processing coefficients K, chaotic signal models with telegraphic and uniform distributions are more noise-resistant in relation to the model with a Gaussian distribution (Fig. 4).

Real chaotic signals have a limited amplitude and do not have long tails, as in the Gaussian distribution. Therefore, estimates relating to a uniformly distributed signal and to a random telegraphic signal are more fair for them.

Calculations show that with an increase in the processing coefficient, the dependence of the simulation results on the type of distribution of the chaotic signal model is leveled, and when evaluating the characteristics, one can limit oneself to some one type of signal, for example, a signal model with a Gaussian distribution.

When the processing coefficient is increased to $K = 50$, the minimum value of E_b/N_0, which ensures the probability of error $P < 10^{-3}$, is 15.3 dB (Fig.5).

A chaotic signal is modeled by a random signal with a Gaussian distribution. As K increases, the values of the error probability per bit, depending on the level of external noise for different distributions, become close to each other and fall on the same curve (Fig.6).

Of particular interest is the study of the capabilities of a communication system with very high processing coefficients. So, in Fig. 7 the calculation results for the processing coefficient $K = 10000$ are given. They show that the communication system in this case can work with an error probability of 10^{-3} at the ratio level of $E_b/N_0 = 22.2$ dB. However, if we go directly to the dependence on the signal-to-noise ratio (S/W — SNR) (Fig. 7, *b*), then you can see that due to the high processing coefficient, the required C/W level (SNR) in this case is less than -10 dB, which indicates that the communication system is operational at a signal level much lower than the noise level in the channel connections.

Further, computer modeling was additionally carried out in order to identify the optimal value of the processing coefficient, which allows for a given probability of error per bit (in this case, $P = 10^{-3}$) with a minimum value of the ratio of the average energy of the chaotic signal of the chaotic radio pulse E_b to the spectral density of the Gaussian bandpass noise N_0 (fig. 8).

The results shown in Fig. 8, show that the graph of the dependence of E_b/N_0 on K has an extremum at $K = 100$, in which the minimum value of $E_b/N_0 = 15.3$ dB is reached. Further, as K increases, the value of E_b/N_0 increases, providing error probabilities of $P = 10^{-3}$ in the communication system, which makes its operation with such parameters less energetically advantageous. However, despite this, it should be noted that, according to Fig. 8, *b*, with an increase in the value of the processing coefficient, the necessary SNR ratio decreases, providing the probability of error in the communication system $P = 10^{-3}$, which makes it advantageous to use signals with a large base in a straightforward relative information transmission scheme in terms of secrecy and operation below the noise level.

Conclusion

The paper investigates the statistical characteristics of a new scheme of relative information transmission based on chaotic radio pulses, which uses delays with a duration determined by the attenuation time of the autocorrelation function of a chaotic signal. This is the fundamental difference between the considered scheme and the classical relative DCSK scheme, in which the delay time is determined by the length of the transmitted bit. Numerical modeling has been performed and compared with previously obtained analytical estimates concerning the dependence of error probabilities per bit on the ratio of energy per bit to the spectral density of white Gaussian bandpass noise. The patterns associated with the use of chaotic signals with different statistical distributions of instantaneous values are studied.

It is shown that at high processing values ($K > 30$), the considered communication scheme works effectively both in a channel without external fluctuation interference and in a channel with white Gaussian noise. At the same time, with an increase in the processing coefficient, the dependence of noise immunity on a specific type of statistical distribution of a chaotic signal is leveled. This pattern is largely due to the fact that at low processing values for chaotic signals with Gaussian or uniform distribution, the variability of the received energy from pulse to pulse

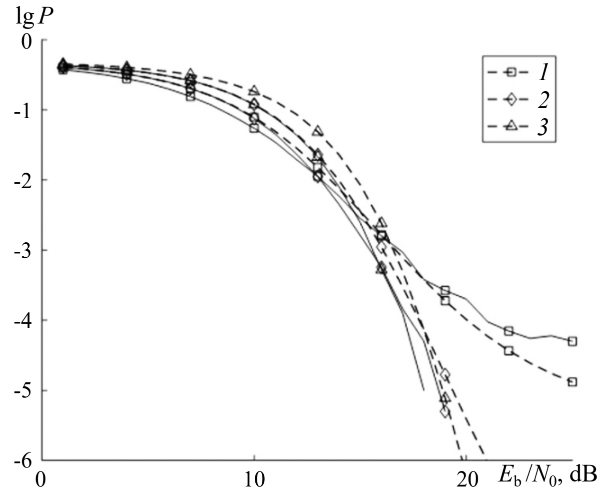
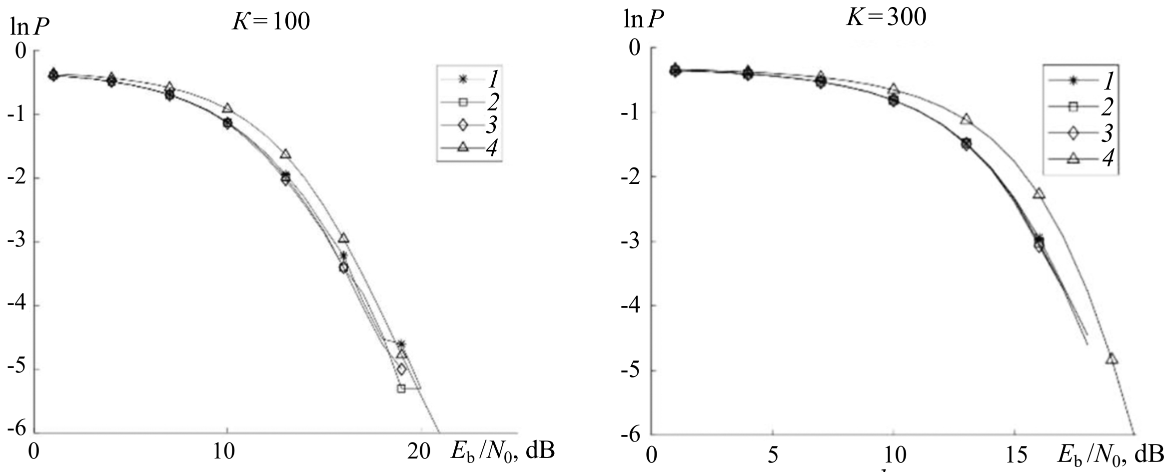
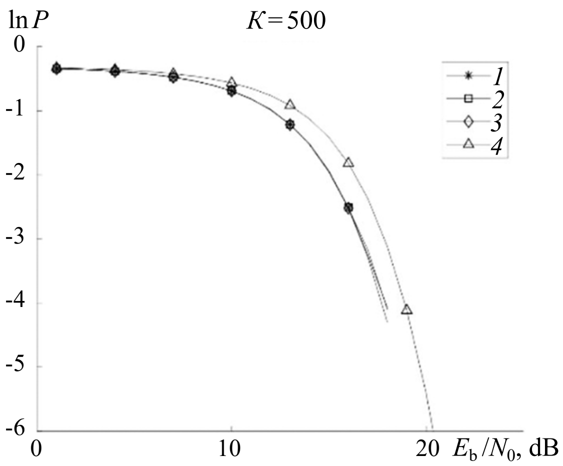


Fig. 5. Bit error probability as a function of E_b/N_0 obtained for $K = 15; 100; 200$ (corresponding to curves *1*, *2*, *3*), solid lines correspond to experimental results and dashed ones — to analytical estimate



a

b



c

Fig. 6. Bit error probability as a function of E_b/N_0 obtained for different processing coefficient: K = 100 (*a*); 300 (*b*); 500 (*c*). Line *1* corresponds to computer modeling with gaussian distribution, *2* — to uniform distribution, *3* — to telegraph; line *4* corresponds to analytical estimate

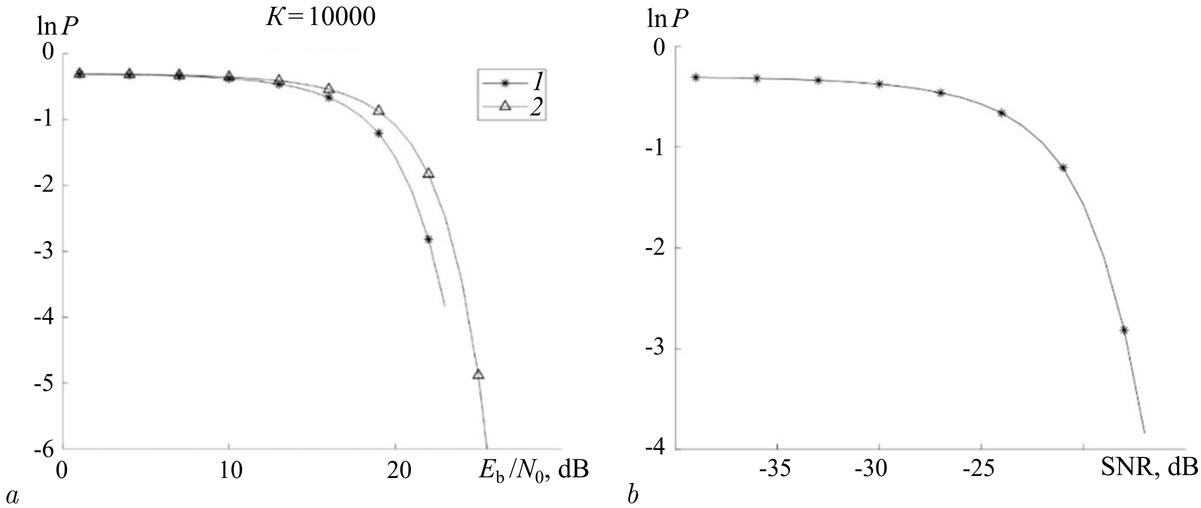


Fig. 7. Error probability as a function of: *a* — E_b/N_0 with $K = 10000$; *b* — signal to noise ratio (SNR). Figure *a* shows line *1* corresponding to computer modeling for gaussian distribution, and line *2* corresponding to analytical estimate

is significant, and this worsens the probability of correct reception. At the same time, with a telegraphic distribution corresponding in the initial system to a chaotic signal with a constant amplitude (for example, phase chaos), the pulse energy in the stream is the same even with small processing. With large processing, the variability of energy from pulse to pulse tends to zero for all three types of distributions.

The totality of the results obtained is the basis of the statistical theory of the scheme of relative direct chaotic transmission. Further development of this theory is possible using methods for measuring and evaluating the parameters of noise-like signals against a background of noise and interference [34].

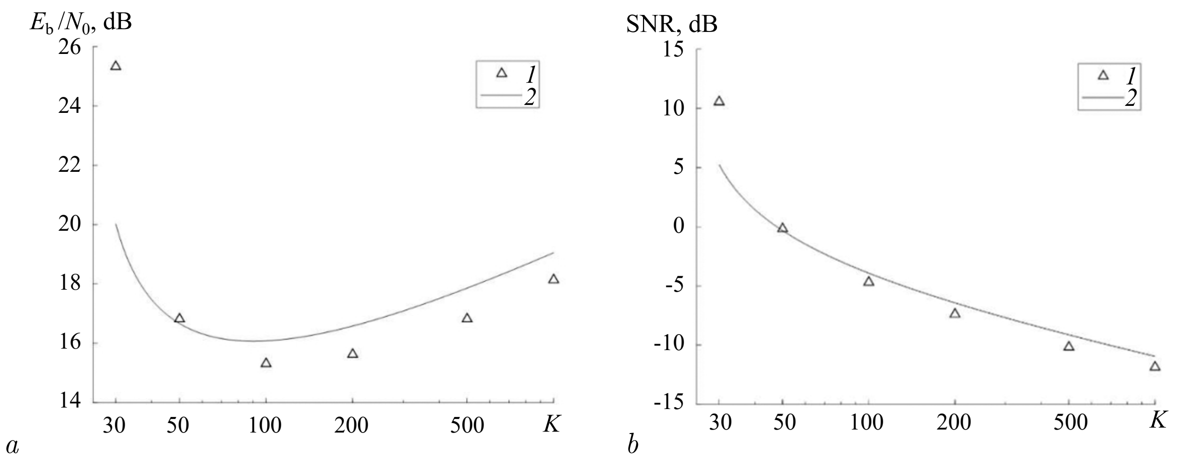


Fig. 8. *a* — E_b/N_0 versus processing coefficient graph, where error probability $P = 10^{-3}$ is reached; *b* — SNR versus processing coefficient graph, where error probability $P = 10^{-3}$ is reached. Line *1* corresponds to computer modeling for gaussian distribution, and line *2* corresponds to analytical estimate

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