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Analysis of the accuracy of the signal processing algorithm of the differential phase polarimeter

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Abstract. The *purpose* of this work is to analyze the effect of the polarimeter signal processing algorithm on the results of measurements of the optical rotation angle of the polarization plane to improve the accuracy of measurements in differential polarimetry. *Methods.* The paper considers the methods of polarimetry used for the analysis of optically active substances, based on the methods of phase measurements used to calculate the optical rotation angle. The expediency of using the Fourier transform to calculate the phase difference of differential polarimeter signals is noted. To analyze the error of the algorithm, mathematical modeling of the measurement information processing for various signal parameters is applied. *Results.* The results of the study of the effect of the bit depth of the analog-to-digital converter, the number of samples over the period of the signal and the accumulation time on the accuracy of restoring the phase difference are presented. The influence of the ratio of signal amplitudes and the level of amplitude and phase noise caused by the imperfection of the measuring system has also been investigated. *Conclusion.* The obtained results make it possible to optimize the operating mode and improve the accuracy of the optical rotation angle measurements using a differential phase polarimeter based on the Fourier transform.

Keywords: polarimetry, optical rotation angle, phase measurements, harmonic signal analysis, amplitude and phase noise, Fourier transform.

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Introduction

Polarimetric research methods are based on measuring the optical rotation angle of light transmitted through an optically active medium [1]. The optical activity of substances consists in changing the plane of polarization of light and is due to the structure of the crystal lattice or molecules, therefore, by measuring the optical rotation angle, it is possible to determine the composition of the substance, the configuration of molecules, the concentration of solutions, etc.

In optical devices for various purposes, polarization materials are often used, for quality control of which high-precision measurements are required [2]. Polarization measurements are widely used in astronomy [3], earth sounding from space [4], the food industry to determine the sugar content [5], identification of oils and other optically active liquids [6], in medicine — for quality control of medicines [7], ophthalmology [8], etc. In more complex cases, for the analysis of mixtures of several optically active substances, [9] spectropolarimetric methods are used, in which the change in the rotation angle of the polarization plane is investigated depending on the wavelength of light.

1. Methods of measuring the angle of rotation of the plane of polarization of light

Among the methods used to measure the characteristics of optically active substances, two groups can be distinguished. The first group includes zero methods in which the minimum intensity (attenuation) of the luminous flux is achieved by changing the relative angular position of the optical elements — polarizer and analyzer [10]. The disadvantages of such methods are the need to use high-precision angle measuring devices, the strong influence of electrical noise and fluctuations in the intensity of the light source, since the moment of quenching must be determined by measuring the minimum signal level. In the methods of the second group, modulation of the polarization of the luminous flux is used [10], in this case, both the amplitude and phase characteristics of the signals can be used to calculate the desired values. Methods based on the analysis of phase characteristics are less sensitive to noise and have been developed in differential polarimetry [11], in which the modulation of the polarization of rays is usually carried out using a rotating analyzer with constant speed. The essence of differential polarimetry consists in passing the first (object) modulated polarized beam through a cuvette with the test substance, and the second (reference) — bypassing this cuvette (Fig. 1).

In this case, the first beam undergoes an additional optical rotation angle due to the optical activity of the substance under study, so the signals on the photodetectors will differ in phase. Thus, the measurement of the rotation angle of the polarization plane is reduced to the calculation of the phase difference of harmonic signals. To analyze the spectral dependence of the rotation angle of the polarization plane, radiation sources with different wavelengths are used.

However, the existing methods of differential polarimetry have disadvantages in the field of circuit solutions and implemented algorithms for calculating measurement results, so their improvement is an urgent task. The purpose of this work is to improve the accuracy of differential polarimetry by analyzing the influence of the signal processing algorithm on the results of the optical rotation angle measurements.

2. Methods of phase measurements

Various methods of phase measurements [12] have been developed and widely used. In the simplest case, the conversion of the phase difference of the signals into a constant voltage or time interval is used. The disadvantage of such methods is that only a small part of the information signal is used to calculate the phase in the areas of its transition through a certain level, so noise and signal fluctuations will have a noticeable effect on the accuracy of measurements.

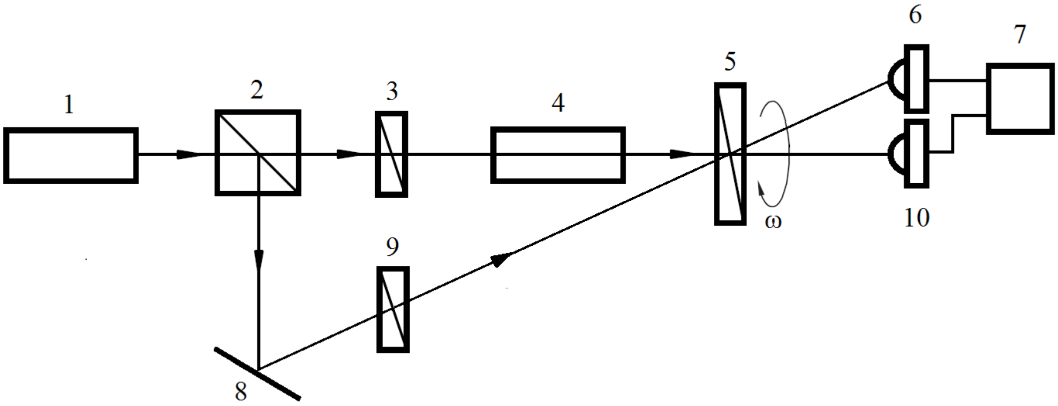

Fig. 1. Scheme of differential polarimeter: 1 — source of monochromatic radiation; 2 — beam splitter; 3, 9 — polarizers; 4 — cuvette with the test substance; 5 — rotating analyzer; 6, 10 — photodetectors; 7 — phase meter; 8 — fixed mirror

To use the entire array of phase information, you can use correlation (integral) methods that are used for image analysis [13]. These methods are usually implemented in the frequency domain via the Fourier transform, but only the maximum of the correlation function contains information about the phase shift, the calculation of which in discrete form with high accuracy is a rather difficult task [12].

If we turn again to image processing methods, then we can use a wealth of experience in the field of decoding interferograms, and solve the problem of measuring the phase difference of harmonic signals using digital interferometry methods. To calculate the wavefront phase in the analysis of interferograms, the phase step method or the method based on the Fourier transform (Fourier method) [14] is usually used.

The method of phase steps in various modifications is very popular, but requires the analysis of at least three interferograms, and therefore it is difficult to apply it to work with fast-flowing processes. The Fourier method allows us to calculate the phase difference from one interferogram, but with a lower resolution, which is due to the limitation of the signal spectrum [15]. However, since the measuring signals of a differential polarimeter have a very narrow spectrum, it is advisable to use the Fourier method to calculate the phase difference and use an analog-to-digital converter instead of a phase meter.

3. The algorithm for calculating the phase shift based on the signals of the differential phase polarimeter

Consider the process of obtaining measurement information using a differential phase polarimeter (see Fig. 1). In a differential phase polarimeter, a polarizer rotating at a constant speed is used to modulate the beam, while the light intensity at its output is described by the Malus law [16]. Then the signals on the photodetectors can be described by the equations

$$S_1(t) = M_1 + \sigma_1\eta(t) + A_1\cos(\omega_0 t + \sigma_{\phi 1}\eta_\phi(t) + \phi_{01}), \quad (1)$$

$$S_2(t) = M_2 + \sigma_2\eta(t) + A_2\cos(\omega_0 t + \sigma_{\phi 2}\eta_\phi(t) + \phi_{02}), \quad (2)$$

where $S_1(t)$ is signal from the object photodetector; $S_2(t)$ is signal from the reference photodetector; M_1, M_2, σ_1, σ_2 are mean values and standard deviation of the constant component A_1, A_2 are signal amplitudes; $\eta(t)$ is additive amplitude noise caused by the variability of the intensity of the radiation source; $\omega_0 = 2\pi/T$ is cyclic frequency of the signal (determined by the period T); $\sigma_{\phi 1}$, $\sigma_{\phi 2}$, $\eta_\phi(t)$ are phase standard deviation and phase noise caused by uneven rotation of the polarizer; ϕ_{01}, ϕ_{02} are initial phases; t is time.

It is assumed that the noise $\eta(t)$ and $\eta_\phi(t)$ have a normal distribution, but low-frequency filtering is applied to the phase noise.

We describe the stages of signal processing using the proposed algorithm.

1) The original reference and object signals (1) and (2) are multiplied by an apodizing function: a modified fourth-order Blackman-Harris window [17] having an analytical representation in the form:

$$w(n) = a_0 - a_1 \cos(2\pi n/N) + a_2 \cos(4\pi n/N) - a_3 \cos(6\pi n/N), \quad 0 \leqslant n \leqslant N, \tag{3}$$

where $a_0 = 0.3635819$, $a_1 = 0.4891775$, $a_2 = 0.1365995$, $a_3 = 0.0106411$, N is the total number of counts.
2) A direct Fourier transform is performed.
3) First-order bandpass filtering is performed in the spectrum of the received signals at stage 2 by the fifth-order Butterworth filter, described by the equation

$$F(\omega) = \frac{1}{1 + \left(\dfrac{\omega - \omega_1}{\Delta\omega_c}\right)^{2k}}, \tag{4}$$

where $\omega_c = 0.1\omega_1$ is cutoff frequency, k is filter order.
4) The inverse Fourier transform of the filtered signals is performed.
5) The phase difference $\Delta\phi(t)$ of the reference and object signals is calculated by the formula

$$\Delta\phi(t) = \arg[S_1^{‘} \cdot \ S_2^{‘*}], \tag{5}$$

where the sign $*$ means a complex-conjugate value; arg is a function for calculating the argument of a complex number; $S_1^{‘}$, $S_2^{‘}$ are signals received at the stage 4.
6) The average value of the obtained phase distribution $\Delta\phi(t)$ is calculated, which represents the desired value the rotation angle of the polarization plane.

4. Analysis of the influence of various parameters on the algorithm errors

We present the results of evaluating the effect on the error of the algorithm for calculating the phase shift based on the signals of the differential phase polarimeter of the following parameters: the bit depth of quantization levels, the number of samples over the period and duration of signal accumulation, the ratio of signal amplitudes, amplitude noise and phase noise caused by uneven rotation of the analyzer. Mathematical modeling was carried out in the MATLAB [18] environment. The purpose of this simulation is to select the parameters of the polarimeter circuit and the optimal mode of its operation.

The error analysis of the algorithm was carried out as follows.

1) The object and reference signals were set according to the formulas (1) and (2) with different parameters. In this case, the phase shift between the original signals is constant for all implementations, $\Delta\phi_0 = \phi_{01} - \phi_{02}$, where $\phi_{01} = 0^\circ$ and $\phi_{02} = 45^\circ$ are the initial phases of the object and reference signals, respectively.

2) The phase shift $\Delta\phi(t)$ between the signals was calculated using the proposed algorithm.
3) The absolute error of the algorithm was estimated by the difference between the specified and calculated phase shift: $\Delta\phi = \Delta\phi(t) - \Delta\phi_0(t)$. Additionally calculated the standard deviation $\sigma_{\Delta\phi}$.

4.1. The effect of the bit depth of quantization levels. When digitizing signals, a quantization error occurs (quantization noise), which depends on the number of quantization levels associated with the bit depth of the ADC used. Therefore, to choose the optimal value of the quantization levels, consider its effect on the measurement error. The parameters of the mathematical model of signals are given in Table. 1, where Q is the maximum value of the signal in a given system bit depth.

The simulation results are shown in Fig. 2, *a, b*.

As follows from the analysis of the obtained dependencies, the bit depth of the quantization levels has a negligible effect on the error and the standard deviation, and inexpensive 12-bit ADCs can be used to digitize the differential polarimeter signal.

Table 1. Parameters values of the mathematical model of signals

Parameter	Value
ADC bit rate	8...16
The average value of the object signal	$Q/2$
The average value of the reference signal	$Q/2$
The amplitude of the object signal	$Q/2$
The amplitude of the reference signal	$Q/2$
Signal period, c	0.1
Number of counts per period	1000
Duration of signal accumulation, c	20

4.2. The influence of the number of samples over the period and the duration of signal accumulation. Next, let's consider the influence of the number of samples over the period and the duration of signal accumulation (Fig. 3, *a, b*). The parameters of the mathematical model of signals are given in Table. 2.

As follows from Fig. 3, *a* and , *b*, the time of signal accumulation (that is, the number of analyzed periods) has the greatest influence on the error and the standard deviation. At the same time, the values of these parameters change slightly after an accumulation time of more than 30 seconds (or 300 periods), so you can limit the amount of information received taking into account these results.

4.3. Influence of the ratio of signal amplitudes. Since the intensity of the object and reference rays may not be the same, let's consider the effect of the ratio of signal amplitudes on the error and the standard deviation. To do this, it is possible to simulate at different values of the attenuation coefficient of the amplitude of the signals K_i. The parameters of the mathematical model are given in Table. 3.

Based on the simulation results (Fig. 4), it can be concluded that a change in the ratio of signal amplitudes within 15% will not have a significant effect on the calculation error.

4.4. The influence of amplitude noise. Modeling according to clauses 4.1–4.3 was carried out for the case of the absence of noise of random components $\eta(t)$ and $\eta_\phi(t)$ — amplitude

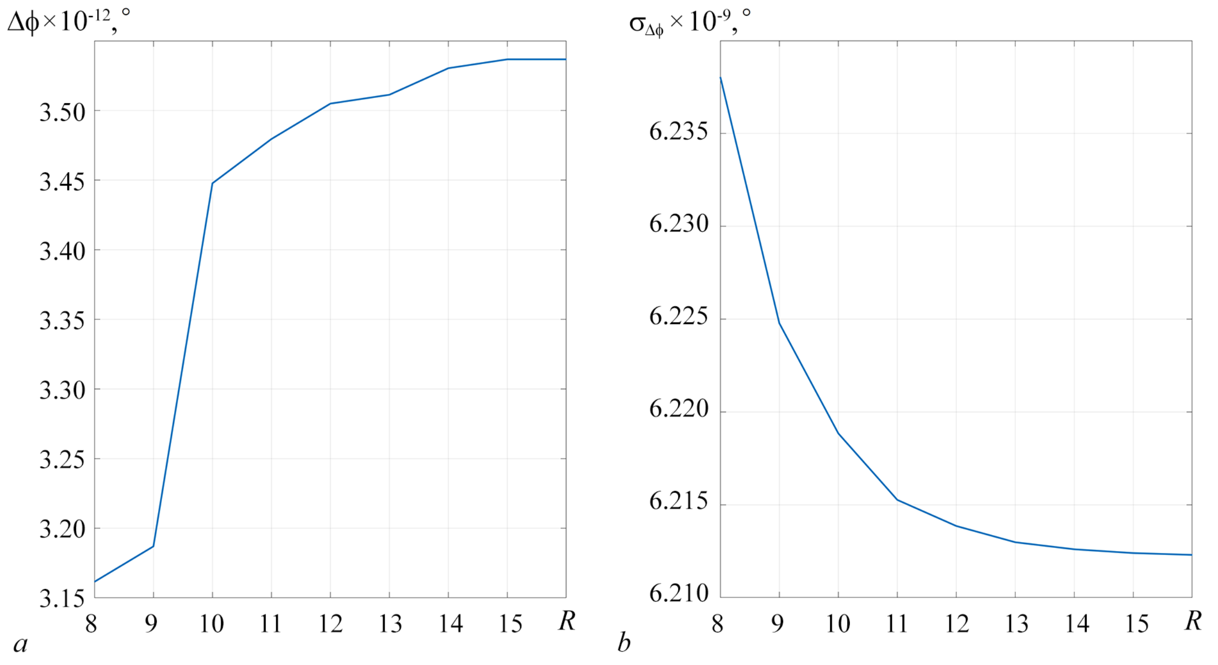

Fig. 2. The effect of the analog-to-digital converter bit rate R on the error $\Delta\phi$ (a) and the standard deviation $\sigma_{\Delta\phi}$ (b) of the restored phase difference

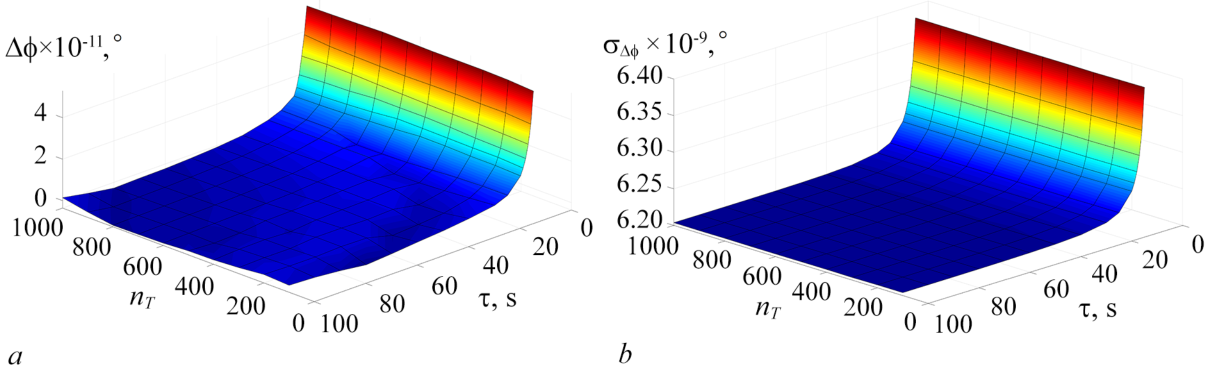

Fig. 3. The effect of the number of samples for the period n_T and the time of accumulation of the signal τ on the error $\Delta\phi$ (a) and the standard deviation $\sigma_{\Delta\phi}$ (b) of the restored phase difference (color online)

and phase noise. Next, we will consider the effect of these parameters on the error of the algorithm.

The amplitude noise level was set by changing the standard deviation of the object and reference signals σ_1 and σ_2 using a dimensional coefficient η, varying in the range 0.001...0.01. The signal amplitudes are taken as 95% of the maximum value. The values of the error and the standard deviation will be considered depending on the signal-to-noise ratio μ_A, which for the reference signal is calculated by the formula

$$\mu_A = 20\lg(M_1/\sigma_1), \tag{6}$$

The parameters of the mathematical model are given in Table. 4.

Thus, the value of the amplitude noise (Fig. 5, a, b) has a noticeable effect on the error of

Table 2. Parameters values of the mathematical model of signals

Parameter	Значение
ADC bit rate	12
The average value of the object signal	2048
The average value of the reference signal	2048
The amplitude of the object signal	2048
The amplitude of the reference signal	2048
Signal period, c	0.1
Number of counts per period	100...1000
Duration of signal accumulation, c	5...100

Table 3. Parameters values of mathematical model of signals

Parameter	Value
ADC bit rate	12
The average value of the object signal	$K_1 \cdot 2048$
The average value of the reference signal	$K_2 \cdot 2048$
The amplitude of the object signal	$K_1 \cdot 2048$
The amplitude of the reference signal	$K_2 \cdot 2048$
Signal period, c	0.1
Number of counts per period	1000
Duration of signal accumulation, c	30

Table 4. Parameters values of mathematical model of signals

Parameter	Value
ADC bit rate	12
The average value of the object signal	2048
The average value of the reference signal	2048
The amplitude of the object signal	$0.95 \cdot 2048$
The amplitude of the reference signal	$0.95 \cdot 2048$
The standard deviation of the object signal noise	$\eta \cdot 4096$
The standard deviation of the reference signal noise	$\eta \cdot 4096$
Signal period, c	0.1
Number of counts per period	1000
Duration of signal accumulation, c	30

the algorithm, and it is desirable to increase the signal-to-noise ratio in the measuring system to a level of 50 dB or higher.

4.5. The effect of phase noise. Let's also consider the influence of phase noise caused by the unevenness of the analyzer rotation, which can be estimated through the coefficient of

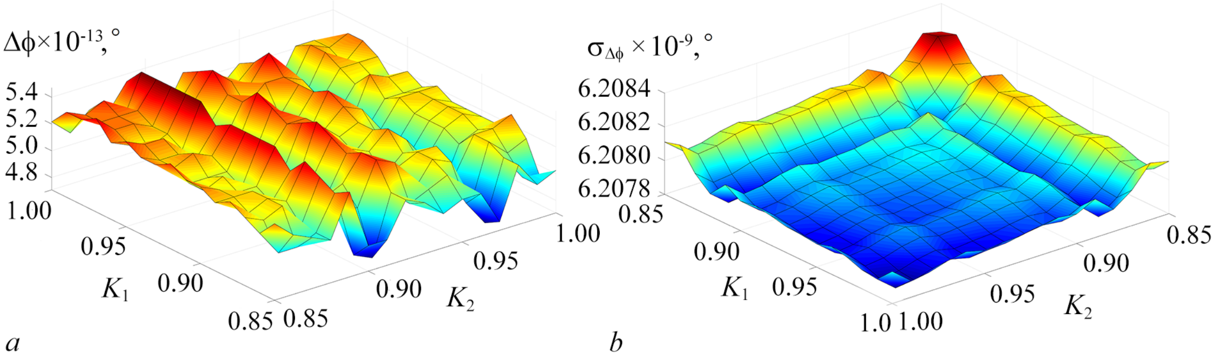

Fig. 4. The effect of the signal attenuation coefficient K_i on the error $\Delta\phi$ (a) and the standard deviation $\sigma_{\Delta\phi}$ (b) of the restored phase difference (color online)

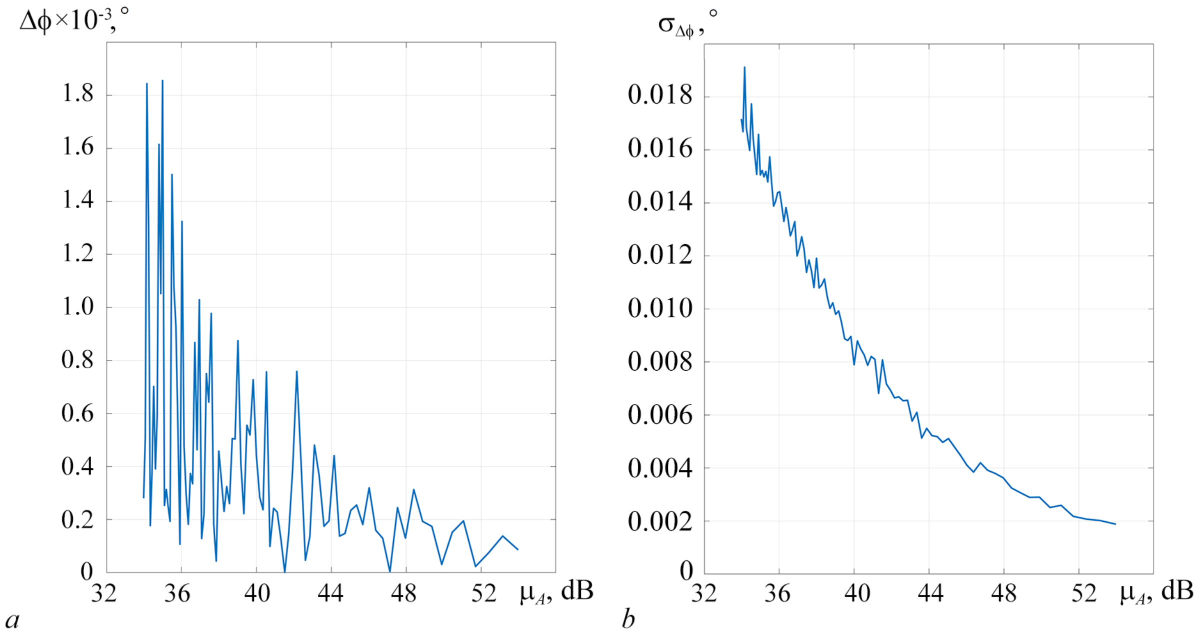

Fig. 5. The effect of the signal-to-noise ratio μ_A on the error $\Delta\phi$ (a) and the standard deviation $\sigma_{\Delta\phi}$ (b) of the restored phase difference

unevenness of rotation δ [19], determined by the formula

$$\delta = (\omega_{\max} - \omega_{\min})/\omega_0, \tag{7}$$

where $\omega_{\max}$, $\omega_{\min}$, ω_0 are the maximum, minimum and average values of angular velocity for 1 cycle. This coefficient depends on the type of electric motor rotating the polarizer, and the typical values of δ are 0.001...0.01 [19], therefore, the effect of rotation irregularity was simulated in the presence of low-frequency phase noise in the range from 0 to $0.01\omega_0$. The parameters of the mathematical model are given in Table.5.

From the results obtained, it can be concluded (Fig. 6) that the unevenness of the analyzer rotation within $0...0.01\omega_0$ is compensated by the processing algorithm and has little effect on the measurement results.

Table 5. Parameters values of mathematical model of signals

Parameter	Value
ADC bit rate	12
The average value of the object signal	2048
The average value of the reference signal	2048
The amplitude of the object signal	2048
The amplitude of the reference signal	4096
The amplitude of the phase noise of the object signal	$0...0.01\omega_0$
The amplitude of the phase noise of the reference signal	$0...0.01\omega_0$
Signal period, c	0.1
Number of counts per period	1000
Duration of signal accumulation, c	30

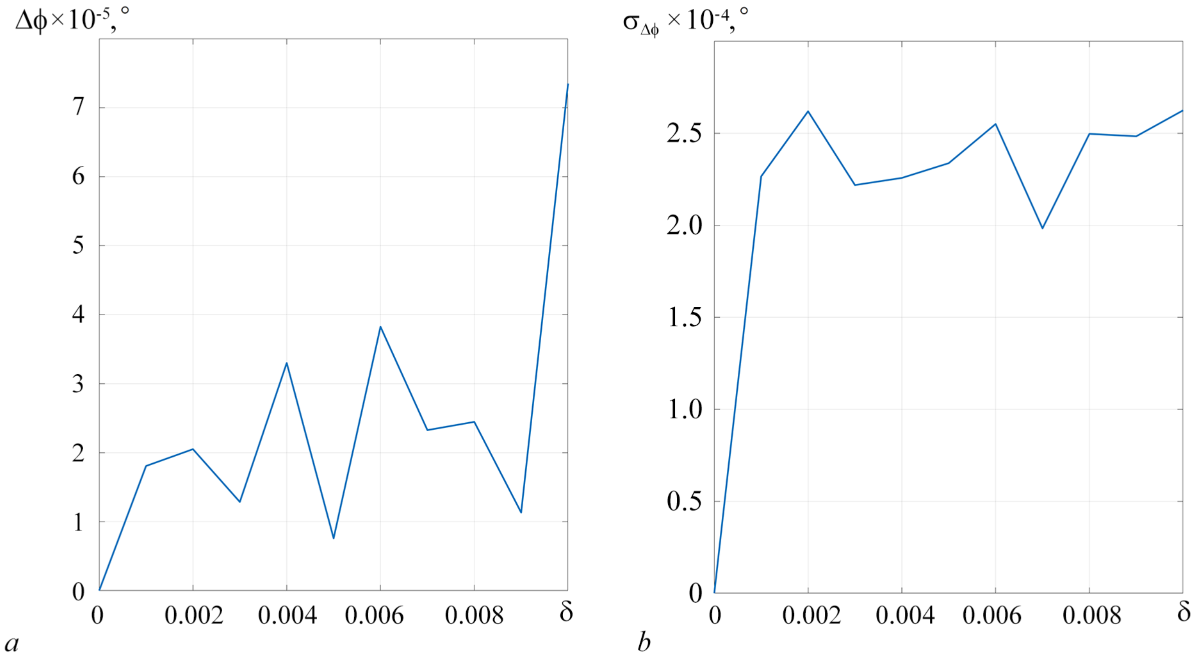

Fig. 6. The effect of the coefficient of rotation unevenness of the polarizer δ on the error $\Delta\phi$ (*a*) and the standard deviation $\sigma_{\Delta\phi}$ (*b*) of the restored phase difference

Conclusion

The paper presents the results of error analysis of the signal processing algorithm of the differential polarimeter, in which angular measurements are replaced by phase measurements. Due to the rejection of angular measurements of the analyzer position, the requirements for the rotary device and photodetectors are significantly reduced, which reduces the cost of the polarimeter device. A signal processing algorithm based on the Fourier transform is proposed to calculate the optical rotation angle. To estimate the error of the algorithm, mathematical modeling of the process of processing measurement information for various signal parameters was carried out. The results obtained allow us to evaluate the requirements for the polarimeter elements (the bit depth and sampling frequency of the ADC, the speed and unevenness of the motor rotation, the signal-to-noise ratio), as well as to optimize its operation mode.

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