Izv.VUZ «AND», vol. 6, № 4, 1998

GAP SOLITONS

S.V. Polyakov, A.P. Sukhorukov

In this paper we present the studies of the problem of slow and immobile solitons which are excited inside the stop bands of inhomogeneous media with either quadratic or cubic nonlinearity. We found an integral criterion of slow soliton formation, determined the main properties of gap solitons. The dynamics of parametric gap soliton formation and propagation in quadratic media is illustrated by means of computer simulations and compared with such solitons due to cubic nonlinearity.

Introduction

It is well known what solitons can be excited in wide range of nonlinear media due to the balance of dispersive and nonlinear effects [1]. The properties of solitons in the media with third-order nonlinearity were studied very intensively. In particular, the phenomenon of slow and immobile solitons forming near the bounds of stop bands in the medium with stop bands was discovered and studied theoretically and numerically by A.C.Newell, Yu.I. Voloshchenko et. al., W. Chen., D.L. Mills and others [2-4]. Experimental evidence of slow solitons was reported by B.J. Eggleton et. al. [5]. This phenomenon is based on the bound frequency shift due to self-action effects. Thus, the non-dumping signals on «forbidden» frequencies may propagate through the material. The phenomenon of non-dumping propagation was named nonlinear tunneling. Solitons that propagate in forbidden bands are known as gap solitons.

In recent years the phenomenon of parametric solitons draws the attention of many scientists all around the world. These solitons were predicted theoretically by Yu.N.Karamzin and A.P. Sukhorukov in 1974 [6]. The parametric soliton consists of three frequencies ω_1, ω_2 and $\omega_3=\omega_1+\omega_2$ or in degenerated case of two frequencies ω - fundamental frequency (FF) and 2ω - second harmonic (SH). It is clear, that on quadratic nonlinearity the wave packet with narrow band can not affect itself. Nevertheless, it can interact with second and subharmonic (mutual interaction). Thus the soliton due to quadratic nonlinearity consists of two or three colours. In 1995 a scientific group directed by G.I. Stegeman proved the existence of parametric solitons in experiments with the beams [7]. Since then, many works and publications were devoted to the phenomenon of optical parametric processes (see [8-11] and the references within). Nevertheless, the investigations and discussions of problems of immobile and slow solitons were started only recently [12, 13]. There are still a lot of questions on the excitation and properties of gap parametric solitons in periodical structures. In order to excite this type of solitons one can use media with two dispersion curves with a non-transmission gap between them, because both FF and SH have to be disposed near critical frequencies - bounds between transmission and non-transmission bands. In particular, this phenomenon can be observed in the chain of oscillatory circuits with asymmetric constants of linear coupling. In this paper we present theoretical outline and results of numerical simulation, that show the main properties of gap solitons both in $\chi^{(2)}$ and $\chi^{(3)}$ inhomogenous media, illustrating some analogies between parametric solitons and ones due to cubic nonlinearity. We found that for both $\chi^{(2)}$ and $\chi^{(3)}$ cases it is possible to modify Hamiltonian I_3, and thus to obtain a good indicator of nonlinear processes domination.

Basic equations of $\chi^{(3)}$ tunneling

Let us consider the wave propagation in the system with a stop band. In the neighborhood of the boundary frequency ω_b one can approximate the dispersion relation, taking into account the cubic nonlinearity as follows:

$$-\Omega = Dk^2 - \sigma|A|^2, \qquad (1)$$

where D is a dispersion coefficient, σ is a coefficient of nonlinearity, $\Omega=\pm(\omega-\omega_b)$ is the detuning of the impulse's frequency from the boundary ω_b. Here, plus must be chosen for stop-band, that lays under the limit frequency and minus corresponds to reverse situation. It is clear, that in the linear case a signal can not propagate into the medium if its frequency falls into the stop band, as the wave number becomes imaginary. Nevertheless, the boundary frequency can vary due to the presence of a nonlinear term, namely if $\sigma>0$, it shifts into the forbidden (in linear approximation) band and if $\sigma<0$ - it shifts to the reverse direction.

The dispersion relation (1) what conforms to the propagation of narrow-band pulse can be obtained by substitution $E={}^1/_2 A(z,t)\exp(\pm i\omega_b t)$+c.c., where A is a slow-varying envelope. The envelope of a signal into nonlinear medium can be described by NLS:

$$\partial A/\partial t=iD(\partial^2 A/\partial z^2) + i\sigma|A|^2 A, \qquad (2)$$

what gives the dispersion relation (1). All variables and parameters of (2) are supposed to be dimensionless.

The integrals of motion (see for example [10]) are very useful in the theoretical analysis of nonlinear tunneling. In the absence of waves at the boundaries of the system, the full energy $I_1=\int_0^L |A|^2 dz$ doesn't vary in time. There are also two Hamiltonians and

$$I_2=\int_0^L [A^*(\partial A/\partial z) - A(\partial A^*/\partial z)]dz,$$
$$I_3=\int_0^L [D|\partial A/\partial z|^2 - \sigma|A|^4/2 \ ]dz]. \qquad (3)$$

We found that it is possible to modify I_3, so the sign of this Hamiltonian indicates the domination of nonlinear processes on dispersion, or, in other words, the negativeness of modified I_3 . Let us make a conclusion on slow solitons formation. If the envelope could be presented as

$$A=\tilde{A}\exp(-imz), \qquad (4)$$

where $\tilde{A}$ is real, it is clear that I_3 acquires the «superfluous» term Dm^2I_1, what is implied with the energy transfer. Even if nonlinearity dominates on dispersion, this positive «superfluous» term can make I_3 positive and doesn't permit us to determine analytically which of the processes is stronger. To eliminate the influence of energy transfer on I_3 we need to subtract Dm^2I_1 from the Hamiltonian. Let us remark that $|I_2|$ after substitution A from (4) gives $2mI_1$, and if there is no energy transfer I_2=0. Thus, we can modify the Hamiltonian I_3 as follows:

$$\tilde{I}_3=I_3 - D|I_2|^2/(4I_1). \qquad (5)$$

We would like to notice that (5) in the absence of nonlinearity is always positive. Indeed, the assumption α=0 gives:

$$\tilde{I}_3 = D(\int(\partial A/\partial z)\cdot(\partial A^*/\partial z)dz - I_2^2/4\int AA^*dz). \qquad (6)$$

The sign of $\tilde{I}_3$ is determined by the sign of expression inside the brackets. It can be determined by substitution of I_2 in (6). So, the brackets can be given in the form:

$$(4\int(\partial A/\partial z)\cdot(\partial A^*/\partial z)dz\int AA^*dz - 4|\int A^*(\partial A/\partial z)dz|^2)/(4I_1).$$

The obtained expression is positive due to Cauchy - Schwartz inequality

$$|\int f_1f_2dz|^2 \leq \int |f_1|^2dz \int |f_2|^2dz \text{ , where in this case } f_1=A^*, f_2=\partial A/\partial z.$$

This permits us to make a suggestion that the sign of $\tilde{I}_3$ could be used as an indicator of nonlinearity domination which is not disturbed by the energy transfer processes. We will show below that the numerical experiments confirm this conclusion. The same modification method could be used to determine the domination of nonlinear effects to study the propagation of inclined wave beams. In order to do this, one can apply the time-spatial analogy to (2), and change t for the propagation coordinate and z for the transverse one.

It is possible to perform the same analysis for $\chi^{(2)}$ medium and to obtain modification similar to (5) [14].

Let us remind that NLS (2) has an analytical soliton solution [15]:

$$A=a\text{sech}[(z-ut)/l]\exp(i\Omega t-iqz), \qquad (7)$$

where a is a peak amplitude of the slow soliton, u is a velocity, l is an extension, Ω is a detuning of the soliton from the boundary frequency and q is an addition to wave number.

Substitution of (7) to (2) gives that the parameters of a soliton are coupled by the following correspondences [16]

$$u=[2D(\sigma a^2 - 2\Omega)]^{1/2}, \qquad (8a)$$

$$l=[2D/(\sigma a^2)]^{1/2}, \qquad (8b)$$

$$q=[(\sigma a^2 - 2\Omega)/2D]^{1/2}. \qquad (8c)$$

The analysis of (8) gives a simple result, that in order to excite a soliton in the stop-band it is necessary to excite the medium edge by the soliton (7) with a peak intensity $a^2>2\Omega/\sigma$.

Computer modeling of $\chi^{(3)}$ nonlinear tunneling

To examine the dynamics of formation and interactions between slow and immobile gap solitons we applied numerical simulations. To study the propagation of complex input signals including long impulses we numerically solved the boundary problem for a nonlinear medium, described by (2). The left edge of a medium is exited by the input signal $A(t,z=0)=E_1(t)$. The initial energy distribution is described by $A(t=0,z)=E_0(z)$. In particular, for a tunneling process the initial energy inside the system is equal to zero. As the observation of the propagation of a signal in the nonlinear media is most interesting for us, the closeness of the right edge is very undesirable, due to unavoidable reflection of impulses' tails. To control the correctness of the computations we watched for the conservation of integrals of motion $I_{1,2,3}$.

First, we examined the boundary excitation process of exact soliton solutions (7). These excited solitons propagate without shape distortion with the velocities, predicted by (8b). It is necessary to underline, that the integral of motion I_3 in all our numerical experiments remains positive, that doesn't indicate strong nonlinear selfaction. However, the sign of modified integral $\tilde{I}_3$ points to a soliton propagation process.

To show the insensitivity of the modified integral to the phase modulation, we carried out a great number of computer experiments with various initial amplitude distributions and boundary excitations. These experiments demonstrate, that the sign of $\tilde{I}_3$ can be used as a keen indicator of nonlinear tunneling effect. In particular, for the initial distribution $A|_{t=0}=A_G\exp(-z^2/l_G^2)\exp(iMz)$ we measured the dependence of I_3 and $\tilde{I}_3$ on phase modulation constant M. It is clear that $\tilde{I}_3=I_3$ then $M=0$ (immobile impulse). We found, that the variation of integral $\tilde{I}_3$ is approximately 100 times less than that of I_3 for $0<M<5$, (for example, in the nonlinear medium $D=5\cdot10^{-3}$, $\alpha=1$ for $A_G=0.593861$, $l_G=0.2$, and $M=0$ we obtain $I_3=\tilde{I}_3\approx10^{-8}$; if $M=1$, we observe $I_3=4.4\cdot10^{-4}$ and $\tilde{I}_3=2.24\cdot10^{-6}$). It must be stressed, that the calculations method we used does not provide a precise I_2 conservation due to boundary influence to the phase of the signal. That is why we expect better insensitivity of $\tilde{I}_3$ for the scheme, that does not take into account the boundary problems i.e. for the ring-like system with connected boundaries.

It is interesting to observe the dynamics of tunneling process with boundary exciting pulses of different forms. Upon entering into the medium, these pulses split into one or more slow solitons depending on the form of the pulse energy.

For the particular case of rectangular pulse with career frequency equal to ω_b $E_1(t)=A_0\{\text{th}[100(t-b)]+\text{th}[100(b-t)]\}$, with $A_0=1.5$, $b=2.5$ propagating in the nonlinear dispersive medium $D=5\cdot10^{-3}$, $\sigma=1$ we carried out detailed studies of the evolution of the signal shape. This rectangular pulse generates three slow solitons into the medium. Their velocities could be determined by analyzing the graphs showing the time dependence of the positions of the vertices. Thus obtained velocities, namely $u_1=0.2$, $u_2=0.165$ and $u_3=0.16$ and durations differ noticeably (approximately by one with a half) from the velocities and durations calculated from the peak intensities ($A_1^2=10$, $A_2^2=9$, $A_3^2\approx14$). This inconsistency leads us to the suggestion that the frequencies of new formed pulses are different from initial ones. So, propagating solitons possess nonlinear frequency shifts. Indeed, the analysis of the phase modulation of tunneling solitons has shown the presence of such shifts: $\Omega_1=2.5$, $\Omega_2=2.6$, $\Omega_3=4$. These results are in a good agreement with theoretical calculations using formula (8).

The study of mutual interactions between two pulses was conducted for initial (Cauchy) value problems and boundary value problems. In all cases there are two interesting tasks: the mutual interactions between formed solitons and interactions between weak underthreshold soliton-like pulses (with positive $\tilde{I}_3$ for single pulse) that in some cases may generate one rigorous soliton.

The interactions of initially assigned pairs of identical solitons were investigated for energy distributions given by:

$$A|_{t=0}=\Sigma_{i=1}^{2}a\mathrm{sech}[(z-z_{0i})/l]\exp(i\Phi_i), \tag{10}$$

where a and l are selected according to (8). The distance between the solitons $\Lambda=|z_{01}-z_{02}|$ varies taking into account the necessity of tails overlapping. The phase difference $\Delta=\Phi_1-\Phi_2$ determines the interaction scenario. If $\Delta=0$ we observe the beating effect, when the solitons periodically merge into one powerful pulse with two symmetrical spikes on tails. Soon after formation, this strong pulse falls apart and the system returns into its initially state. If the phase difference is equal to $\pi/2$, the interaction scenario changes dramatically. One of the solitons sucks energy from the other, so one pulse becomes stronger. After the energy exchange, the solitons repulse each other. Finally, if $\Delta=\pi$ one can observe the repulsion without energy exchange.

The energy fusion we observed for a pair of exact solitons in phase leads us to the interesting problem of two underthreshold soliton-like pulses fusion and the capability of soliton generation. The initial conditions are set in form (10) with $a_u=0.4a$ and $l_u=l$, where a, l are the parameters of sech soliton according to (8). It is clear that a pulse alone is spreading due to dispersion. However, a pair of such impulses can form a soliton.

The interaction dynamics of signals initiated by boundary excitation of the medium was studied for pairs of sech soliton-like pulses:

$$A|_{x=0}=\Sigma_{i=1}^{2}a\mathrm{sech}[(t-t_{0i})/\tau]\exp(i\Phi_i).$$

The process of interaction of two exact solutions of (2) is presented in Fig. 1. Fig. 1, *a*

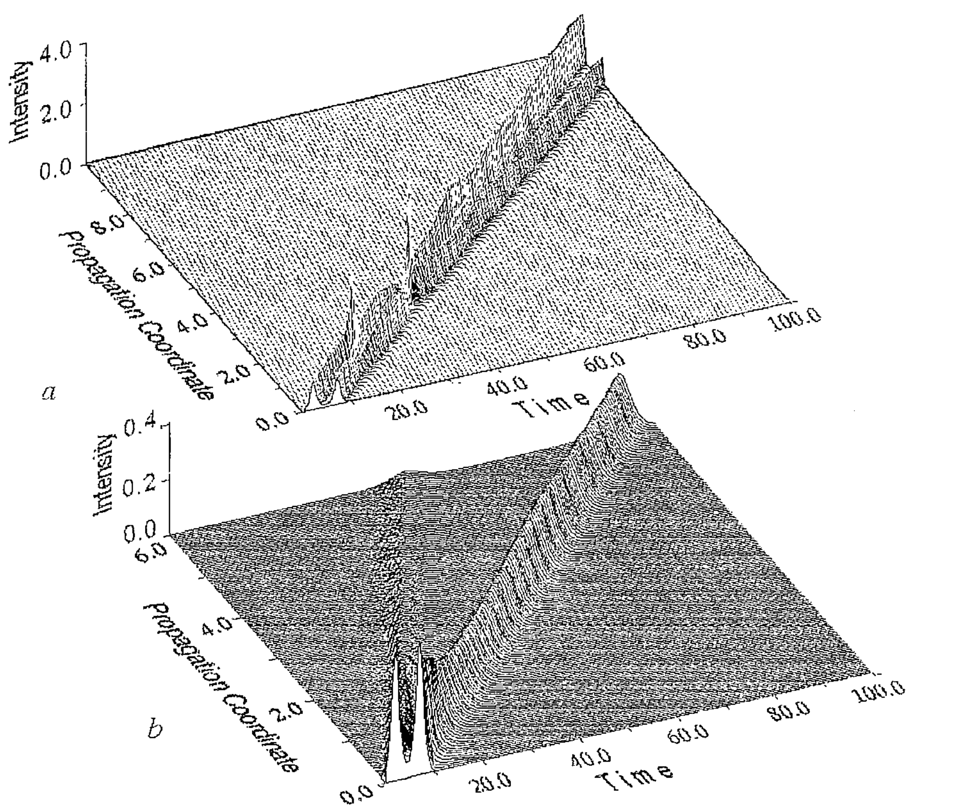

Fig 1. Tunneling of pairs of solitons (*a*) and underthreshold pulses (*b*) in cubic medium

represents the propagation of two overlapping solitons in phase. It is clearly seen, that these solitons attract each other and collide. The collision results in energy redistribution, that affects the velocity of each soliton (8a). This velocity mismatch leads to separation of solitons. In antiphase the same solitons repulse each other.

The boundary excitation by underthreshold in-phase pulses with a_u=0.7a leads to formation of a pulsating soliton. It is easy to notice that some energy leaks off during the process of this soliton producing (see Fig. 1, *b*).

The outlined consideration shows that it is possible to use the pulses with noticeably lesser amplitude than that of slow soliton to generate one during the process of nonlinear tunneling.

Basic equations of $\chi^{(2)}$ tunneling

Let us consider a periodically inhomogenious medium with a dispersion characteristic, illustrated in Fig. 2.

The fundamental and the second harmonics of the parametric soliton are coupled by nonlinearity [6]. In order to excite slow or immobile soliton the second harmonic would have the frequency $\omega_2 \approx \omega_a$ near the top of the upper curve and the fundamental one would consist of two waves with the frequency $\omega_1=\omega_2/2$ and cross-oriented wave vectors $k_{11} \approx -k_{12}$ (see Fig. 2).

The envelopes of these waves are found to obey the following equations [16]:

$$\partial B_2/\partial t=iD_2(\partial^2 B_2/\partial z^2)-i\Theta_2 B_2+i\beta_2 B_{11}B_{12},$$
$$\partial B_{11}/\partial t=iD_1(\partial^2 B_{11}/\partial z^2)-i\Theta_1 B_{11}+i\beta_1 B_{12}^* B_2, \quad (11)$$
$$\partial B_{12}/\partial t=iD_1(\partial^2 B_{12}/\partial z^2)-i\Theta_1 B_{12}+i\beta_1 B_{11}^* B_2,$$

where z is the propagation coordinate, $D_{1,2}$ are the dispersion coefficients near the extreme points on the dispersion curve, $\beta_{1,2}$ are the coefficients of quadratic nonlinearity. Terms $\Theta_1=(\omega_1-\omega_c)$ and $\Theta_2=(\omega_2-\omega_a)$ correspond to frequency detuning between frequencies of soliton and critical frequencies.

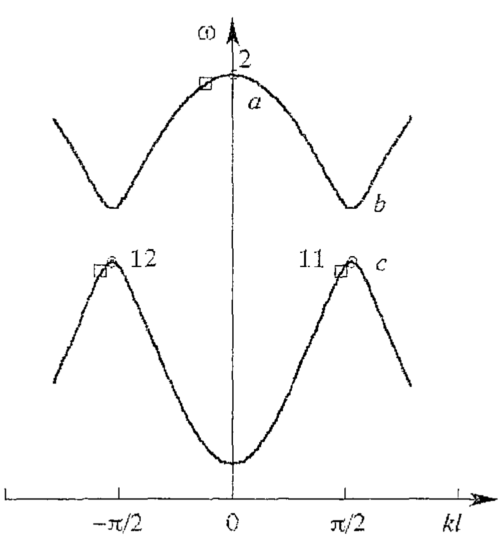

Fig 2. Dispersion curve of the system (linear regime); *a*, *b*, *c* mark the boundaries of pass-bands; circles correspond to excitation of immobile soliton and squares - of slow one

To examine the properties of slow solitons, let us substitute

$$B_{11,12,2}=A_{11,12,2}(\xi)\exp(-iq_{11,12,2}z),$$

where $\xi=z-v_{1,2}t$, $v_{1,2}$ are velocities of FF and SH respectively, q are additions to wave numbers, into (11):

$$D_2(\partial^2 A_2/\partial\xi^2)=(\Theta_2+D_2 q_2^2)A_2-\beta_2 A_{11}A_{12},$$
$$D_1(\partial^2 A_{11}/\partial\xi^2)=(\Theta_1+D_1 q_{11}^2)A_{11}-\beta_1 A_2 A_{12}, \quad (12)$$
$$D_1(\partial^2 A_{12}/\partial\xi^2)=(\Theta_1+D_1 q_{12}^2)A_{12}-\beta_1 A_2 A_{11}.$$

In addition, we obtain from (11) that the velocity of FF and SH is determined by the following formula:

$$v_j=-2q_j\ D_j, \quad j=1,2 \tag{13}$$

and the additions to wave numbers satisfy to: $q_2=q_{11}+q_{12}$. The requirement of FF and SH velocity equality leads to the dispersion coefficient ratio:

$$D_1=2D_2 \tag{14}$$

and additions to wave vectors parity: $q_{11}=q_{12}=q_1$, therefore

$$q_2=2q_1. \tag{15}$$

It is easy to normalize equations (12), taking into account (14) and (15):

$$\begin{aligned} A_1''&=A_1-A_1A_2, \\ A_2''&=\alpha A_2-A_1^2, \end{aligned} \tag{16}$$

where α is the dimensionless constant

$$\alpha=D_1(\Theta_2+D_2q_2^2)/[D_2(\Theta_1+D_1q_1^2)]=2(\Theta_2+v^2/2D_1)/(\Theta_1+v^2/4D_1). \tag{17}$$

The extension of soliton is determined by:

$$l=[D_1/(\Theta_1+v^2/4D_1)]^{1/2}, \tag{18}$$

and slow soliton's velocity can be found from:

$$v^2=4D_1(\alpha\Theta_1-2\Theta_2)/(4-\alpha). \tag{19}$$

It is necessary to notice, that in case $\alpha=1$ it is easy to find an exact sech^2 soliton solution. As it follows from (17), it is possible to excite a sech^2 soliton in wide range of parameters $\Theta_1>2\Theta_2$, D and β, imposing correct velocity by initial or boundary conditions. The characteristics of such solitons could be found by substitution the exact solution $A=a\,\mathrm{sech}^2(\xi/2l)$ into (12), which gives us the extension and peak amplitude:

$$l^2={}^3/_2D_1(2\Theta_1-\Theta_2)^{-1}={}^3/_2D_1(\omega_a-2\omega_c)^{-1}, \tag{20}$$

$$a_{11}=a_{12}=(\omega_a-2\omega_c)/(2\beta_1\beta_2)^{1/2}, \quad a_2=(\omega_a-2\omega_c)/\beta_1. \tag{21}$$

Let us note, that the condition on detunings $\Theta_1>2\Theta_2$ guarantees the positivity of left part in (20), so the extension remains real. The next interesting feature is that the dispersion has no effect on peak amplitude. Nevertheless, the increasing of dispersion increases energy trapped into soliton by enlarging its extension (20). The parameters of slow soliton (20), (21) are not depended on excitation frequency and determined by the properties of media, on the other hand, the velocity of slow sech^2 soliton decreases with tuning the frequencies deeper into the gap. In the limit case, the velocity of soliton is equal to zero. The properties of immobile solitons are examined theoretically in [12, 17]. The method of excitation of such solitons was presented and numerically proved in [18].

Computer modeling of $\chi^{(2)}$ tunneling

To support the theoretical investigations and to explore the dynamics of gap soliton tunneling we developed the algorithm of numerical solution of boundary value problem.

First, we used the simulation program to prove the possibility of boundary sech^2 soliton excitation. We found that the soliton in question could be excited by irradiating the edge of the nonlinear chain with FF and SH waves with soliton shapes. This soliton propagates freely trough the system, as it was predicted theoretically.

Second, we were interested in evolution of long input impulses consisted of both FF and SH. We chose the following detunnings: $\Theta_1=0.9$, $\Theta_2=0.1$. The system was determined by dispersion coefficients $D_1=10^{-2}$, $D_2=5\cdot10^{-3}$ and constants of quadratic nonlinearity $\beta_1=1$, $\beta_2=2.2$. We observed the effect of splitting of plane waves into slightly-oscillating slow solitons with close peak intensities, extensions, velocities and intervals. The variations of SH input intensity did not affect critically on the process of soliton formation. Moreover, the numerical experiment showed that it is possible to generate slow soliton series by irradiating the chain with FF only. It means that FF, that is exposed to bragg reflection and can't propagate alone, generates SH due to nonlinear effect. The SH wave is unable to propagate trough the system too, because it is situated in the stop-band. Thus, energy of SH concentrates near the edge of the system. Then it grows strong enough it starts to interact with FF. This mutual interaction leads the soliton-like formation to «tear off» from the edge zone of nonlinear system. The dynamics of slow soliton generation and tunneling is presented in Fig. 3.

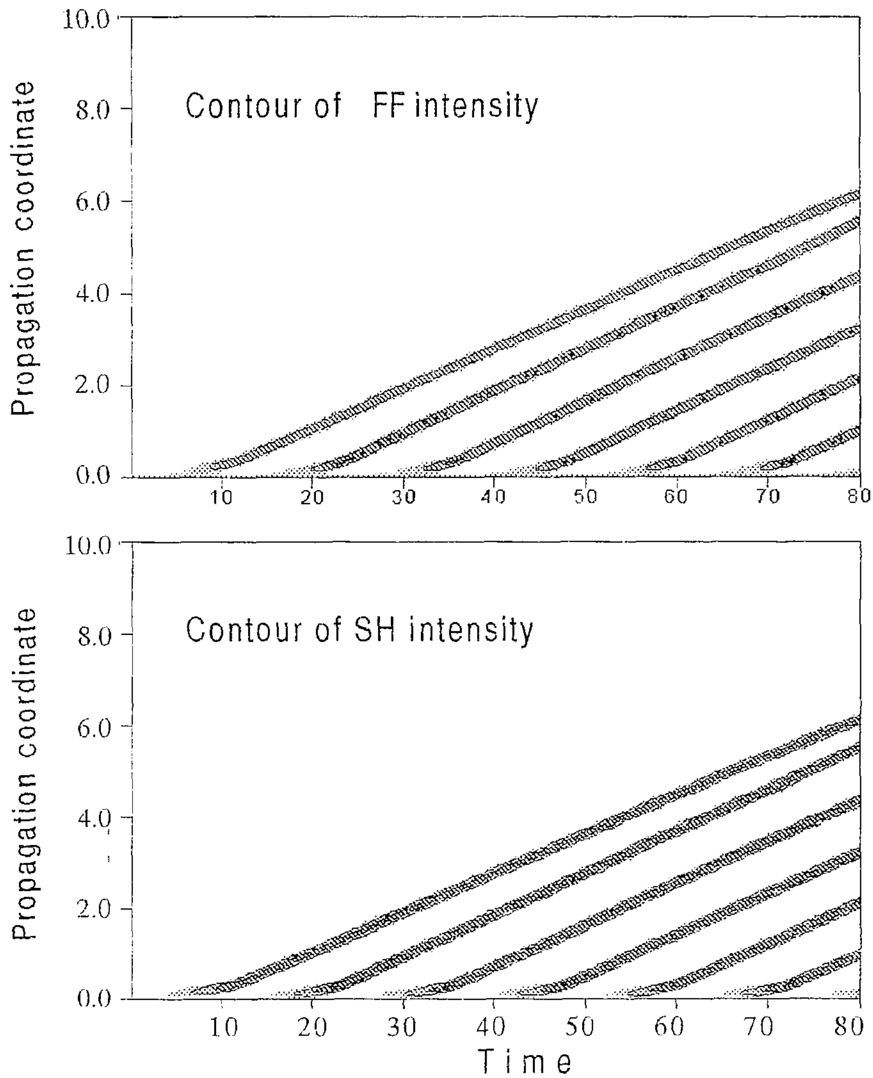

Fig 3. Generation of slow parametric gap solitons at the fundamental frequency and second harmonic frequency by input CW radiation at the fundamental frequency only

The dynamics of interactions between two parametric sech^2 solitons, injected into the media from the bound is also of interest. This process was simulated for the medium with dispersion coefficients $D_1=10^{-2}$, $D_2=5\cdot10^{-3}$ and coefficients of nonlinearity $\beta_1=3$, $\beta_2=5$. The typical interaction scenario is presented in Fig. 4. At first stage two solitons without phase mismatch propagate almost in a parallel way. At the critical point they start to fuse. At the collision point some energy is dropped, but the main part of it organizes the oscillating formation. The interval between solitons ejection determines the time between first collision and the «deepness» of pulsation.

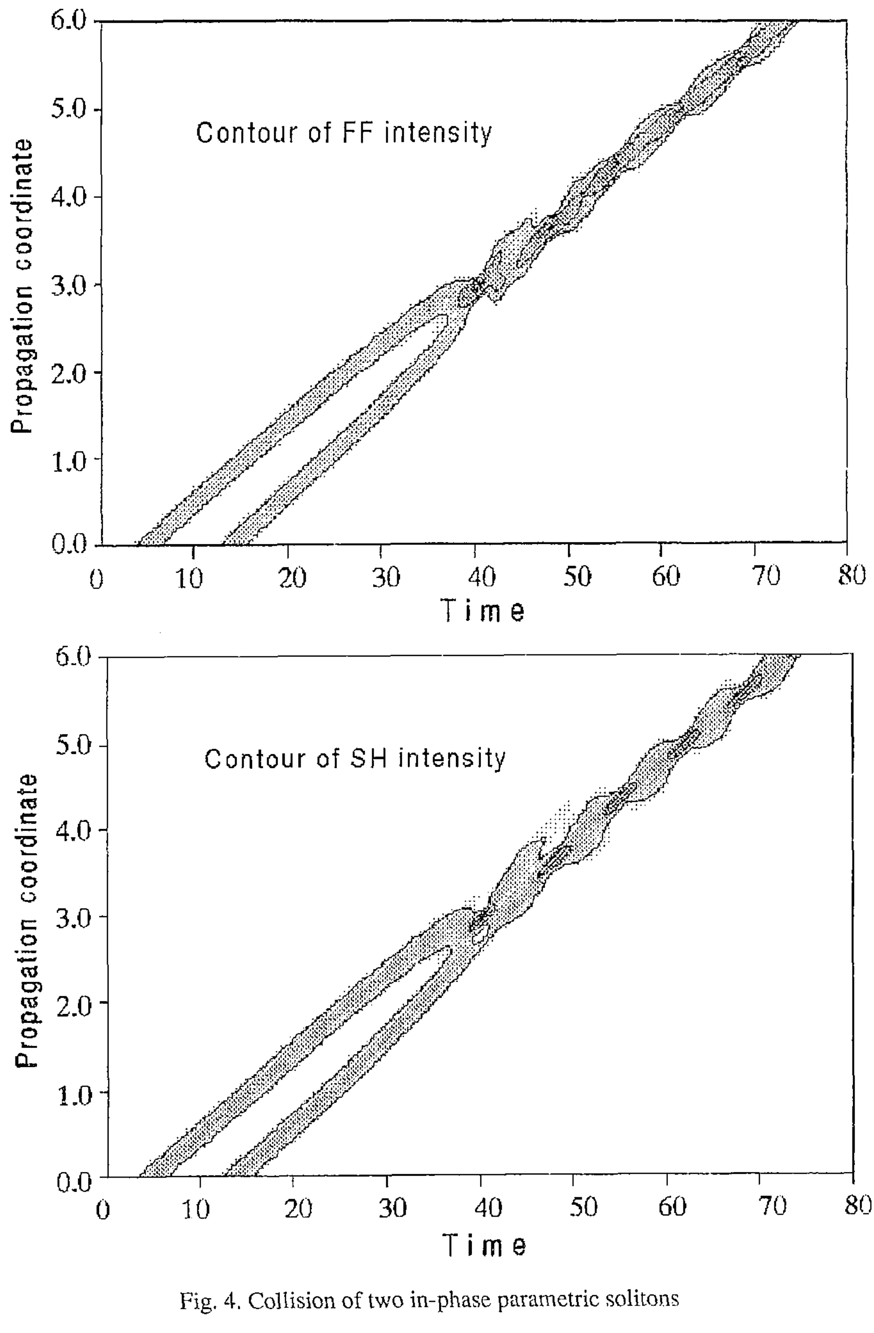

Fig. 4. Collision of two in-phase parametric solitons

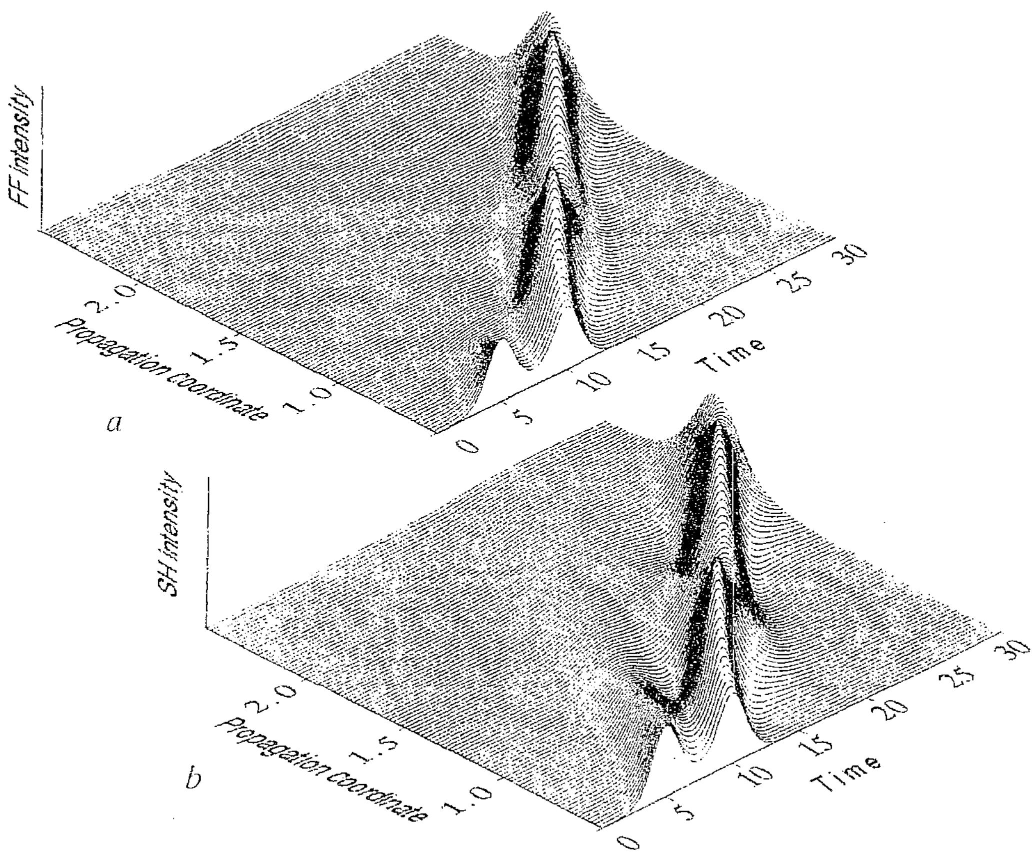

Fig. 5. Tunneling of overlapping underthreshold synphase bicolour signals, that leads to formation of oscillating parametric soliton

The fusion effect described above leads us to the question of possibility of decreasing the peak intensity of input sech^2 solitons to obtain underthreshold impulses, that cannot propagate through the chain alone, while the pair of these solitons can fuse and generate soliton-like signal that propagates on great distances without spreading. This regime was observed in numerical simulations. We used previous parameters of the medium and detunings and altered the peak amplitude of sech^2 solitons and intervals between them. The typical behavior of the pair of pulses in «underthreshold fusion» regime is pictured in Fig 5. The collision that leads to fusion occurs near the edge of the nonlinear system. The generated pulsing soliton slowly propagates along the chain. The increasing of interval between two initial impulses as well as decreasing of the peak amplitude leads to absence pulsing soliton formation.

Conclusions

In this work we investigated the phenomenon of gap $\chi^{(2)}$ and $\chi^{(3)}$ soliton tunneling both theoretically and numerically. The original theory of slow solitons was developed. The exact soliton solutions were found and its properties were investigated theoretically. We presented the modification of integral of motion I_3 that converted it to the sufficient condition of slow soliton formation. The same modification method could be applied to analysis of inclined wave beam propagation in nonlinear media.

The dynamics of slow soliton formation and main scenarios of interactions between solitons were explored by means of computer simulations. The tunneling of long impulses was investigated. In particular, parametric soliton generation regime from pure FF wave was discovered. The collisions of in-phase solitons were found to lead to the fusion and formation of oscillating soliton.

The research is supported by RFBR, Program «Russian Scientific Schools» and INTAS (grant 97-0581).

References

1. *Vinogradova M. V., Rudenko O. V., Sukhorukov A. P.* Theory of waves. Moscow: Nauka, 1990 (in Russian).

2. *Newell A. C.* Nonlinear tunneling // J. Math. Phys. 1978. Vol. 19 (5). P. 1126.

3. *Voloshchenko Yu. I., Rizhov Yu. N., Sotin V. E.* // JTF. 1981. Vol. 51. P. 902.

4. *Chen W., Mills D. L.* Gap solitons and the nonlinear optical response of superlattices // Phys. Rev. Lett. 1987. Vol. 58. P. 160.

5. *Eggleton B. J., Slusher R. E., de Sterke C. M., Krug P. A., Sipe J. E.* Bragg Grating Solitons // Phys. Rev. Lett. 1996. Vol. 76. P. 1627.

6. *Karamzin Yu. N., Sukhorukov A. P.* Nonlinear interaction of diffracted light beams in a medium with quadratic nonlinearity: mutual focusing of beams and limitation of the efficiency of optical frequency converters // JETP Lett. 1974. Vol. 20. P. 339.

7. *Torruellas W. E., Wang Z., Hagan D. J., Van Stryland E. W., Stegeman G. I., Torner L., Menyuk C. R.* Observation of two-dimensional spatial solitary waves in quadratic medium // Phys. Rev. Lett. 1995. Vol. 74. P. 5036.

8. *Stegeman G. I., Hagan D. J., Torner L.* Cascading phenomena and their applications to all-optical signal processing, mode locking, pulse compression and solitons // J. of Opt. and Quantum Electronics. 1996. Vol. 28 (12). P. 1691.

9. *Sukhorukov A. P.* Nonlinear wave interactions in optics and radiophysics. Moscow: Nauka, 1988 (in Russian).

10. *Steblina V. V., Kivshar Yu. S., Lisak M., Malomed B. A.* Self-guided beams in diffractive quadratic medium: variational approach // Opt. Commun. 1995. Vol. 118. P. 345.

11. *Karamzin Yu. N., Sukhorukov A. P., Trofimov V. A.* Mathematical modeling in nonlinear optics. Moscow: Moscow University Publishing, 1989 (in Russian).

12. *Kivshar Yu. S.* Gap solitons due to cascading // Phys. Rev. E. 1995. Vol. 51 (2). P. 1613.

13. *Conti C., Trillo S., Assanto G.* Bloch function approach for parametric gap solitons // Opt. Lett. 1997. Vol. 22 (7). P. 445.

14. *Polyakov S. V., Sukhorukov A. P.* Formation and Properties of Bicolored Gap Solitons // Bull. Russ. Acad. Sci. Phys. 1998. Vol. 62 (12).

15. *Akhmanov S. A., Visloukh V. A., Chirkin A. S.* Optics of femto-second laser pulses. Moscow: Nauka, 1988 (in Russian).

16. *Karamzin Yu. N., Polyakov S. V., Sukhorukov A. P.* Nonlinear tunneling in waveguids // Moscow University Phys. Bull. 1996. № 5. P. 70.

17. *Polyakov S. V., Sukhorukov A. P.* Slow and Immobile Parametric Solitons in the Media with Periodic Inhomogenity // Bull. Russ. Acad. Sci. Phys. 1997. Vol. 61 (12). P. 2353.

18. *Conti C., Trillo S., Assanto G.* Trapping of slowly moving or stationary two-color gap solitons // Opt. Lett. 1998. Vol. 23 (5). P. 334.

Department of Physics,
Moscow State University

Received 13.07.98

УДК 621.3.09:621.373.1

ЩЕЛЕВЫЕ СОЛИТО́НЫ

С. В. Поляков, А. П. Сухоруков

В статье представлены исследования, посвященные проблеме медленных и неподвижных солитонов, возбуждаемых внутри запрещенных зон неоднородной среды как с квадратичной, так и с кубичной нелинейностью. Найден интегральный критерий формирования медленных щелевых солитонов, позволяющий определить их основные свойства. Динамика распространения и взаимодействия параметрических щелевых солитонов в квадратичной среде иллюстрируется с помощью численного моделирования и сравнения с солитонами такого же типа на кубичной нелинейности.

Sergey V. Polyakov was born in Moscow (1974), graduated Moscow State University (1998) and currently studying in MSU as Ph.D. student. The field of scientific interests includes nonlinear optics, soliton generation and interaction, numerical simulations of optical phenomena.

Anatoly Petrovich Sukhorukov was born in Moscow (1935). In 1961 he graduated from Moscow State University, Physics Department. He received Ph.D. degree in physics (1968) and joined Physics Department of MSU the same year. He received Doctor of Sciences degree (1974) and got the title of professor (1977). At present he is the Head of Radiophysics Division. His scientific interests cover various aspects of nonlinear optics and radiophysics. In particular, he made important contributions to the theory of self-focusing and self-action of laser beams and short pulses. He also investigated wide range of parametric processes, predicted the parametric solitons in quadratic media in his pioneer works in 1974. He is co-author of «Wave Theory» textbook and author of monograph «Nonlinear Wave Interactions in Optics and Radiophysics» which was published in 1988. A. P. Sukhorukov was awarded by Lenin's and State Prizes in 1988 and 1984 respectively. respectively. He is a member of the Optical Society of America, SPIE, Russian Optical Society. He supervised 60 masters degree works, 25 Ph.D. works. Five of his students got Doctor degree by now. E-mail: aps@nls.phys.msu.su.