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CHAOTIC OSCILLATIONS IN A MODEL OF VOCAL SOURCE

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A model of human vocal folds in the form of two spring-loaded plates, which can collide with one another, is considered. It is shown that due to air flow between the plates excitation of chaotic self-oscillations occurs. In this process the shape of oscillations of flow volume velocity was found to be close to observed experimentally.

Introduction

Nowadays many different models of voice production are known. A review of some of these models is given by Sorokin [1]. These models are of paramount importance both for the best understanding the mechanism of voice production and in certain practical applications, for example, in speech synthesis and recognition [2], in voice pathology [3], and so on. The heart of any such model is a model of vocal source, i.e., of vocal folds located in the human (or animal) larynx. The structure of vocal folds is rather complicated. It is described, in particular, in [1]. The schematic vertical cross-section of the larynx, taken from [4] and somewhat simplified, is shown in Fig. 1, *a*. The vocal folds are formed from cords of two muscles (the vocal muscle *1* and the aryten-thyroid muscle *2*) and conjunctive tissues. Above the vocal folds there are located the so-called false vocal folds *3*. They are free from internal muscles but play an important role in the formation of hissing sounds. As true vocal folds are resected, their functions are sometimes taken over by the false vocal folds [5]. Between the true and false vocal folds there is some expansion of the larynx called Morgan's ventricles *4*. Variations of stiffness, length and shape of the vocal folds, as their internal muscles contract, cause a certain change of the fundamental frequency of the self-oscillations excited.

A variety of models of vocal folds, both complicated [6-8] and moderately simple [9-12] are known. The techniques for the calculation of aerodynamic forces determine mainly the extent to which one or the other of these models is complicated. For example, in the widely known two-mass model by Ishizaka and Flanagan [9] aerodynamic forces were calculated in the quasi-static approximation using the Bernoulli law. In so doing, the velocity of motion of the glottis' walls, the viscosity and inertia of air were ignored. Such a technique was also used in a simplified version of this model suggested by Herzel and Knudsen [3].

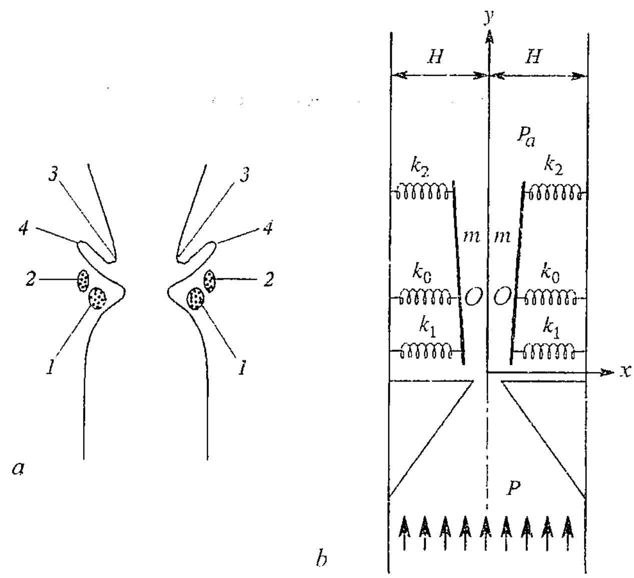

Fig. 1. *a* - The schematic vertical cross-section of the larynx: the vocal muscles are labelled *1*, the arytenthyroid muscles *2*, the false vocal folds *3*, and Morgan's ventricles *4*; *b* - the model of the vocal folds

Another model of vocal folds, which by its nature is close to Ishizaka and Flanagan's model, was suggested in [13, 14]. Contrary to the first, this model permits the calculation of aerodynamical forces not only in quasi-static approximation. However, the study of this model with such aerodynamical forces, even numerically, is very complicated problem. Therefore, we consider below the results of numerical simulation of this model with aerodynamical forces calculated in quasi-static approximation.

1. The structure of the model and its equations

The model involves two absolutely rigid plates suspended by springs to the walls of a tube with right-angled (for simplicity) cross section (see Fig. 1, *b*). Air enters the tube from a reservoir of a sufficiently large volume V due to the pressure drop $\Delta P = P - P_a$ and can cause self-oscillations of the plates. It can be shown that the excitation of self-oscillations is possible if each plate has at least two degrees of freedom, i.e., it can both move progressively in the direction orthogonal to the air flow and turn about axis O, passing through its centre of mass. In this regard the excitation of self-oscillations of the plates is akin to the excitation of the bending-torsion flutter of an aeroplane wing [15- 17].

Assuming the motion of the plates to be completely symmetric relative to the tube mid-plane, the motion equations for each of the plates can be written as

$$\begin{aligned} m\ddot{h}_a + \alpha\dot{h}_a + k(h_a - \tilde{h}_a) + \kappa(\varphi - \tilde{\varphi}) &= F, \\ J\ddot{\varphi} + \beta\dot{\varphi} + K(\varphi - \tilde{\varphi}) + \kappa(h_a - \tilde{h}_a) &= M, \end{aligned} \tag{1}$$

where h_a is the displacement of plate's center of mass, φ is the angle of rotation clockwise about the axis passing through the center of mass, $\tilde{h}_a$ and $\tilde{\varphi}$ are the values of h_a and φ for undeformed springs, m is the plate mass, J is the plate moment of inertia about the axis passing through the center of mass, $k = k_1 + k_2 + k_0$ is the total rigidity factor of the springs, $K=k_1a^2+k_2(b-a)^2$ is the total rigidity factor with respect to the rotation, a is the distance between the lower edge of the plate and its center of mass, b is the plate length along the flow, $\kappa=k_1a-k_2(b-a)$ is the coupling factor characterizing the effect of the displacement of the plate center of mass on the plate rotation, and vice versa, α and β are the damping factors,

$$F = l\int_0^b p(x)dx,$$
$$M = l\int_0^b (x-a)p(x)dx \tag{2}$$

are the aerodynamic force and moment acting on the plate which are created by the air flow, $p(x)$ is the difference between the air pressure in the inter-plate slot and atmospheric pressure P_a, and l is the plate transverse dimension. It should be noted that Eqs (1) can be obtained by the Galerkin method as a discrete model of bending-torsion oscillations of a beam lying on an elastic base.

Eqs (1) are conveniently written in another form:

$$\ddot{S}_0+2\delta_{11}\dot{S}_0+2\delta_{12}\dot{S}_b+\omega_1^2(S_0-\tilde{S}_0)+\kappa_1(S_b-\tilde{S}_b) = F_0,$$
$$\ddot{S}_b+2\delta_{21}\dot{S}_0+2\delta_{22}\dot{S}_b+\omega_2^2(S_b-\tilde{S}_b)+\kappa_2(S_0-\tilde{S}_0)=F_b, \tag{3}$$

where

$$\delta_{11}=[(b-a)/b]\alpha/(2m)+(a/b)\beta/(2J),$$
$$\delta_{12}=(a/b)(\alpha/(2m)-\beta/(2J)),$$
$$\delta_{21}=[(b-a)/a]\delta_{12},$$
$$\delta_{22}=(a/b)\alpha/(2m)+[(b-a)/b]\beta/(2J),$$
$$\omega_1^2=[(b-a)/b]k/m+(a/b)K/J+[\kappa/(mJb)][J+ma(b-a)],$$
$$\omega_2^2=(a/b)k/m+[(b-a)/b]K/J-[\kappa/(mJb)][J+ma(b-a)], \tag{4}$$
$$\kappa_1=(a/b)(k/m-K/J)-[\kappa/(mJb)](J-ma^2),$$
$$\kappa_2=[(b-a)/b](k/m-K/J)+[\kappa/(mJb)][J-m(b-a)^2],$$

$$\tilde{S}_0=2l(\tilde{h}_a+a\tilde{\varphi}),\quad \tilde{S}_b=2l[\tilde{h}_a-(b-a)\tilde{\varphi}],$$
$$F_0=2l(F/m+aM/J),\quad F_b=2l(F/m-(b-a)M/J).$$

It should be remembered that the vocal folds, executing self-oscillations, collide with one another. This process plays a great role in voice production because results in the generation of pulses containing a number of high harmonics which enrich speech. Therefore we have to add impact conditions to Eqs (3). These conditions may be obtained as follows: because the plates are assumed to be absolutely rigid, then only the plate edges can collide. For definiteness, first we consider the collision of the plate edges corresonding to $x=0$. Let an impulse of force $F\Delta t$, causing changes of the plate momentum $m\Delta\dot{h}_a$ and of the plate angular momentum $J\Delta\dot{\varphi}$, arise as a result of the collision. Thus, $F\Delta t=m\Delta\dot{h}_a$, $Fa\Delta t=J\Delta\dot{\varphi}$. From this it follows that

$$\Delta\dot{\varphi}=(ma/J)\Delta\dot{h}_a. \tag{5}$$

On the other hand, it can be easily shown that

$$\Delta\dot{h}_a+a\Delta\dot{\varphi}=\Delta\dot{h}_0. \tag{6}$$

If the velocity restitution coefficient after impact is $R\leq 1$, then

$$\Delta\dot{h}_0=\dot{h}_0^{+}-\dot{h}_0^{-}=-(1+R)\dot{h}_0^{-}, \tag{7}$$

where $\dot{h}_0^{+}$ is the value of $\dot{h}_0$ after impact, and $\dot{h}_0^{-}$ is the value of $\dot{h}_0$ before impact. From (5) - (7) we obtain the following conditions for the collision of the plate edges under consideration:

$$\Delta\dot{h}_a=-(1+R)\dot{h}_0^{-}/(1+ma^2/J), \quad \Delta\dot{\varphi}=-(ma/J)[(1+R)\dot{h}_0^{-}]/(1+ma^2/J). \tag{8}$$

In a like manner we obtain the conditions for the collision of the plate edges corresponding to $x=b$:

$$\Delta\dot{h}_a=-(1+R)\dot{h}_b^{-}/[1+m(b-a)^2/J], \quad \Delta\dot{\varphi}=[m(b-a)/J][(1+R)\dot{h}_b^{-}]/[1+m(b-a)^2/J] \tag{9}$$

From (8) and (9) we can find the conditions for the collision of the plate edges corresonding to $x=0$ and $x=b$ in terms of S_0 and S_b: for $x=0$

$$\Delta\dot{S}_0=\dot{S}_0^{+}-\dot{S}_0^{-}=-(1+R)\dot{S}_0^{-},$$
$$\Delta\dot{S}_b=\dot{S}_b^{+}-\dot{S}_b^{-}=-(1+R)\dot{S}_0^{-}[J-ma(b-a)]/(J+ma^2), \tag{10}$$

and for $x=b$

$$\Delta\dot{S}_b=-(1+R)\dot{S}_b^{-}, \quad \Delta\dot{S}_0=-(1+R)\dot{S}_b^{-}[J-ma(b-a)]/[J+m(b-a)^2]. \tag{11}$$

We note that for sufficiently small R the impact may be quasi-plastic [19], i.e. the duration of the contact of the plate edges during the impact may be finite.

It can be shown that the sound pressure above the slot is

$$p_s(t)=\rho c_0(u-u_0)/(Hl), \tag{12}$$

where c_0 is the velocity of sound, u is the volume velocity, u_0 is the steady state value of u, and H is the width of the tube.

As opposed to the flow of a wing, we are dealing here with one-sided flow. Therefore we cannot use the expressions for aerodynamic forces which are known for a flatter problem. Let us calculate these forces in quasi-static approximation with using the Bernoulli theorem.

First of all, we use the formula for a dynamical pressure drop as flow is constricted gradually [18]. In a zero approximation with respect to h/H we obtain

$$p(0)=\Delta P-\zeta_1\rho u^2/(2S_0^2), \tag{13}$$

where $1<\zeta_1<2$ is the coefficient depending on the bevel shape at the slot input and, in general, on the Reynolds number.

According to the Bernoulli theorem we find $p(x)$:

$$p(x)=p(0)-\rho u^2/(2S_0^2)\{S_0^2/[S_0-(S_0-S_b)x/b]^2-1\}. \tag{14}$$

Unknown volume velocity u can be found from the formula for a dynamic pressure drop as flow diverges abruptly at the slot output [18]

$$p(b)=\zeta_2\rho u^2/(2S_b^2), \tag{15}$$

where ζ_2 is the coefficient depending on the velocity profile at the slot's output and on the Reynolds number **Re** (for **Re**$>10^3$, if the velocity profile is uniform then $\zeta_2=0$, and if the velocity profile is Poiseuille's then $\zeta_2\approx0.6$). Using further the expression (14), in view of (13), for $x=b$ we obtain the following equation for the volume velocity u:

$$\rho u^2/2=S_0^2S_b^2\Delta P/[(\zeta_1-1)S_b^2+(1+\zeta_2)S_0^2]. \tag{16}$$

Substituting (14) in (2) and taking into account (13) we find

$$F=bl[\rho u^2/(2S_b^2)](1+\zeta_2-S_b/S_0),$$

$$M=rF-\rho b^2lu^2/[2(S_0-S_b)^2][\ln(S_b/S_0)+(S_0^2-S_b^2)/(2S_0S_b)],$$

where $r=b/2-a$.

In addition to the explanation of the excitation of self-oscillations, the model allows us to explain some other experimental facts as well. For example, certain results of the experiments performed with human vocal folds *in vivo* are presented by Kaneko in [20]. In these experiments human vocal folds were excited by a mechanical vibrator over the frequency range of 30-300 Hz. As a result of investigation of 17 men and 19 women, it was found that in the case of excitation of unstrained folds there is one resonance both for men and for women, the resonant frequency being in the range of 91-145 Hz for men (average value 128 Hz) and 115-167 Hz for women (average value is 136 Hz). For men, the average resonant frequency (128 Hz) approximately coincides with their average fundamental frequency at phonation (129 Hz), while for women the average resonant frequency (136 Hz) is significantly lower than their average fundamental frequency at phonation (240 Hz). Measurements of frequency responses in the case of excitation of the strained folds, ready to be voiced with a fundamental frequency f_0, showed their essential distinction both between men and women and between low and high frequency f_0. For men, if the frequency f_0 was low (approximately 100 Hz) then, as in the first case, one resonance was observed at the frequency f_0; but if the frequency f_0 was high then two resonances were observed: one at a frequency of about 100 Hz, and another at the frequency f_0. For women two resonances were always observed: one also at a frequency of about 100 Hz, and another at the frequency f_0. These facts imply that vocal folds possess at least two natural frequencies, one of them depending only slightly on the extent of fold strain and another being completely determined by it. This behaviour can be easily explained in the framework of the model considered if we assume that the cord of the vocal and the aryten-thyroid muscles, whose straining determines the fundamental frequency of phonation, passes close to the centre of mass of the fold. Under this assumption the change of strain of these muscles is associated with a change of the rigidity k_0 in the model, which has to affect slightly the value of the natural frequency close to the partial frequency of rotational plate oscillations. At the same time, it has to change considerably the natural frequency close to the partial oscillation frequency of the plate centre of mass. If both frequencies are sufficiently close to each other then under an external periodic action only one resonance can be observed.

2. Results of numerical simulation of the model equations

The parameters which are necessary for the simulation of self-oscillations of the human vocal folds can be estimated on the basis of the information are presented by Sorokin [1]. We set the following values of the parameters: $\zeta_1=1.37$, $\zeta_2=0.2$, $\rho=1.3\cdot10^{-3}$ g/cm^3, m = 0.15g, J = 0.004 g·cm^2, $k=8\cdot10^4$ g/s^2, K=2400 g·cm^2/s^2, κ = 10^3 g·cm/s^2, α=40 g/s,

β=1 g/s, a=0.15 cm, b=0.5 cm, l=1.6 cm, $\tilde{h}_0$=0.07 cm, $\tilde{h}_b$=0.06 cm. The parameters ΔP and R are varying.

A steady state solution of Eqs (3) is determined from the following algebraic equations:

$$\omega_1^2(S_0-\tilde{S}_0)+\kappa_1(S_b-\tilde{S}_b)=F_0, \quad \omega_2^2(S_b-\tilde{S}_b)+\kappa_2(S_0-\tilde{S}_0)=F_b. \tag{17}$$

(Here the subscripts 'st' are omitted for brevity.)

Taking into account (4), (16) and (17), we can find the dependence of steady-state values of the cross-section area of the slot at its input and output and of the volume flow velocity u_0 on the pressure drop ΔP. This dependence is given in Fig. 2.

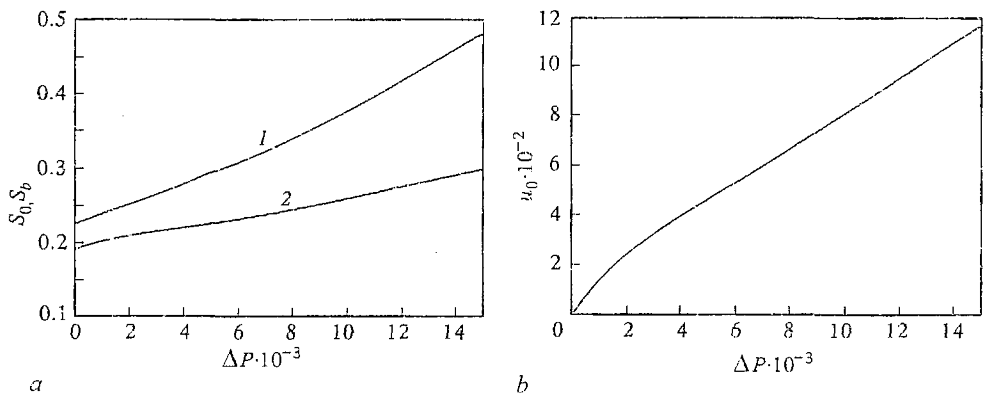

Fig. 2. a - The plots of S_0^{st} (*1*) and S_b^{st} (*2*) versus ΔP; b - the plot of u_0 versus ΔP for the values of the parameters given above

Writing linearized equations for small deviations from the steady state values found, one can find a condition for the self-excitation of the system and the self-oscillation frequency in the neighbourhood of the boundary of self-excitation. It can be shown that the condition of self-excitation is satisfied for $\Delta P \geq \Delta P_{cr} \approx$ 5650 g/cm·s^2≈ ≈5.7 cm of water. The critical value of u associated with this value of ΔP is $u_{cr} \approx$ 527 cm^3/s. The self-oscillation frequency in the neighbourhood of the self-excitation boundary is $f=\omega/2\pi\approx$120 Hz. The results obtained correspond to known experimental data.

Numerical simulation of Eqs (3), with using (17) and the impact conditions (10), (11), shows that excitation of self-oscillations is hard. The character of the transition through the self-excitation threshold essentially depends on the restitution coefficient R. For small R a chaotic attractor exists on each side of the transition point, but for $\Delta P^*<\Delta P<\Delta P_{cr}$, where ΔP^*<3700 g/cm·s^2, in addition to this attractor the steady state is also an attractor. For R close to unity and $\Delta P<\Delta P_{cr}$ no chaotic attractor has been found. In this case the transition through the self-excitation threshold occurs via intermittency.

The shape of oscillations of S_0 for R=1 and different values of ΔP are given in Fig. 3, a, b and c. The shape of oscillations of S_b and u is similar to that for S_0. For ΔP=6000 the oscillations resemble in their shape stochastic oscillations in Neymark's pendulum that is an example of stochastic generator [21, 22]. Power spectra of the volume velocity u for all indicated values of ΔP contain only a single line, but this line is moderately wide. An example of the power spectrum is shown in Fig. 3, d.

For small values of the restitution coefficient R stable self-oscillations are possible down to ΔP^*. For $\Delta P^*<\Delta P<\Delta P_{cr}$ their existance depends on initial conditions. In this range of ΔP the oscillations excited in their shape are close to periodic with the fundamental frequency about 120 Hz. An example of such oscillations is illustrated in Fig. 4.

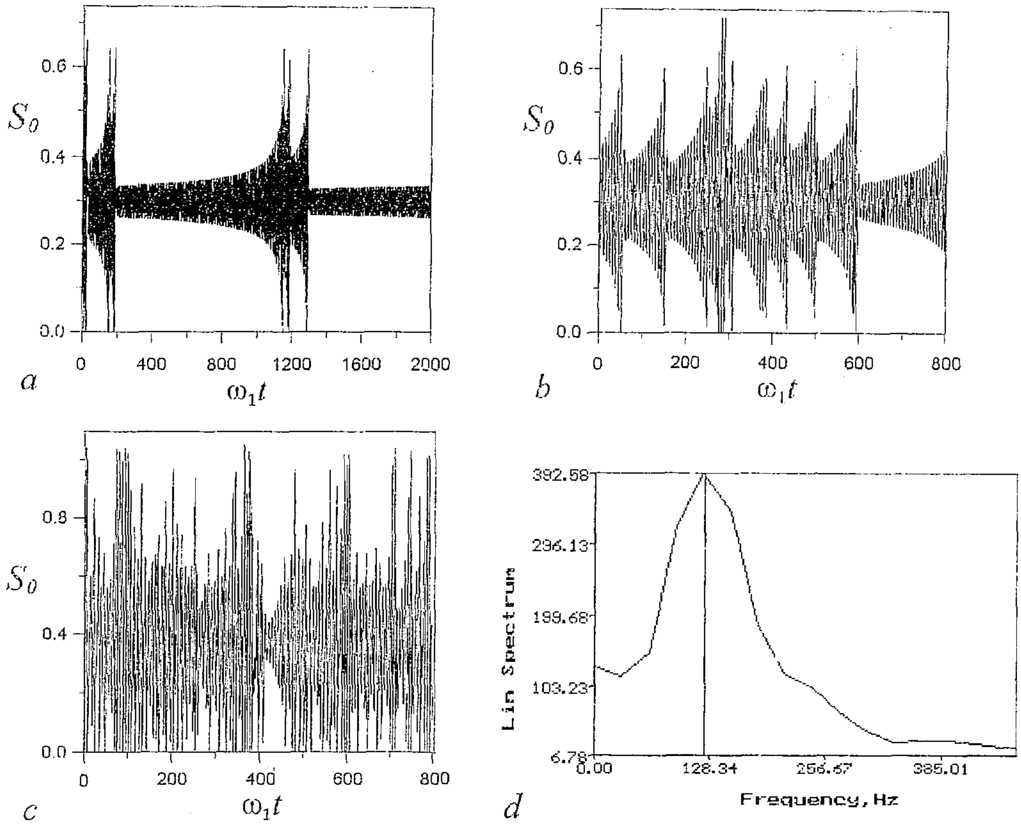

Fig. 3. The dependences of S_0 on $\omega_1 t$ for R=1, ΔP=5710 (*a*), ΔP=6000 (*b*) and ΔP=10000 (*c*); the power spectrum of the volume velocity u for ΔP=10000 (*d*)

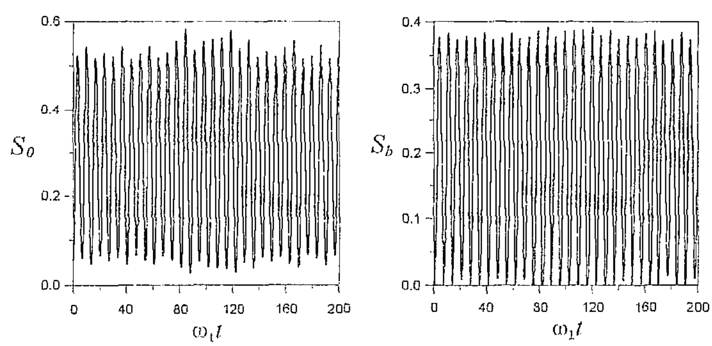

Fig. 4. The dependence of S_0 and S_b on $\omega_1 t$ for ΔP=4000, R=0

As ΔP increases the oscillations become more chaotic and the duration of the contact of the plate edges during the impact increases. This is illustrated in Figs 5, 6. Power spectra of u contain a number of moderately narrow lines at the frequencies divisible by the fundamental frequency (see Figs 5, *d*; 6, *d*).

It is interesting that the phase shift between oscillations of the lower and upper edges of the plates is close to observed in experiments. This is illustrated in Fig. 7, where oscillations of S_0 and S_b both numerical (for ΔP= 6000, R=0) and experimental [23] are shown.

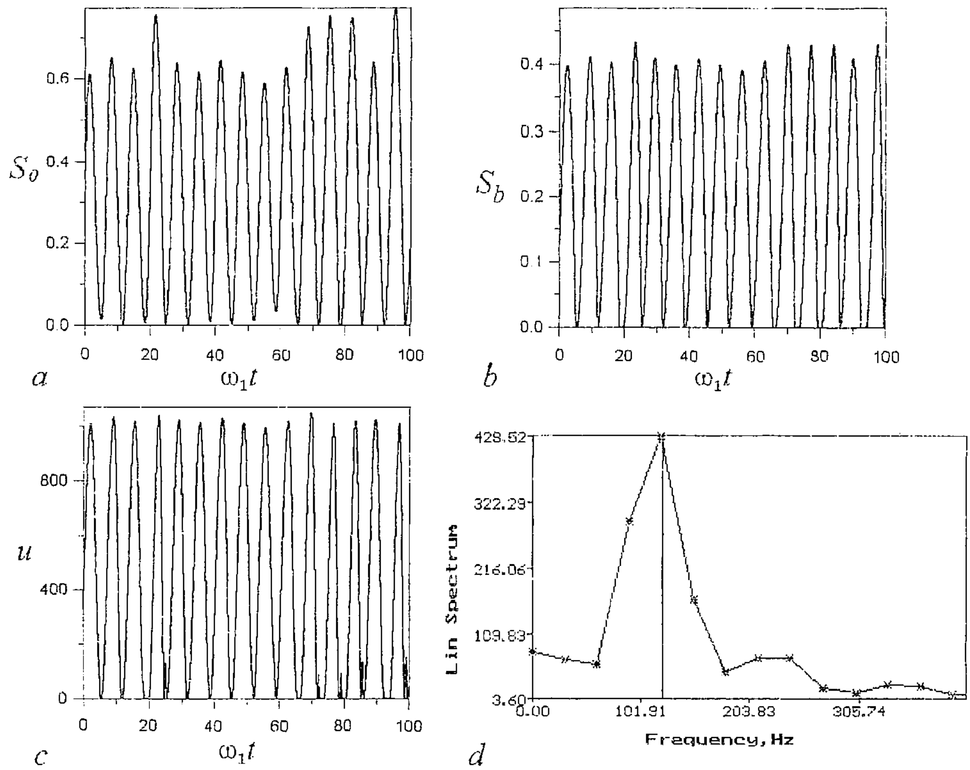

Fig. 5. The dependences of S_0 (a), S_b (b), u (c) on $\omega_1 t$ and the power spectrum of u (d) for ΔP=6000, R=0

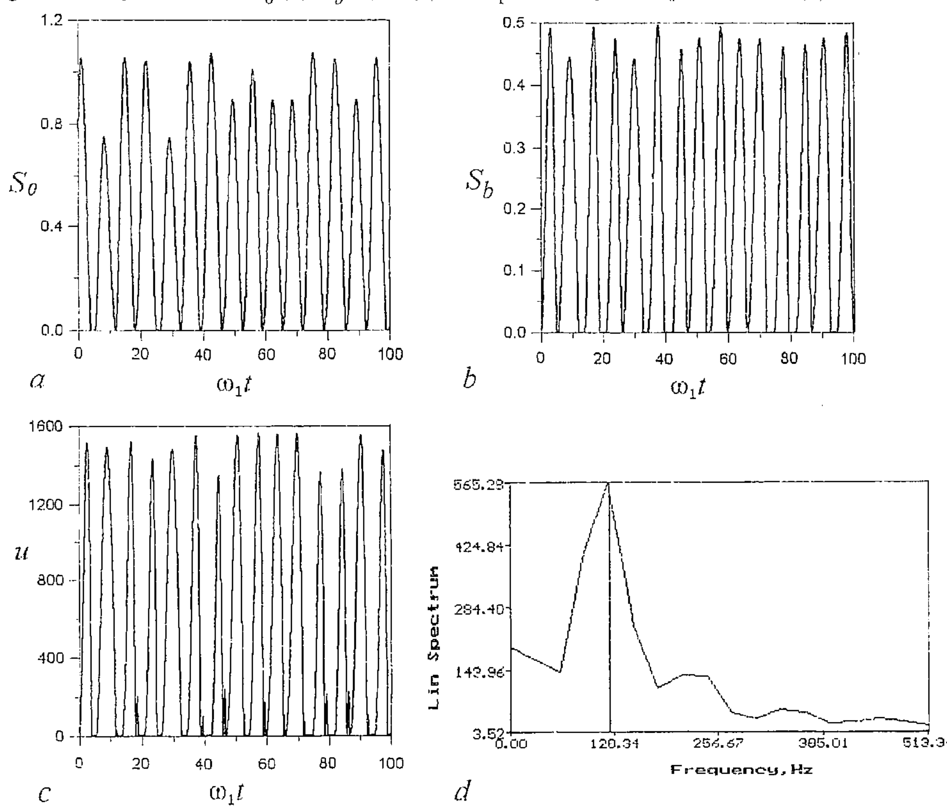

Fig. 6. The dependences of S_0 (a), S_b (b), u (c) on $\omega_1 t$ and the power spectrum of u (d) for ΔP=10000, R=0

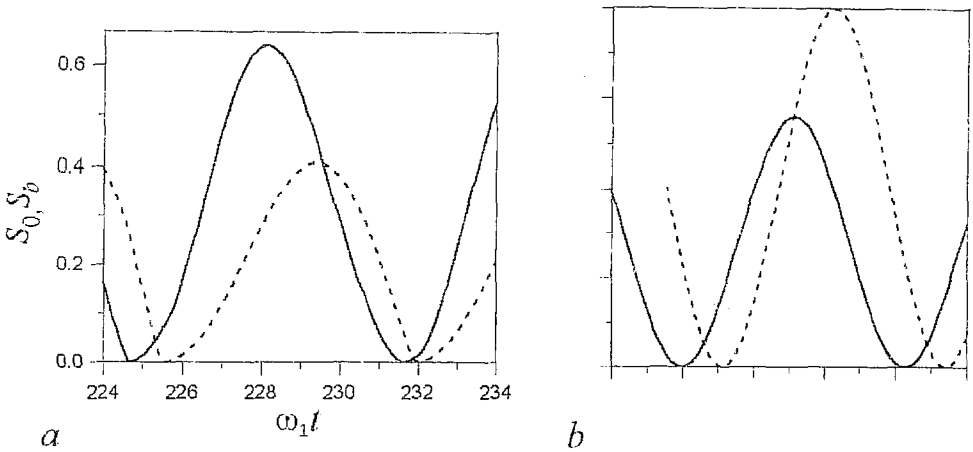

Fig. 7. Plots of S_0 (solid lines) and S_b (dashed lines) versus time for numerical simulation (*a*) and an experiment (*b*)

In more detail the shape of the air flow volume velocity pulses is given in Fig. 8 for R=0, ΔP = 6000 and ΔP=10000. It corresponds to known experimental data [1].

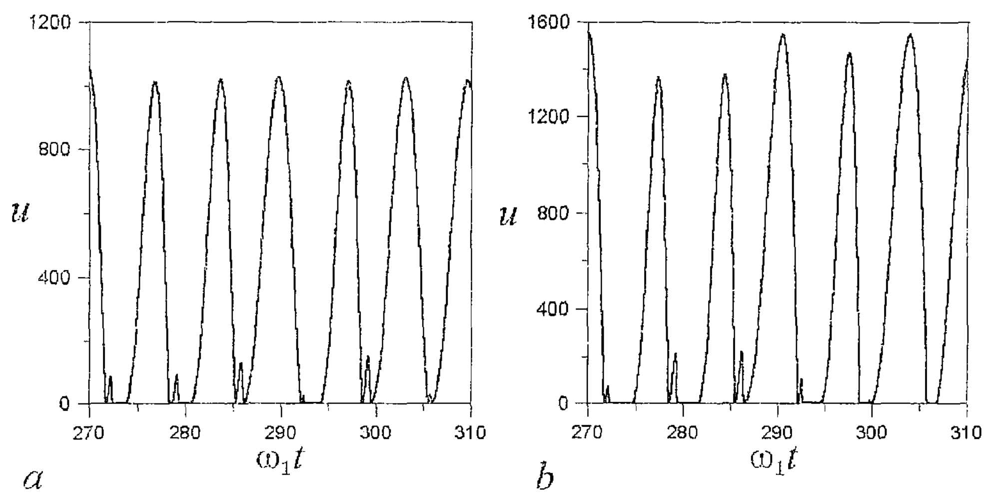

Fig. 8. The shape of the pulses of volume velocity for R=0, ΔP=6000 (*a*) and ΔP=10000 (*b*)

Conclusion

So, we have shown that the model considered allows us to simulate chaotic self-oscillations of human vocal folds taking into account collisions between them. Experiments with speech synthesators, where have used periodic vocal sources, show that for better sounding it is necessary to add small noise [1]. It seems likely that in the Nature slight chaos plays a role of this noise.

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ХАОТИЧЕСКИЕ КОЛЕБАНИЯ В МОДЕЛИ ГОЛОСОВЫХ СКЛАДОК

П.С. Ланда

Рассматривается модель голосовых складок человека в виде двух пластин, прикрепленных пружинами к стенкам трубы. Предполагается, что в процессе колебаний пластины могут соударяться друг с другом. Показано, что под действием потока воздуха происходит возбуждение хаотических колебаний пластин и скорости воздушного потока. При определенных значениях параметров форма колебаний объемной скорости потока оказалась близкой к экспериментально наблюдаемой.

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