Innovations in applied physics

Izvestiya Vysshikh Uchebnykh Zavedeniy. Applied Nonlinear Dynamics. 2024;32(2)

Article DOI: 10.18500/0869-6632-003091

Peculiarities of the dynamics of a viscous liquid with a free boundary under periodic influences

V. L. Sennitskii

Lavrentyev Institute of Hydrodynamics SB of the RAS, Novosibirsk, Russia
E-mail: ✉sennitskii@yandex.ru

Received 29.08.2023, accepted 23.10.2023, available online 8.02.2024, published 29.03.2024

Abstract. *Purpose* of the work is revealing and researching of peculiarities of a motion of a viscous liquid having a free boundary and undergoing periodic in time influences which are characterized by the absence of a predominant direction in space. *Methods.* The analytic investigation methods of non-linear problems, of boundary problems for the system of Navier−Stokes and continuity equations are used that are the method of perturbations (the method of a small parameter) the method of Fourier (the method of a separation of variables), an averaging, a construction and studying of asymptotic formulas. *Results.* A new problem on the motion of a viscous liquid is formulated and solved. Asymptotic representations of the found solution are constructed and explored. New hydromechanical effects are revealed. *Conclusion.* The work is fulfilled in the development of a perspective direction in liquid mechanics that is of researching the dynamics of hydromechanical systems under periodic influences. The obtained results can be used in particular in further investigations of a non-trivial dynamics of hydromechanical systems, under working for the methods of a control of hydromechanical systems.

Keywords: viscous liquid, free boundary, periodic in time influences, predominant direction in space, stationary motion.

For citation: Sennitskii VL. Peculiarities of the dynamics of a viscous liquid with a free boundary under periodic influences. Izvestiya VUZ. Applied Nonlinear Dynamics. 2024;32(2):197–208. DOI: 10.18500/0869-6632-003091

Introduction

Theoretical and experimental study of the dynamics of hydromechanical systems under periodic time effects is one of the promising areas in liquid mechanics. A number of non-trivial results have been obtained in this direction (see the works [1–31] and [32–38]). The conducted studies allowed us to prove the existence of the phenomenon of predominantly unidirectional motion of compressible inclusions in a vibrating liquid [1,2,9,26,38]; to construct a mathematical model of the hydromechanical analogue of the "Kapitsa pendulum" [17, 39]; detect the effects of paradoxical behavior of a solid body in a vibrating liquid [26, 32, 33, 35–37], "levitation" of liquid [31], "spontaneous" transition of a solid inclusion in an oscillating liquid to a position with a given orientation in space [23].

In this paper, we consider the problem of the motion of a viscous liquid caused by the translational periodic motion of a flat wall and a flat plate with a boundary permeable to the liquid. The liquid fills two regions of space. The motion of the liquid in these regions occurs under substantially different hydromechanical conditions: the liquid in one region has only solid boundaries, while in the other region it has a solid and free boundary. New hydromechanical effects have been discovered. In particular, the presence of an effect has been established, consisting in the fact that against the background of oscillations the liquid in one region is at rest, while in the other region it performs a stationary motion.

1. Formulation of the problem

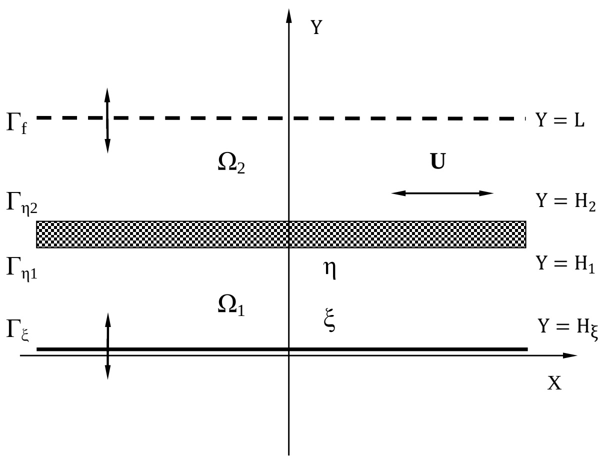

Fig 1. Hydromechanical system

There is a hydromechanical system consisting of an incompressible viscous liquid, a gas, an absolutely rigid plate η and an absolutely rigid wall ξ (Fig. 1). The liquid borders the gas, the plate and the wall. The plate boundary η is permeable to the liquid. The plate moves translationally relative to the inertial rectangular coordinate system X, Y, Z with velocity $\mathbf{U} = \{U_X, 0, 0\}$. Velocity U_X changes periodically with time t in a given manner, with a period T, ($U_X = \widetilde{U}\sin(2\pi t/T); \widetilde{U} > 0$ — constant). The wall ξ performs a given translational motion along the Y axis. The boundary Γ_ξ of the wall ξ is the plane $Y = H_\xi$; $-\infty < X < \infty$, $-\infty < Z < \infty$ ($H_\xi = \widetilde{H}\sin(2\pi t/T + \varphi)$; $\widetilde{H} > 0$, φ are constants). The boundary $\Gamma_{\eta 1}, \Gamma_{\eta 2}$ of the plate η is the planes $Y = H_1$, $Y = H_2$; $-\infty < X < \infty$, $-\infty < Z < \infty (H_2 > H_1, H_1 > \widetilde{H}$ are constants, the difference $H_2 - H_1$ is the plate thickness η). The free boundary Γ_f of the liquid is characterized by the relations $Y = L$; $-\infty < X < \infty$, $-\infty < Z < \infty$ ($L = \widehat{L} + H_\xi$; $\widehat{L} > H_2 + \widetilde{H}$ — constant). The regions $\Omega_1 : H_\xi < Y < H_1$ and $\Omega_2 : H_2 < Y < L$ ($-\infty < X < \infty$, $-\infty < Z < \infty$) are filled with liquid.

It is required to determine the periodic planar motion of a liquid.

Let $\tau = t/T$; $x = X/\widehat{L}$; $y = Y/\widehat{L}$; $z = Z/\widehat{L}$; $\varepsilon = \widetilde{H}/\widehat{L}$; $u = TU_X/\widehat{L} = \widetilde{u}\sin(2\pi\tau)$; $\mathbf{e}_x = \{1, 0, 0\}$; $\mathbf{e}_y = \{0, 1, 0\}$; ρ and $\mathbf{V}$ — the density and velocity of the liquid, respectively; $\mathbf{v} = T\mathbf{V}/\widehat{L} = v_x(\tau, y)\mathbf{e}_x + v_y(\tau, y)\mathbf{e}_y$; P — pressure in liquid; $p = T^2P/(\rho\widehat{L}^2) = p(\tau, y)$; P_g — gas pressure on liquid; $p_g = T^2P_g/(\rho\widehat{L}^2) = p_g(\tau)$; $h_\xi = H_\xi/\widehat{L} = \varepsilon\sin(2\pi\tau + \varphi)$; $h_1 = H_1/\widehat{L}$; $h_2 = H_2/\widehat{L}$; $Re = \widehat{L}^2/(\nu T)$ — Reynolds number.

The problem of liquid motion consists of the Navier–Stokes equation, the continuity equation and the conditions on the free and solid boundaries of the liquid:

$$\frac{\partial \mathbf{v}}{\partial \tau} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla p + \frac{1}{Re}\Delta\mathbf{v} \qquad \text{in} \quad \Omega_1,\ \Omega_2; \tag{1}$$

$$\nabla\cdot\mathbf{v} = 0 \qquad \text{in} \quad \Omega_1,\ \Omega_2; \tag{2}$$

$$v_y = \frac{dh_\xi}{d\tau}, \quad p - \frac{2}{Re}\frac{\partial v_y}{\partial y} = p_g, \quad \frac{\partial v_x}{\partial y} = 0 \qquad \text{on} \quad \Gamma_f; \tag{3}$$

$$v_x = 0, \quad v_y = \frac{dh_\xi}{d\tau} \qquad \text{on} \quad \Gamma_\xi; \tag{4}$$

$$v_x = u, \quad v_y = \frac{dh_\xi}{d\tau} \qquad \text{on} \quad \Gamma_{\eta 1},\ \Gamma_{\eta 2}. \tag{5}$$

2. The solution of the problem

According to (2)–(5) we have

$$v_y = 2\pi\varepsilon[\cos(2\pi\tau + \varphi)] \quad \text{in} \quad \Omega_1,\ \Omega_2. \tag{6}$$

From (1), (3), (6) it follows

$$\begin{aligned} &p = 4\pi^2\varepsilon[\sin(2\pi\tau + \varphi)]y + p' \quad \text{in} \quad \Omega_1, \\ &p = 4\pi^2\varepsilon[\sin(2\pi\tau + \varphi)(y - 1 - h_\xi) + p_g \quad \text{in} \quad \Omega_2, \end{aligned} \tag{7}$$

where p' is a function of τ.

Using (1), (3)–(6), we define the tasks

$$\frac{\partial v_x}{\partial \tau} + 2\pi\varepsilon[\cos(2\pi\tau + \varphi)]\frac{\partial v_x}{\partial y} = \frac{1}{Re}\frac{\partial^2 v_x}{\partial y^2} \quad \text{in} \quad \Omega_1, \tag{8}$$

$$v_x = 0 \quad \text{at} \quad y = h_\xi, \tag{9}$$

$$v_x = u \quad \text{at} \quad y = h_1 \tag{10}$$

и

$$\frac{\partial v_x}{\partial \tau} + 2\pi\varepsilon[\cos(2\pi\tau + \varphi)]\frac{\partial v_x}{\partial y} = \frac{1}{Re}\frac{\partial^2 v_x}{\partial y^2} \quad \text{in} \quad \Omega_2, \tag{11}$$

$$v_x = u \quad \text{at} \quad y = h_2, \tag{12}$$

$$\frac{\partial v_x}{\partial y} = 0 \quad \text{at} \quad y = 1 + h_\xi. \tag{13}$$

We will consider problems (8)–(10) and (11)–(13) for values of ε that are small compared to unity. We will apply the method of expansion in powers of the small parameter [40, 41]. We will assume that

$$v_x \sim v_0 + \varepsilon v_1 \quad \text{at} \quad \varepsilon \to 0. \tag{14}$$

Using (8)–(14), in the ε^N-approximation ($N = 0, 1$) we obtain

$$\frac{\partial v_N}{\partial \tau} + 2N\pi[\cos(2\pi\tau + \varphi)]\frac{\partial v_0}{\partial y} = \frac{1}{Re}\frac{\partial^2 v_N}{\partial y^2} \quad \text{in} \quad \bar{\Omega}_1, \tag{15}$$

$$v_N = -N[\sin(2\pi\tau + \varphi)]\frac{\partial v_0}{\partial y} \quad \text{at} \quad y = 0, \tag{16}$$

$$v_N = (1 - N)u \quad \text{at} \quad y = h_1, \tag{17}$$

$$\frac{\partial v_N}{\partial \tau} + 2N\pi[\cos(2\pi\tau + \varphi)]\frac{\partial v_0}{\partial y} = \frac{1}{Re}\frac{\partial^2 v_N}{\partial y^2} \quad \text{in} \quad \bar{\Omega}_2, \tag{18}$$

$$v_N = (1 - N)u \quad \text{at} \quad y = h_2, \tag{19}$$

$$\frac{\partial v_N}{\partial y} = -N[\sin(2\pi\tau + \varphi)]\frac{\partial^2 v_0}{\partial y^2} \quad \text{at} \quad y = 1, \tag{20}$$

where $\bar{\Omega}_1$ and $\bar{\Omega}_2$ are regions respectively $0 < y < h_1$ and $h_2 < y < 1$ ($-\infty < x < \infty, -\infty < z < \infty$).

Let $N = 0$. Problem (15)–(17) has a solution

$$v_0 = \widetilde{u}\ \mathrm{Imag}\left(\frac{\mathrm{sh}\ qy}{\mathrm{sh}\ qh_1}e^{2\pi i\tau}\right) \quad \text{for} \quad 0 \leqslant y \leqslant h_1, \tag{21}$$

problem (18)–(20) has a solution

$$v_0 = \widetilde{u}\ \mathrm{Imag}\left[\frac{\mathrm{ch}\ q(1-y)}{\mathrm{ch}\ q(1-h_2)}e^{2\pi i\tau}\right] \quad \text{for} \quad h_2 \leqslant y \leqslant 1, \tag{22}$$

where $q = (1 + i)\sqrt{\pi Re}$.

Let $N = 1$. We perform averaging (15)–(20) over dimensionless time τ. As a result, we obtain

$$2\pi\left\langle[\cos(2\pi\tau + \varphi)]\frac{\partial v_0}{\partial y}\right\rangle = \frac{1}{Re}\frac{d^2\bar{v}}{dy^2} \quad \text{in} \quad \bar{\Omega}_1, \tag{23}$$

$$\bar{v} = -\left\langle[\sin(2\pi\tau + \varphi)]\frac{\partial v_0}{\partial y}\right\rangle \quad \text{at} \quad y = 0, \tag{24}$$

$$\bar{v} = 0 \quad \text{at} \quad y = h_1, \tag{25}$$

$$2\pi\left\langle[\cos(2\pi\tau + \varphi)]\frac{\partial v_0}{\partial y}\right\rangle = \frac{1}{Re}\frac{d^2\bar{v}}{dy^2} \quad \text{in} \quad \bar{\Omega}_2, \tag{26}$$

$$\bar{v} = 0 \quad \text{at} \quad y = h_2, \tag{27}$$

$$\frac{d\bar{v}}{dy} = -\left\langle[\sin(2\pi\tau + \varphi)]\frac{\partial^2 v_0}{\partial y^2}\right\rangle \quad \text{at} \quad y = 1. \tag{28}$$

Here $\langle ... \rangle = \int_\tau^{\tau+1} ...\ d\tau'$; $\bar{v} = \langle v_1 \rangle$. Problem (15)–(17) has a solution

$$v_1 = \bar{v} + \mathrm{Real}[v^{(1)}e^{4\pi i\tau}] \quad \text{for} \quad 0 \leqslant y \leqslant h_1, \tag{29}$$

Problem (18)–(20) has a solution

$$v_1 = \bar{v} + \mathrm{Real}[v^{(2)} e^{4\pi i\tau}] \qquad \text{for} \quad h_2 \leqslant y \leqslant 1, \tag{30}$$

here $v^{(1)}$, $v^{(2)}$ — functions y.

From (21)–(28) it follows

$$\bar{v} = \sqrt{\frac{\pi}{2} Re}\ \widetilde{u}\ \mathrm{Real}\Big[\frac{(\mathrm{ch}\, q h_1) y - h_1 \,\mathrm{ch}\, q y}{h_1 \,\mathrm{sh}\, q h_1}\ e^{i(\frac{\pi}{4} - \varphi)}\Big] \qquad \text{for} \quad 0 \leqslant y \leqslant h_1, \tag{31}$$

$$\bar{v} = \sqrt{\frac{\pi}{2} Re}\ \widetilde{u}\ \mathrm{Real}\Big[\frac{\mathrm{sh}\, q(1-y) - \mathrm{sh}\, q(1-h_2)}{\mathrm{ch}\, q(1-h_2)}\ e^{i(\frac{\pi}{4} - \varphi)}\Big] \qquad \text{for} \quad h_2 \leqslant y \leqslant 1. \tag{32}$$

The formulas

$$v_x = v_0 + \varepsilon v_1 \tag{33}$$

и (6), (7), (22), (23), (29)–(32) define an approximate solution to the problem (1)–(5). This solution indicates the presence of an effect consisting in the fact that (against the background of oscillations) the liquid performs a stationary motion.

Let us consider the question of the average time flow of a liquid for values of Re that are small compared to unity. Using (6), (21), (22), (29)–(33), we obtain

$$\langle \mathbf{v} \rangle \ \sim \ -\ \frac{1}{2}\varepsilon \widetilde{u} (\cos\varphi) \frac{h_1 - y}{h_1^2}\ \mathbf{e}_x \quad \text{for} \quad 0 \leqslant y \leqslant h_1, \tag{34}$$

$$\langle \mathbf{v} \rangle \ \sim \ -\ \pi \varepsilon \widetilde{u} Re (\sin\varphi)(y - h_2)\ \mathbf{e}_x \quad \text{for} \quad h_2 \leqslant y \leqslant 1 \tag{35}$$

at $Re \to 0$.

According to (34), (35) (against the background of oscillations) the following takes place. In the region $\bar{\Omega}_1$ at $\cos\varphi > 0$ the liquid moves in the direction opposite to the direction of the X axis; at $\cos\varphi < 0$ the liquid moves in the direction coinciding with the direction of the X axis; at $\cos\varphi = 0$ the liquid is at rest. In the region $\bar{\Omega}_2$ at $\sin\varphi > 0$ the liquid moves in the direction opposite to the direction of the X axis; at $\sin\varphi < 0$ the liquid moves in the direction coinciding with the direction of the X axis; at $\sin\varphi = 0$ the liquid is at rest. When $(\sin\varphi)\cos\varphi > 0$ the liquid in the regions $\bar{\Omega}_1$, $\bar{\Omega}_2$ moves (along the X-axis) in the same directions; when $(\sin\varphi)\cos\varphi < 0$ the liquid in the regions $\bar{\Omega}_1$, $\bar{\Omega}_2$ moves (along the X-axis) in mutually opposite directions; when $(\sin\varphi)\cos\varphi = 0$ the liquid in one of the regions $\bar{\Omega}_1$, $\bar{\Omega}_2$ is at rest, and in the other it moves in the direction coinciding with the direction of the X-axis or in the direction opposite to the direction of the X-axis.

Using (34), (35), we find that for $(\sin\varphi)\cos\varphi \neq 0$ the relation

$$(\cos\varphi)\Big(\frac{1}{1-h_2}\int\limits_{h_2}^{1} \langle \mathbf{v} \rangle\, dy\Big) \ = \ 2\pi Re (\sin\varphi) h_1 (1-h_2) \Big(\frac{1}{h_1}\int\limits_{0}^{h_1} \langle \mathbf{v} \rangle\, dy\Big). \tag{36}$$

According to (36) for small values of Re (in (34), (35) $Re \to 0$) and any (admissible) values of $h_1, 1-h_2$ when the liquid moves in both regions $\bar{\Omega}_1$, $\bar{\Omega}_2$, the fluid in region $\bar{\Omega}_2$, on average, moves significantly slower than in region $\bar{\Omega}_1$.

Let us consider the question of the average time flow of liquid in the regions $\bar{\Omega}_1, \bar{\Omega}_2$ for values small compared to unity $h_1, 1-h_2$. Let $\sigma_1 = (h_1 - y)/h_1$ ($0 \leqslant \sigma_1 \leqslant 1$ at $0 \leqslant y \leqslant h_1$); $\sigma_2 =$

$(y-h_2)/(1-h_2)$ ($0 \leqslant \sigma_2 \leqslant 1$ at $h_2 \leqslant y \leqslant 1$). Using (6), (21), (22), (29)–(33), we obtain

$$\langle \mathbf{v} \rangle \;\sim\; -\frac{1}{2}\varepsilon\widetilde{u}(\cos\varphi)\frac{\sigma_1}{h_1}\,\mathbf{e}_x \quad \text{for} \quad 0 \leqslant y \leqslant h_1, \tag{37}$$

$$\langle \mathbf{v} \rangle \;\sim\; -\pi\varepsilon\widetilde{u}Re(\sin\varphi)\sigma_2(1-h_2)\,\mathbf{e}_x \quad \text{for} \quad h_2 \leqslant y \leqslant 1 \tag{38}$$

at $h_1 \to 0,\ 1-h_2 \to 0$ (and fixed $Re, \widetilde{u}, \varphi$).

The expressions on the right-hand sides of (37), (38) coincide with the expressions on the right-hand sides of (34), (35, respectively. Unlike formulas (34), (35), which are suitable for small $Re > 0$ and any (admissible) $h_1, 1-h_2$, formulas (37), (38) are suitable for small $h_1, 1-h_2$ and any (fixed) $Re > 0$.

From (37), (38) follows a relation coinciding with (36), according to which for small values of $h_1,\ 1-h_2$ (in (37), (38) $h_1 \to 0, 1-h_2 \to 0$) and any value of $Re > 0$, when the fluid moves in both regions $\bar{\Omega}_1,\ \bar{\Omega}_2$, the fluid in region $\bar{\Omega}_2$, on average, moves significantly slower than in region $\bar{\Omega}_1$.

Let us dwell on the question of the time-average force action from the liquid on the plate η in the direction of the X axis along which the plate η moves. Let $\Delta\eta$ be a body, a part of the plate η, at an (arbitrary) moment of time $t = t^*$ occupying the region $\Omega_{\Delta\eta} : X^* < X < X^* + D_X$, $H_1 < Y < H_2$, $Z^* < Z < Z^* + D_Z$ ($X^*, Z^*, D_X > 0, D_Z > 0$ are constants). Let us determine the time-average force F acting from the liquid on the body $\Delta\eta$ in the direction of the X axis. Using (6), (21), (22), (29)–(33), we find

$$F = \varepsilon\frac{\rho\widehat{L}^4}{Re\,T^2}\Big[-\Big(\frac{d\bar{v}}{dy}\Big)_{|y=h_1} + \Big(\frac{d\bar{v}}{dy}\Big)_{|y=h_2}\Big]s = -\sqrt{\frac{\pi}{2}}\frac{\varepsilon}{\sqrt{Re}}\frac{\rho\widehat{L}^4\widetilde{u}}{T^2 h_1}\mathrm{Real}\Big[(\mathrm{cth}\,qh_1)e^{i(\frac{\pi}{4}-\varphi)}\Big]s, \tag{39}$$

where $s = D_X D_Z / L^2$.

According to (39), the average time force action from the liquid on the plate η in the direction of the X axis does not depend on the “thickness” $1-h_2$ of the $\bar{\Omega}_2$ region.

From (39) it follows

$$f = \frac{FT^2}{\rho\widehat{L}^4} \sim -\sqrt{\frac{\pi}{2}}\frac{\varepsilon}{\sqrt{Re}}\frac{\widetilde{u}}{h_1}\cos\Big(\varphi - \frac{\pi}{4}\Big) \tag{40}$$

at $Re \to \infty$ (and fixed $h_1, \widetilde{u}, \varphi$).

Formula (40) demonstrates that for Reynolds numbers large compared to unity (in the approximation under consideration), for $\cos(\varphi - \pi/4) = 0$ the force F is zero, and there is no time-averaged force action in the direction of the X axis from the liquid on the plate η; for $\cos(\varphi - \pi/4) \neq 0$, with increasing Re the modulus of the force F decreases according to the law $Re^{-1/2}$.

Conclusion

The conducted research led to the discovery of new effects of unusual liquid motion under periodic time effects. The behavior of a viscous liquid caused by effects that do not have a distinguished direction in space is considered. From what is presented in the work it follows that such effects are capable of generating qualitative changes in the liquid motion. The reason for the discovered effects is the consistency (with each other) of the effects exerted on the liquid.

A hydromechanical system, subjected to periodic impacts in time that do not have a specific direction in space, produces responses (reactions to impacts) that are characterized by the presence of a specific direction in space and are expressed in the fact that the free parts of the system (parts of the system whose movement is not specified) — for example, liquid layers — perform an average movement against the background of oscillations.

This is in direct connection with the following generalized principle of average motion: the fundamental reason that periodic in time (oscillatory, vibrational) effects on a hydromechanical system that do not have a specific direction in space generate average in time motion of the free parts of the system is the possibility of the free parts of the system performing motion in different directions in space under different conditions (see also [26]).

The obtained results can be used in conducting targeted experimental studies of non-trivial dynamics of hydromechanical systems; in developing promising methods of controlling hydromechanical systems; in creating hydromechanical systems with prescribed properties, for example, systems that respond in a given way to periodic effects.

References

1. Sennitskii VL. Predominantly unidirectional motion of a gas bubbe in a vibrating liquid. Proceedings of the Academy of Sciences of the USSR. 1991;319(1):117–119 (in Russian).
2. Sennitskii VL. Predominantly unidirectional motion of a compressible solid body in a vibrating liquid. Journal of Applied Mechanics and Technical Physics. 1993;34(1):96–97. DOI: 10.1007/BF00851812.
3. Lyubimov DV. New approach in the vibrational convection theory. In: Proc. 14 IMACs Congresson Computational and Applied Mathematics. Atlanta, Georgia, USA: Georgia Institute of Technonogy; 1994. P. 59–68.
4. Lyubimov DV. Thermovibrational flows in nonuniform systems. Microgravity Quarterly. 1994;4(1):221–225.
5. Kozlov VG. Solid-body dynamics in cavity with liquid under high-frequency rotational vibration. Europhysics Letters. 1996;36(9):651–656. DOI: 10.1209/epl/i1996-00282-0.
6. Lyubimov DV, Lyubimova TP, Meradji S, Roux B. Vibrational control of crystal growth from liquid phase. Journal of Crystal Growth. 1997;180(3–4):648–659. DOI: 10.1016/S0022-0248(97)00294-7.
7. Ivanova AA, Kozlov VG, Evesque P. Dynamics of a cylindrical body in a liquid-filled sector of a cylindrical layer under rotational vibration. Fluid Dynamics. 1998;33(4):488–496. DOI: 10.1007/BF02698213.
8. Luybimov DV, Perminov AV, Cherepanov AA. Generation of mean flows in vibrational field near the interface. In: Vibration Effects in Hydrodynamics. Perm: Perm State University Publishing; 1998. P. 204–221 (in Russian).
9. Sennitskii VL. Motion of a pulsating rigid body in an oscillating viscous liquid. Journal of Applied Mechanics and Technical Physics. 2001;42(1):72–76. DOI: 10.1023/A:1018808628235.
10. Lyubimov DV, Lyubimova TP, Cherepanov AA. Dynamics of Interfaces in Vibrationalfields. Moscow: Fizmatlit; 2003. 216 p. (in Russian).
11. Ivanova AA, Kozlov VG, Kuzaev AF. Vibrational lift force acting on a body in a fluid near a solid surface. Doklady Physics. 2005;50(6):311–314. DOI: 10.1134/1.1958123.
12. Lyubimov D, Lyubimova T, Vorobev A, Mojtabi A, Zappoli B. Thermal vibrational convection in near-critical fluids. Part 1. Non-uniform heating. Journal of Fluid Mechanics. 2006;564:159–183. DOI: 10.1017/S0022112006001418.
13. Hassan S, Lyubimova TP, Lyubimov DV, Kawaji M. Motion of a sphere suspended in a vibrating liquid-filled container. J. Appl. Mech. 2006;73(1):72–78. DOI: 10.1115/1.1992516.

14. Lyubimov DV, Lyubimova TP, Shklyaev SV. Behavior of a drop on an oscillating solid plate. Phys. Fluids. 2006;18(1):012101. DOI: 10.1063/1.2137358.
15. Shevtsova V, Melnikov D, Legros JC, Yan Y, Saghir Z, Lyubimova T, Sedelnikov G, Roux B. Influence of vibrations on thermodiffusion in binary mixture: A benchmark of numerical solutions. Phys. Fluids. 2007;19(1):017111. DOI: 10.1063/1.2409622.
16. Ivanova AA, Kozlov VG, Kuzaev AF. Vibrational hydrodynamic interaction between a sphere and the boundaries of a cavity. Fluid Dynamics. 2008;43(2):194–202. DOI: 10.1134/S001546280802004X.
17. Sennitskii VL. Pulsating motion of an inhomogeneous solid sphere in a vibrating liquid. Journal of Applied Mechanics and Technical Physics. 2009;50(6):936–943. DOI: 10.1007/s10808-009-0127-6.
18. Ivanova AA, Kozlov VG, Shchipitsyn VD. A light cylinder under horizontal vibration in a cavity filled with a fluid. Fluid Dynamics. 2010;45(6):889–897. DOI: 10.1134/S0015462810060062.
19. Kozlov V, Ivanova A, Schipitsyn V, Stambouli M. Lift force acting on the cylinder in viscous liquid under vibration. Acta Astronautica. 2012;79:44–51. DOI: 10.1016/j.actaastro.2012.04.013.
20. Lyubimov DV, Baydin AY, Lyubimova TP. Particle dynamics in a fluid under high frequency vibrations of linear polarization. Microgravity Sci. Technol. 2013;25(2):121–126. DOI: 10.1007/s12217-012-9336-3.
21. Ivanova AA, Kozlov VG, Shchipitsyn VD. Lift force acting on a cylindrical body in a fluid near the boundary of a cavity performing translational vibrations. Journal of Applied Mechanics and Technical Physics. 2014;55(5):773–780. DOI: 10.1134/S002189441405006X.
22. Alabuzhev AA. Behavior of a cylindrical bubble under vibrations. Computational Continuum Mechanics. 2014;7(2):151–161 (in Russian). DOI: 10.7242/1999-6691/2014.7.2.16.
23. Sennitskii VL. On a prescribed orientation of a solid inclusion in a viscous liquid. Journal of Applied and Industrial Mathematics. 2015;18(1):123–128 (in Russian). DOI: 10.17377/SIBJIM.2015.18.11.
24. Kozlov V, Vlasova O. The repulsion of flat body from the wall of vibrating container filled with liquid. Microgravity Sci. Technol. 2015;27(4):297–303. DOI: 10.1007/s12217-015-9460-y.
25. Kozlov NV, Vlasova OA. Behavior of a heavy cylinder in a horizontal cylindrical liquid-filled cavity at modulated rotation. Fluid Dyn. Res. 2016;48(5):055503. DOI: 10.1088/0169-5983/48/5/055503.
26. Sennitskii VL. Paradoxical motion of a liquid. International Journal of Applied and Basic Research. 2017;(8–1):28–33 (in Russian). DOI: 10.17513/mjpfi.11753.
27. Vlasova OA, Kozlov VG, Kozlov NV. Lift force acting on a heavy solid in a rotating liquid-filled cavity with a time-varying rotation rate. Journal of Applied Mechanics and Technical Physics. 2018;59(2):219–228. DOI: 10.1134/S0021894418020050.
28. Konovalov BB, Lyubimova TP. Numerical study of the effect of vibrations on the interaction in an ensemble of gas bubbles and solid particles in a liquid. Computational Continuum Mechanics. 2019;12(1):48–56 (in Russian). DOI: 10.7242/1999-6691/2019.12.1.5.
29. Shchipitsyn VD. Vibrations of a nonaxisymmetric cylinder in a cavity filled with liquid and performing rotational oscillations. Tech. Phys. Lett. 2020;46(8):771–774. DOI: 10.1134/S1063785020080143.
30. Konovalov VV, Lyubimova TP. Influence of acoustic vibrations on the interaction of a gas bubble and a solid particle in a liquid. In: Lyubimova TP, editor. Perm Hydrodynamical Scientific Readings. Digest of Articles by the Materials of VIII All-Russian Conference

Dedicated for the Memory of Professors G. Z. Gershuny, E. M. Juhovitskii and D. V. Lyubimov. Perm: Perm State National Research University; 2022. P. 254–261 (in Russian).
31. Sennitskii VL. On peculiarities of a liquid flow in a gravity field. Siberian Electronic Mathematical Reports. 2022;19(1):241–247 (in Russian). DOI: 10.33048/semi.2022.19.018.
32. Chelomei VN. Paradoxes in mechanics caused by vibrations. Proceedings of the Academy of Sciences of the USSR. 1983;270(1):62–67 (in Russian).
33. Sennitskii VL. Motion of a circular cylinder in a vibrating liquid. Journal of Applied Mechanics and Technical Physics. 1985;26(5):620–623. DOI: 10.1007/BF00915307.
34. Sennitskii VL. Motion of a sphere in fluid caused by vibrations of another sphere. Journal of Applied Mechanics and Technical Physics. 1986;27(4):501–505. DOI: 10.1007/BF00910190.
35. Lugovtsov BA, Sennitskii VL. Motion of a body in a vibrating liquid. Proceedings of the Academy of Sciences of the USSR. 1986;289(2):314–317 (in Russian).
36. Lyubimov DV, Lyubimova TP, Cherepanov AA. On the motion of a solid body in a vibrating fluid. In: Convective Flows. Perm: Perm Pedagogical Institute Publishing; 1987. P. 61–71 (in Russian).
37. Chelomei VN. Selected Works. Moscow: Mashinostroenie; 1989. 336 p. (in Russian).
38. Sennitskii VL. Motion of a gas bubble in a viscous vibrating liquid. Journal of Applied Mechanics and Technical Physics. 1988;29(6):865–870. DOI: 10.1007/BF00858387.
39. Kapitsa PL. Pendulum with a vibrating suspension. Sov. Phys. Usp. 1951;44(1):7–20 (in Russian). DOI: 10.3367/UFNr.0044.195105b.0007.
40. Krylov NM, Bogoliubov NN. Introduction in Non-linear Mechanics. Moscow-Izhevsk: Research Center Regular and Chaotic Dynamics; 2004. 352 p. (in Russian).
41. Bogoliubov NN, Mitropolsky YA. Asymptotic Methods in the Theory of Non-linear Oscillations. New York: Gordon and Breach; 1961. 537 p.