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NONLINEAR OSCILLATION AND WAVES IN DYNAMICAL SYSTEMS

by P.S. Landa

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PREFACE

A rich variety of books devoted to dynamical chaos, solitons, self-organization made their appearance in recent years. These problems are considered independently of one another. Therefore, many of readers of these books do not suspect that the problems discussed there are divisions of great generalizing science - the theory of oscillations and waves. This science is not some branch of physics or mechanics. It is in its own right. It is meta-science in some sense. In this respect the theory of oscillations and waves is nearest to mathematics. In the book called attention to reader the present-day theory of nonlinear oscillations and waves is carried out. From unified point of view oscillatory and wave processes in the systems of diversified physical nature, both periodic and chaotic, are considered. The relation between the theory of oscillations and waves, nonlinear dynamics and synergetics is discussed. One of purposes of the book is to convince reader of the necessity of thorough study of the theory of oscillations and waves and to show that such popular branches of science as nonlinear dynamics, synergetics, the soliton theory, and so on, are, in fact, constituent parts of this theory.

Primary audience for the book: researchers having to do with oscillatory and wave processes; students and post-graduate students interested in deep study of general laws and applications of the theory of oscillations and waves.

INTRODUCTION

Purpose and subject-matter of the book. Subject of the theory of oscillations and waves. The history of creation and development of this theory.

1. Purpose and subject-matter of the book

Purpose of the book called attention to reader is to give a good indication of the present state of the theory of nonlinear oscillations and waves. A distinguishing feature of this book is unified approch to both oscillatory and wave phenomena as well as to both regular and chaotic processes in dynamical systems.

The book contains introduction, five parts, and two appendixes. In introduction we give the definition of theory of oscillations and waves and describe the subject of its investigation. The history of creation and development of this theory is briefly reviewed too.

The first part deals with notions of dynamical system and its phase space, energy, adiabatic invariants, integrability and so on. Classification of dynamical systems is conducted. Elements of the theory of near-integrable Hamiltonian systems are outlined. The definitions of natural (free), forced and self-oscillations and waves, chaotic and stochastic motions and corresponding attractors are given. The main quantitative characteristics of attractors are also described.

In the second part basic dynamical models studied in theory of oscillations and waves are described. For example, we have considered different models of nonlinear one-degree-of-freedom oscillators, including the «prey-predator» model by Lotka - Volterra; nonlinear chains by Toda and Fermi - Pasta - Ulam; some model equations for waves in dispersive media having solutions of soliton form; the Riemann and the Burgers equations describing wave processes in non-dispersive media; a set of models of self-oscillatory and autowave systems.

Natural oscillations and waves in the models of linear and nonlinear oscillators and chains, and soliton solutions of some model equations are considered in the third part. The notions of normally and anomalously dispersive waves are given and corresponding examples are adduced. Saw-tooth and shock waves in non-dispersive media, and solitary waves for the Burgers equation are considered. Elements of the theory of waves in slightly inhomogeneous, slightly nonstationary and periodically stratified media are also presented.

The fourth part of the book is devoted to consideration of oscillations and waves caused by external actions both forced and parametric. Much attention is given to different resonance problems. Nonlinear phenomena occurring in half-bounded chains and continuous media with harmonic input actions, such as excitation of the second harmonic and decay instability, are analyzed. The formation of saw-tooth and shock waves in nonlinear non-dispersive media and the change in form of harmonic wave in the process of its propagation in nonlinear slightly dispersive medium described by the Korteveg - de Vries equation are considered. Behavior of nonlinear bundles in non-dispersive and dispersive media described by the cubic Schrodinger equation and the Khokhlov - Zabolotzkaya equation is considered too.

Finally, the fifth part of the book, which is the most extensive, is devoted to oscillations and waves in active systems and, in particular, to self-oscillations and autowaves. Forced one-dimensional waves in active (non-equilibrium) media and a possibility of development of so-called burst instability are described. The notion of waves with negative energy is given. A view on turbulence in nonclosed flows as on amplification of fluctuations is discussed. Different mechanisms of self-excitation of oscillations and waves and of limitation of their magnitudes are analyzed. Classification of self-oscillatory and autowave systems is carried out. The energetic criterion for stochastization of self-oscillations is set forth. A great body of examples of self-oscillatory and autowave systems of diversified physical nature, both known and unknown, are adduced and studied. Influence of periodic actions on different self-oscillatory and autowave systems is investigated. In particular, the problems of asynchronous depression and excitation of self-oscillations, of synchronization and chaotization are considered. Interaction between self-oscillatory systems, both periodic and chaotic, is studied; in so doing the emphasis is on synchronization and chaotization problems.

Some mathematical methods used for investigation of oscillatory and wave systems are given in appendixes.

The book concludes with a comprehensive bibliography.

2. Definition and significance of the theory of oscillations and waves. Subject of its investigations. The history of creation and development of this theory. The relation between the theory of oscillations and waves and the synergetics problems

Theory of oscillations and waves is science studying oscillatory and wave motions irrespective of their physical nature. By oscillatory motions any limited changes of body state taking place in a long time interval are meant. Due to the limitation these changes must necessarily be «hither and thither» [Mandelshtam, 1955].

By wave motions oscillatory motions propagating in the space are meant. Such definition of the theory of oscillations and waves is very common. We know that other sciences study spatial-temporal changes of body state too. How does the theory of oscillations and waves differ from their? An answer to this question has been given by L.Mandelshtam [Mandelshtam, 1955]. Contrary to other sciences for which the prime interest is in what takes place with a body in a given space point and at a given moment, the theory of oscillations and waves concerns «general character of a process taken as a whole over a long interval of time».

Basing on knowledge of general laws of oscillatory and wave motions we can profitably predict different phenomena from diversified areas of science. Discovery of light combination scattering by L.Mandelshtam [Mandelshtam, 1947 (1,2)]* is a typical example of such prediction. As for analogy between light combination scattering and usual objects of the oscillation theory,

* In the west literature this effect is usually called the Raman effect.

Mandelshtam has written following [Mandelshtam, 1972]: «From the point of view of theory of oscillations, wireless telephony and light combination scattering are the same. It is modulation. Sound - in radio, atomic oscillations - in combination scattering.» Thus, the availability of analogy between oscillatory and wave systems of diversified physical nature is the basis for prediction. Regarding such analogies, Mandelshtam told students in one of his lectures on the theory of oscillations: «All of you know such systems as a pendulum and an oscillatory circuit, and also know that from oscillatory point of view it is the same. Now all this is trivial, but it is wonderful that this is trivial». These ideas up till now have not become fashioned. In the paper «L.I. Mandelshtam and theory of nonlinear oscillations» A.A. Andronov [Andronov, 1956] has written about lectures and seminars by Mandelshtam: «Lectures and seminars by Mandelshtam have sometimes contained new scientific results which were not published. But, may be, the most significance of these lectures was in methodical inculcating habits of {it oscillatory thinking}, in general rise of {it oscillatory culture}». Unfortunately, many of even prominent scientists, studying concrete problems, are still lacking in «oscillatory culture». For example, if chemistry scientists had got such culture in due time, they would have not argued against principal possibility of oscillatory chemical reactions in homogeneous media, and fortune of B.P.Belousov discovered experimentally such reactions in 1951 [Belousov, 1959] would have been alternate. Furthermore, up till now scientific works, which are absolutely erroneous from the point of view of the oscillation theory, are occurred on occasion. Had their authors general «oscillatory culture», these works could have not appeared. It has historically arisen that the theory of oscillations was strongly attracting to radio engineering and drew from it basic models and objects under investigation. By this fact, the universality of the oscillation theory laws and the necessity to study these laws by specialists in different branches of science were not realized immediately, if at all.

The availability of analogies between oscillatory and wave processes in the systems of diversified physical nature is the reason why the theory of oscillations and waves has got its {it subject} for investigation and thereby it took shape of original science. {it Dynamical system} is such a subject [Neimark, 1988,1992]. Dynamical system is the system whose behavior is predetermined by a set of rules (algorithm). In particular, and it is most often, behavior of dynamical system is described by differential, integral or finite-difference equations. Obviously, dynamical system is a model of a real system. So, we can say that the theory of oscillations and waves studies abstract models, but not concrete systems. Basic models of the theory of oscillations and waves will be described in the Part II.

Contrary to physics, where dynamical models of investigated phenomena have been long worked out and studied, in other sciences this, as a rule, has not taken place. Investigations were concrete and had mainly pure descriptive character. The situation has essentially changed in the last tens of years only. Models have began to be worked out and investigated in chemistry, biology, ecology, meteorology, economics and even medicine. True, exceptions took place in the past too. So, in 1920 A.Lotka proposed a model of hypothetical chemical reaction with oscillations of reacting substances [Lotka, 1920,1925]. Analogous model was later suggested by V.Volterra for explanation of oscillations of numbers of competing species of animals and plants [Volterra, 1931]. At a later time this model has come to be known as «the prey - predator model». In 1928 a dynamical model of heart was proposed by B. van der Pol and M. van der Mark [Van der Pol, 1928]. This model consisted in three coupled relaxation generators. Using this model, authors demonstrated some known heart diseases, such as arhythmia, and even attempted to predict unknown diseases. However, similar models were very few in number and, as a rule, they remained a mystery for the general circle of investigators.

By analyzing different models from diversified areas of science we can detect that these models have much in common. Therefore, these models may be classified by one or other indication; and in so doing we can separate the most typical ones for each class. Such classification, being of considerable importance in the presentation of the theory of oscillations and waves, will be carried out in the Chapter 1.

How and when was created such generalizing science as the oscillation theory? Apparently, it goes back to G.Lagrange works in the field of analytic mechanics published in 1788. By introducing generalized coordinates and momenta Lagrange has in effect digressed from traditional mechanics. The equations derived by him can be applied to systems of any nature. The investigation of properties of solutions of these equations makes possible to obtain general oscillatory and wave laws. It is not accidental that many of fundamental ideas of the present-day theory of oscillations and waves are expounded through the use of the Lagrange equations (or of their counterpart, the Hamilton equations).

The more important step on the road to creation and development of theory of oscillations and waves is associated with famous treatise by Rayleigh (J.W. Strutt) «The Theory of Sound» published in 1877 [Rayleigh, 1945]. In this treatise Rayleigh has first called attention to analogy between acoustic and electrical oscillations. Although the calculations of investigated phenomena in the Rayleigh treatise were largely based on the linear theory, the elements of nonlinear theory, in particular of the self-oscillation theory, were embedded in this book. For example, the equation describing the general laws of self-oscillatory processes was derived there. At the present time this equation is known as the Rayleigh equation. The majority of problems formulated in the Rayleigh treatise were solved in more recent times. Among these are the investigations of self-oscillations of the Froude pendulum [Strelkov, 1933], of thermo-acoustic self-oscillations of the Helmholtz resonator [Teodorchik, 1952], and many others.

Creation of the present-day nonlinear oscillation theory is associated with investigations of H.Poincare [Poincare, 1886,1899], G.Birkhoff [Birkhoff, 1927] and A.M.Lyapunov [Lyapunov, 1950, 1954-1956] laying the mathematical foundations for this theory. True, application of their mathematical methods to the oscillation theory as such has occurred well later, primarily owing to works of A.A. Andronov [Andronov, 1956,1959].

A great contribution to development of nonlinear oscillation theory, especially of applied part of this theory, was made by van der Pol [Van der Pol, 1920, 1926, 1960], studying the operation of electronic generator and proposing his own investigative techniques, viz., the techniques of slightly varying amplitudes.

The next quite considerable step to development of oscillation theory and to transformation of this theory to the branch of science in its own right is associated with works of L.Mandelshtam and his disciples A.Andronov, A.Vitt, G.Gorelik, N.Papalexi, S.Khaikin, S.Strelkov, S.Rytov and other. Mandelshtam was the first scientist who as early as 1930 delivered lectures on the theory of oscillations and waves [Mandelshtam, 1955]. These lectures were given in Moscow University. They, as well as succeeding monograph «Oscillations and Waves» by G.Gorelik [Gorelik, 1959], up till now are examples of a unified approach to oscillatory and wave phenomena.

Almost independently of Mandelshtam, Andronov and other physicists mathematical groundworks for the nonlinear oscillation theory were laid by N. Krylov, N. Bogolyubov, Yu. Mitropolsky and their disciples [Krylov, 1937; Bogolyubov, 1950, 1961; Mitropolsky, 1955,1971,1988]. They worked out the most important methods for analysis slightly nonlinear oscillations: asymptotic method, the methods of averaging and of equivalent linearization. These methods have received further development through works of N.Moiseev [Moiseev, 1981], V.Volosov and B.Morgunov [Volosov, 1971], A.Nayfeh [Nayfeh, 1981], A.Vasilyeva [Vasilyeva, 1973,1990], E.Mischenko [Mischenko, 1975], R.O'Malley [O'Malley, 1974], W. Eckhaus [Eckhaus, 1979], J.Sanders and F.Verhulst [Sanders, 1985] and other.

It was one of the most important Andronov's achievements that he was the first to perceive interrelation between Poincar'e's limit cycles and periodic oscillations of electronic generator studied by van der Pol. Such oscillations were called by Andronov {it self-oscillations}. Injection into oscillation theory by Andronov of the notion of self-oscillations initiated a great current of papers. Authors of these papers let know of detection of self-oscillations in concrete systems. Still more papers were initiated by subsequent discovery of the fact that self-oscillatory processes can be not only periodic, but chaotic as well. A great contribution to this discovery was made by one from Andronov's disciple, Yu.I.Neimark, in his works on homoclinic structure theory [Neimark, 1958, 1972 (1), 1976 (1,2), 1987]. The discovery of chaotic self-oscillations has received unprecedented attention of specialists in the very different branches of science.

In addition to this discovery, widespread attention to the problems of the theory of nonlinear oscillations was caused by the discovery of somewhat opposed trend in the evolution of dynamical systems, viz., trend to ordering, to self-organization**

Owing to this discovery new science by the name «synergetics» arises [Haken, 1978, 1983; Prigogine, 1980; Polak, 1983, Loskutov, 1990]. Although synergetics appears without visible association with the theory of oscillations and waves, subject of its study and general methods of investigation have been just adopted from this theory.

An essential effect on development of theory of oscillations and waves has been exerted by the discovery of special type of waves known as {it solitons}. Experimentally soliton was detected even in the 19th century by J.Scott Russell describing his observations and experiments in [Russell, 1844]. The first equation having solutions of soliton form was derived by D.Korteveg and G. de Vries in 1895 [Korteveg, 1895]. However, the theoretical comprehension of the soliton discovery and the elaboration of mathematical technique for calculation of solitons were happened rather not long ago, in 60s of this century [Zabusky, 1965; Gardner, 1967; Kruskal, 1970]. Today, an extensive literature has evolved which concerns the soliton theory (see, for example, [Whitham, 1974; Zakharov, 1980; Ablowitz, 1981; Eilenberger, 1981; Dodd, 1982; Infeld, 1990]). Un-

** It must be mentioned that the transition of system to chaotic regime, according to Yu.Klimontovich's hypothesis [Klimontovich, 1989,1990], is just associated with ordering of motion in the system. But this hypothesis is not generally recognized.

fortunately, up till now many investigators did not have a clear understanding of how the soliton theory relates to general theory of oscillations and waves, and therefore confusion with using of soliton notion often arises. The term «autosolitons» suggested recently by B.Kerner and V.Osipov [Kerner, 1991, 1994] may help to eliminate this confusion to a degree.

CONTENTS

Вышел в свет сборник трудов Л.А. Вайнштейна

Издательством «Радио и связь» выпущен сборник трудов члена-корреспондента АН СССР, профессора **Льва Альбертовича Вайнштейна.** В книге объемом 600 страниц, изданной под редакцией члена-корреспондента АН СССР С.М. Рытова, объединены оригинальные статьи Л.А.Вайнштейна по теории дифракции, электронике СВЧ и отдельным вопросам радиофизики, в том числе, не вошедшие в его монографии. Эти работы не потеряли актуальности и в настоящее время и могут быть полезны как для студентов, изучающих физику, так и для исследователей, активно работающих в данных областях. В сборник вошли также стихи и переводы Льва Альбертовича, воспоминания его друзей и учеников.

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Клееву А.И.

E-mail: kleev@magnit.msk.su
Факс: (095) 9382030