



## DYNAMICAL INSTABILITIES IN A CO<sub>2</sub> LASER WITH AN EXTERNALLY MODULATED OPTICAL FEEDBACK

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CO<sub>2</sub> lasers are very sensitive to optical feedback and can be driven into a large variety of dynamical states by an external modulation of the feedback parameters. We report on the experimental results concerning instabilities of CO<sub>2</sub> lasers induced by either an additional passive resonator of periodically varying geometrical length or an refractive index modulation in the external cavity by an electro-optical modulator. Experimental results show the presence of stable periodic orbits of different periods, bistability of periodic orbits and chaotic attractors with complex transition scenarios.

### 1. Introduction and background

The transition from regular to irregular behavior in nonlinear dynamical systems has been a topic of both experimental and theoretical interest for over two decades. In the context of laser systems one is interested in the practical problem of the control of the onset of chaos and in decrease of dynamical noises in the laser output intensity. To study the chaos onset mechanisms a number of laser configurations in which chaos is induced in the output have been studied, e.g., a CO<sub>2</sub> laser with internal harmonic modulation of the loss parameter [1-6], CO<sub>2</sub> laser with harmonic modulation of the pumping current [7,8], a single CO<sub>2</sub> lasers with delayed opto-electronic feedback [9-11], and two or more optically coupled lasers [8,12,13].

High-power CO<sub>2</sub> lasers are very sensitive to radiation feedback. Unintentional reflections from the target could be particularly damaging and can destabilize the source, i.e., drive it into a chaotic regime of operation. Therefore it is important to understand how optical feedback induces «instabilities». In this work the extreme sensitivity of CO<sub>2</sub> lasers to optical feedback is employed to induce dynamical instabilities in a CO<sub>2</sub> laser. Modulation of the optical feedback is provided by either geometrical length modulation of an additional passive resonant cavity or by refractive index modulation (and, of course, by optical length modulation) provided by electro-optical modulator placed in an external cavity\*.

\* Впервые режим перехода от регулярных колебаний выходной мощности излучения CO<sub>2</sub> лазера к хаотическому через последовательность удвоения периода модуляции добротности в пассивной части трехзеркального резонатора в диапазоне частот релаксационных колебаний был исследован в работе Г.Г. Акчурина «Экспериментальное исследование перехода в хаос в CO<sub>2</sub> лазере при модуляции добротности и в He-Ne лазере (0.63, 1.15, 3.39 мкм) в автономном режиме». Тезисы докладов XII Международной конференции по когерентной и нелинейной оптике. Москва, 1985. С. 304-305 (примечание Редактора вступка).

Both of the above methods provide the opportunity to investigate the instabilities induced by external optical feedback and complete the picture of dynamical behavior of CO<sub>2</sub> lasers with modulating parameters.

## 2. Experimental procedure

The scheme of the experimental setup is shown in Fig.1. In this scheme an axial flowing gas CO<sub>2</sub> laser operating at a wavelength of  $\lambda = 10.6 \mu\text{m}$  was used. This laser has the resonant cavity 2.5 m long, the diameter of the discharge tube (2) is 6 mm, and the pressure of the conventional gas mixture varies from 15 to 20 mm Hg. An additional mirror (5) provides the radiation feedback into the active region, and together with the laser mirror (3) forms an external passive resonator. Mirror (5) is mounted on a piezoelectric ceramic drive (6) allowing static tuning and/or periodic modulation of the external resonator length.

The length of the external passive resonator does not exceed 30 cm. The laser intensity is measured by a HgCdTe detector (7) monitoring part of the radiation reflected from the diffraction grating (1). The intensity signal from the detector and its time-delayed copy are fed into an oscilloscope (10) operating in the  $x$ - $y$  mode. The delay line (8) has a constant delay of 2.5  $\mu\text{s}$ . This simple scheme allows real time observations of the two-dimensional projections of the system trajectories in the space of delayed signals. A spectrum analyzer (11) of bandwidth 20 Hz - 40 MHz simultaneously displays the power spectra. The electronics used allows one to «freeze» at any instant the phase portrait and power spectra, and digitize the signal using a digitizer (9) with variable sampling frequencies and store the data.

In the first series of experiments, discussed in Section 3.A, the system was modulated by placing an electro-optical modulator (EOM) with a quarter wave voltage of 2.2 kV in the external resonator. The EOM was a Cd-Te crystal having an active length of 48 mm, and an aperture diameter of 3 mm. The voltage applied to the EOM had a DC component of 1.5 kV and the amplitude and frequency of the modulated signal varied within 100-900 V and 50-85 kHz, respectively. In the second series of experiments, discussed in Section 3.B, the intensity and the phase of the feedback radiation were modulated by the induced oscillations of the feedback providing mirror (5) mounted on a piezoelectric drive. The driving frequencies varied from 30 kHz to 60 kHz. The frequency range of these drives is centered on the relaxation frequencies of the considered laser system. The largest amplitude of the external mirror oscillations was of the order of 1  $\mu\text{m}$  for the lowest driving frequencies and estimated as not exceeding 0.5  $\mu\text{m}$  for the highest driving fre-

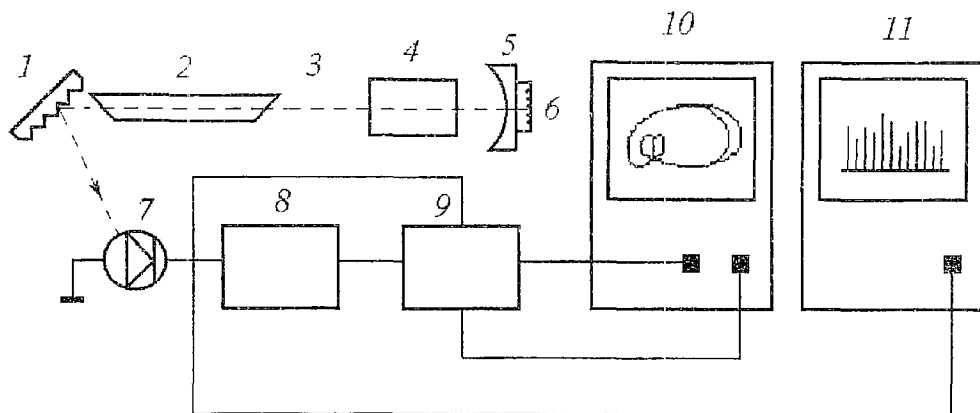


Fig. 1. Experimental setup for obtaining chaos in a CO<sub>2</sub> laser: 1 - diffraction grating; 2 - CO<sub>2</sub> laser tube; 3 - output mirror; 4 - electro-optical modulator (EOM); 5 - additional mirror; 6 - piezo-ceramic drive; 7 - liquid nitrogen cooled HgCdTe detector; 8 - delay line; 9 - digitizer; 10 - oscilloscope; 11 - spectrum analyzer

quencies used. In both series of experiments the external mirror (5) had a low reflectivity on the order of several percent. In both series of experiments the amount of optical feedback radiation was measured to be less than 10%.

### 3. Experimental results

A. *An EOM in the external cavity.* The results presented in Fig. 2 were obtained

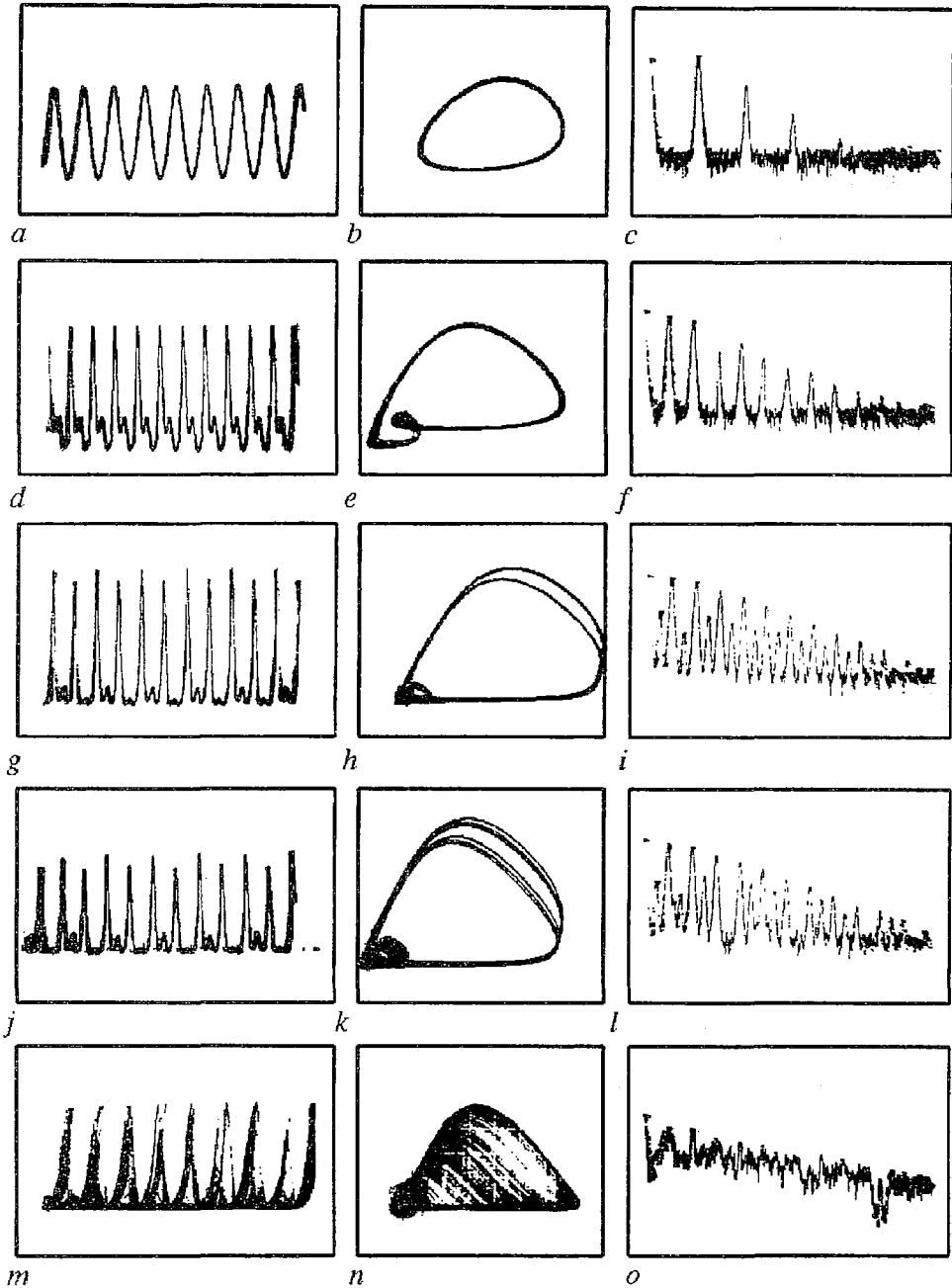


Fig. 2. A representative sequence of laser states for a CO<sub>2</sub> laser obtained by varying the static part of the loss coefficient while all other laser parameters were unchanged. Left column: time courses of the intensity signal; middle column: projection of the phase trajectories onto  $(I(t), I(t+\tau))$  plane in the space of delayed signals; right column: the frequency spectra

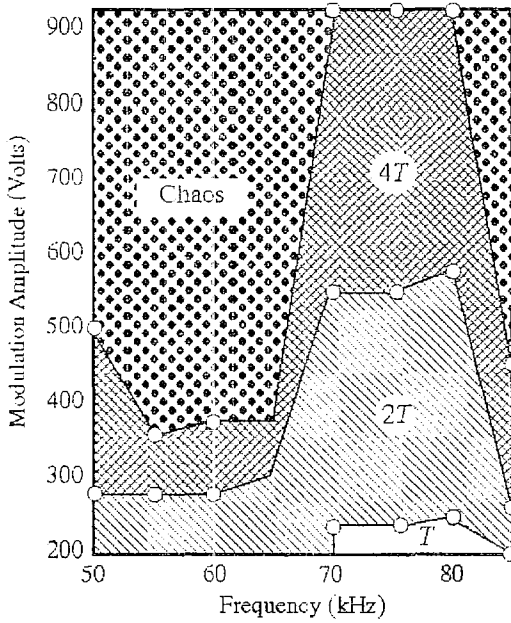


Fig. 3. An exemplary approximate state diagram in the (driving frequency, modulation amplitude) parameter space for a CO<sub>2</sub> laser with a driven electro-optical modulator placed in the external resonator and with a fixed static part of the intensity loss coefficient

by varying the static part of the loss coefficient while all other laser parameters remained unchanged. The static part of the loss coefficient was varied by changing the angle of the output coupler of the laser. In the right column of Fig. 2 the frequency spectra are displayed, the middle column shows the respective phase portraits (Intensity( $t$ ) vs Intensity( $t+\tau$ )) and finally the left column contains the corresponding time series. These spectra were recorded at a constant frequency of 83 kHz and a modulation amplitude of 150 V. For relatively low levels of losses the laser intensity follows the driving signal with a pronounced second and third harmonic present (Fig. 2, c). As the losses are increased, the laser output becomes more and more structured and period 2 oscillations are observed as shown in Figs 2, d, e, f. Subsequently, period 4 is obtained for increasing losses as shown in Figs 2, g, h, i. Similar results were obtained for period 8 Figs 2, j, k, l. Finally the frequency spectrum of the laser output changes into a broad spectrum with a rise in the floor level. This is a definitive indication that the system is driven into chaos, as shown in Figs 2, m, n, o. The chaotic attractor (Fig. 2, n) has a characteristic shape

typical for chaotic states obtained at other driving frequencies. The bifurcation diagram presented in Fig. 3 was obtained by varying the amplitude of ac voltage driving the EOM at a given frequency. The sequence of period doubling bifurcation obtained changing modulated losses by means of the EOM was identical to that obtained by varying the static part of the loss coefficient as described in the paragraph above. In this case modulation amplitude was varied while keeping all other parameters of the laser constant. The results obtained by this method are presented in Fig. 3 in the form of a complete bifurcation diagram. The bifurcation diagram clearly indicates that there is a broad range of frequencies in which the laser system could be driven into instabilities and chaos.

The coexistence of two stable periodic orbits is shown in Fig. 4. Transitions between two orbits of different periods are shown in Figs 4, a, b. The corresponding frequency spectrum (Fig. 4, b) contains fundamental frequencies of both orbits. Similar bi-

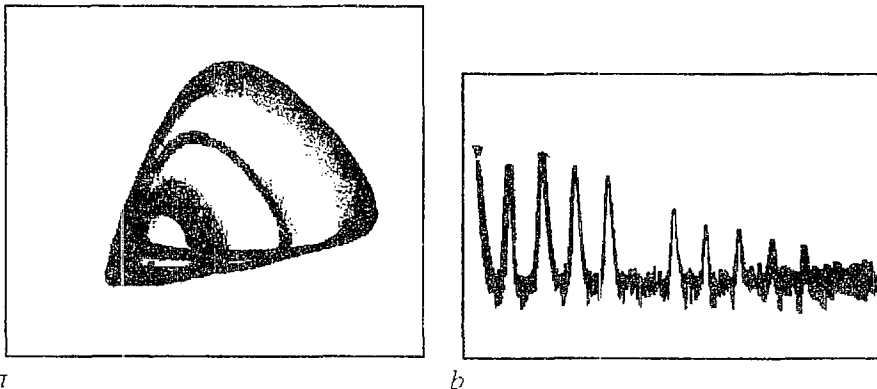


Fig. 4. Bistability in CO<sub>2</sub> laser with externally modulated optical feedback with the help of EOM in external cavity: a - phase portrait, b - spectrum, driving frequency  $f_d = 84$  kHz

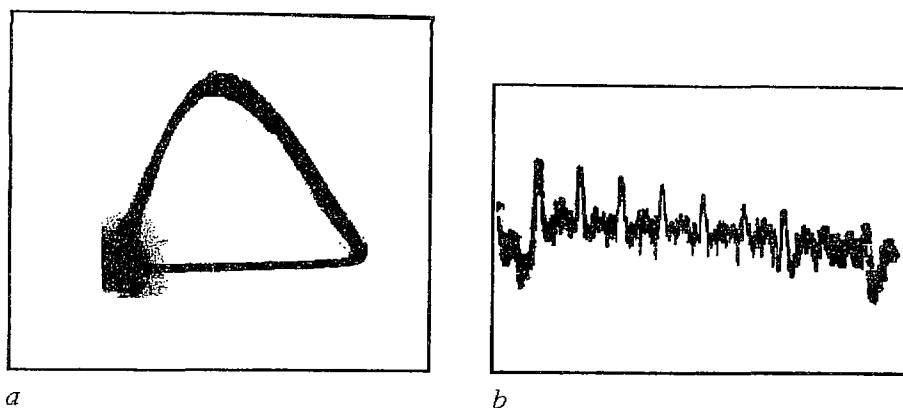


Fig. 5. «Hard transitions» to chaos. A direct transition to chaos without any bifurcation is observed

stabilities were reported previously [1,3] with transitions between orbits of period's 3 and 4.

«Hard transitions» to chaotic orbits also occur in our system (Fig. 5). In this case there is no period doubling cascade, and a direct transition from period 1 oscillations to chaos is observed.

**B. Vibrating mirror in the external cavity.** Instabilities and chaos induced by a vibrating mirror in the external cavity are presented in Fig. 6. The period doubling sequence, bifurcations, and attractor shapes obtained in this series of tests are very similar to those presented in Section 3 A for the EOM placed in the external cavity. Bistable periodic regime are also possible here. Fig.7 shows the coexistence of two period 1 orbits of different amplitudes and the same frequency. This is evidenced by the frequency spectrum shown in Fig. 7, *b* with fundamental frequency and higher harmonics.

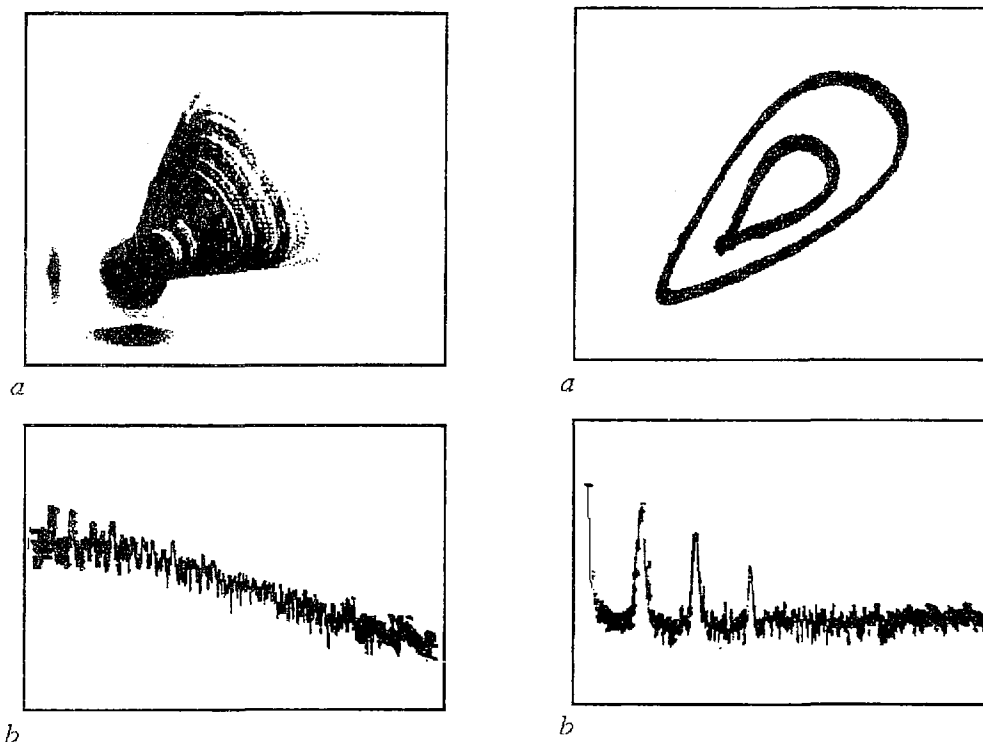


Fig. 6. Chaos induced by a vibrating mirror in the external cavity,  $f_d = 30$  kHz.

Fig. 7. Bistability in  $\text{CO}_2$  laser with vibrating mirror in external cavity: *a* - phase portrait, *b* - spectrum,  $f_d = 60$  kHz

## Summary and conclusions

We showed that a CO<sub>2</sub> laser can be driven into chaotic states by one more, technically very simple, method where one modulates externally the feedback of laser radiation.

Two approaches were used to drive the laser. In the first setup an EOM is placed in an external (passive) resonator, and the second one modulating the length of the passive external resonator. Both of these methods produced instabilities and chaos through period doubling cascades for a broad range of modulation frequencies. Under certain conditions the system also exhibits bistability and hard transitions to chaos.

The results of this work are very similar to those obtained of a CO<sub>2</sub> laser with modulated intercavity losses [1-6]. This is not coincidental, since in the framework of the simplest one-mode theory one can show the equivalence of these two system in the approximation replacing the external cavity with modulated feedback by an effective mirror with modulated transmission / reflection coefficient [14,15].

We would like to add that the simplicity of the external modulation scheme allows straight forward implementation [15] of the chaos control techniques [16,17].

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## ДИНАМИЧЕСКИЕ НЕСТАБИЛЬНОСТИ В $\text{CO}_2$ ЛАЗЕРЕ С ВНЕШНЕЙ МОДУЛЯЦИЕЙ ОПТИЧЕСКОЙ ОБРАТНОЙ СВЯЗИ

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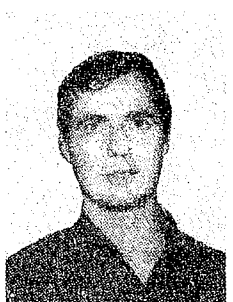
$\text{CO}_2$  лазеры очень чувствительны к оптической обратной связи и отличаются большим разнообразием динамических состояний в условиях внешней модуляции параметров обратной связи. Мы сообщаем о результатах экспериментов, касающихся неустойчивостей  $\text{CO}_2$  лазеров, вызванных либо периодическим изменением геометрической длины дополнительного внешнего пустого резонатора, либо модуляцией коэффициента преломления внешнего резонатора при помощи электрооптического модулятора. Результаты экспериментов демонстрируют наличие устойчивых периодических траекторий с различными периодами, бистабильности периодических траекторий и наличие хаотических аттракторов, характеризующихся сложными сценариями перехода к хаосу.



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